BOARD NOTES

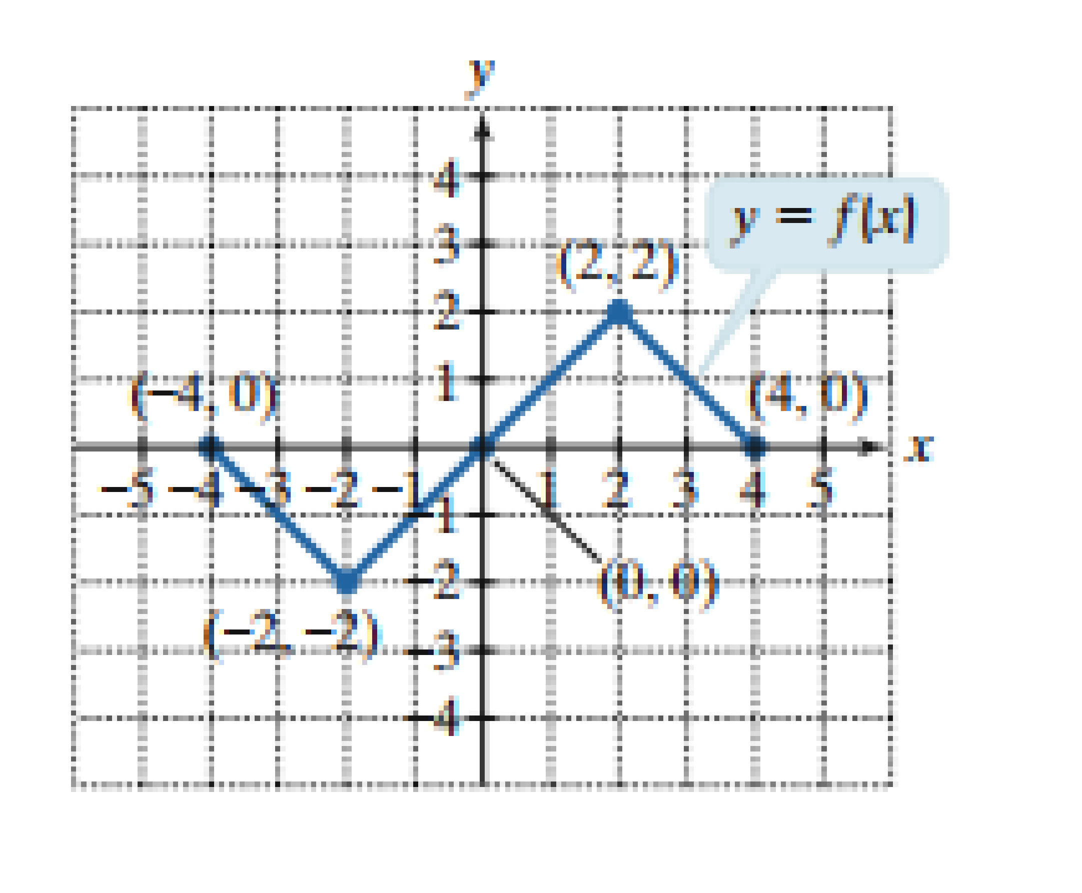
25 SEPTEMBER 2018

CC ALGEBRA
CHAPTER 2 FUNCTIONS & GRAPHS

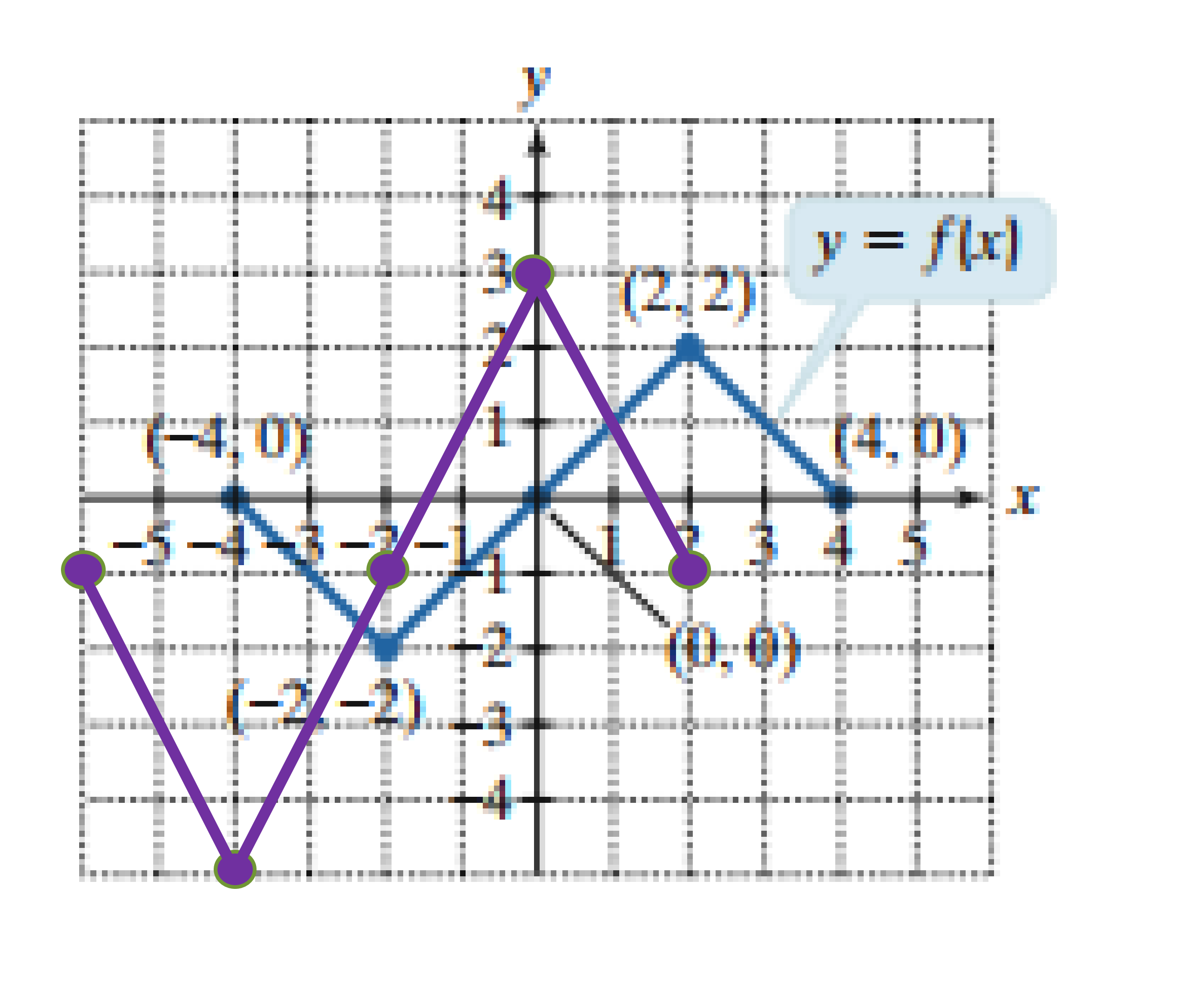
- SECTION 2.6 - COMBINATIONS OF FUNCTIONS; COMPOSITE FUNCTIONS

Objectives:

- Find the domain of a function
- Combine functions using the algebra of functions, specifying the domains
- Form composite functions
- Determine domain for composite functions
- Write functions as compositions



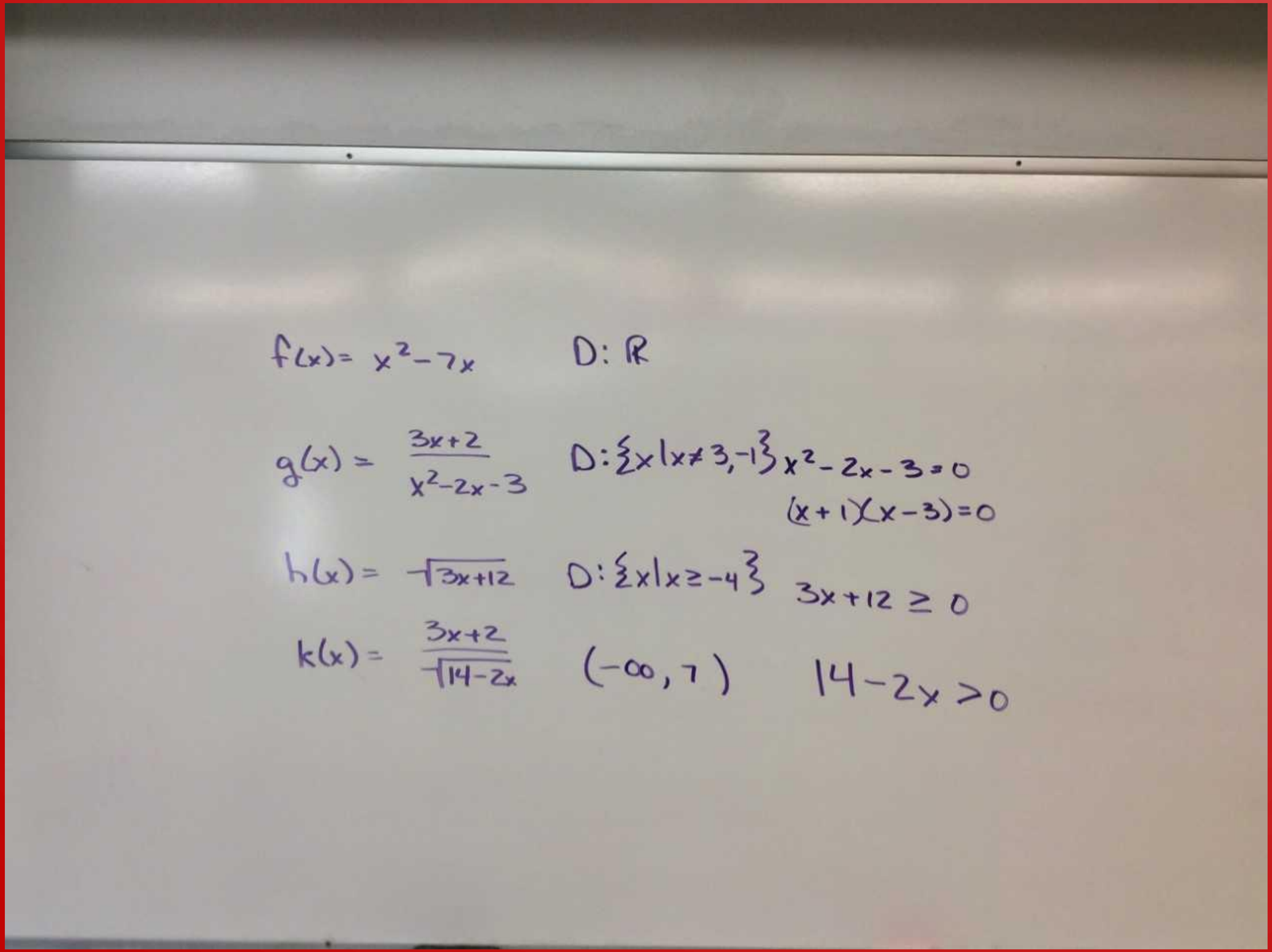
Find $g(x) = 2f(x+2) - 1$



$$g(x) = 2f(x+2) - 1$$

Finding a Function's Domain

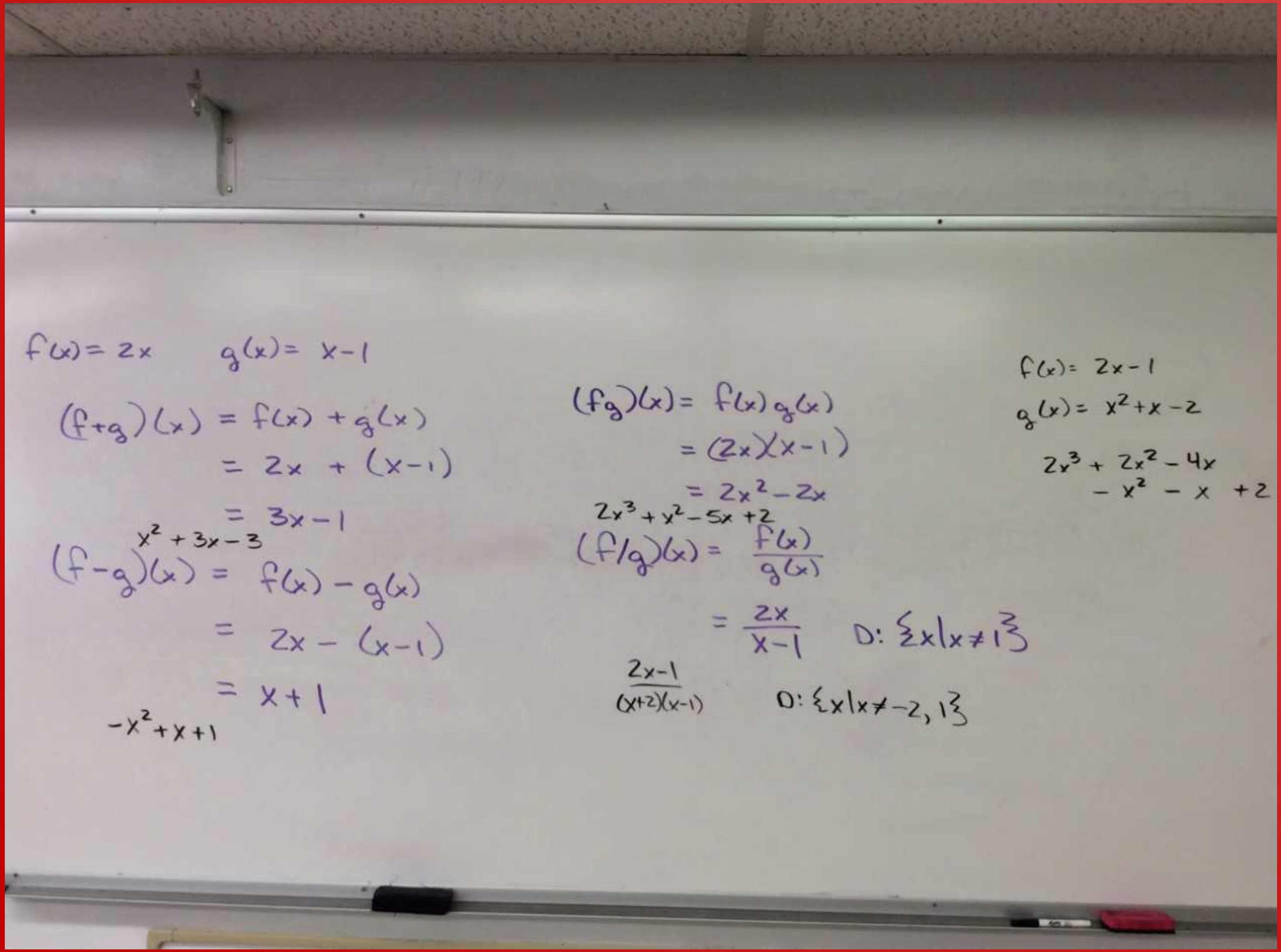
If a function f does not model data or verbal conditions, its domain is the largest set of real numbers for which the value of $f(x)$ is a real number. Exclude from a function's domain real numbers that cause division by zero and real numbers that result in an even root, such as a square root, of a negative number.

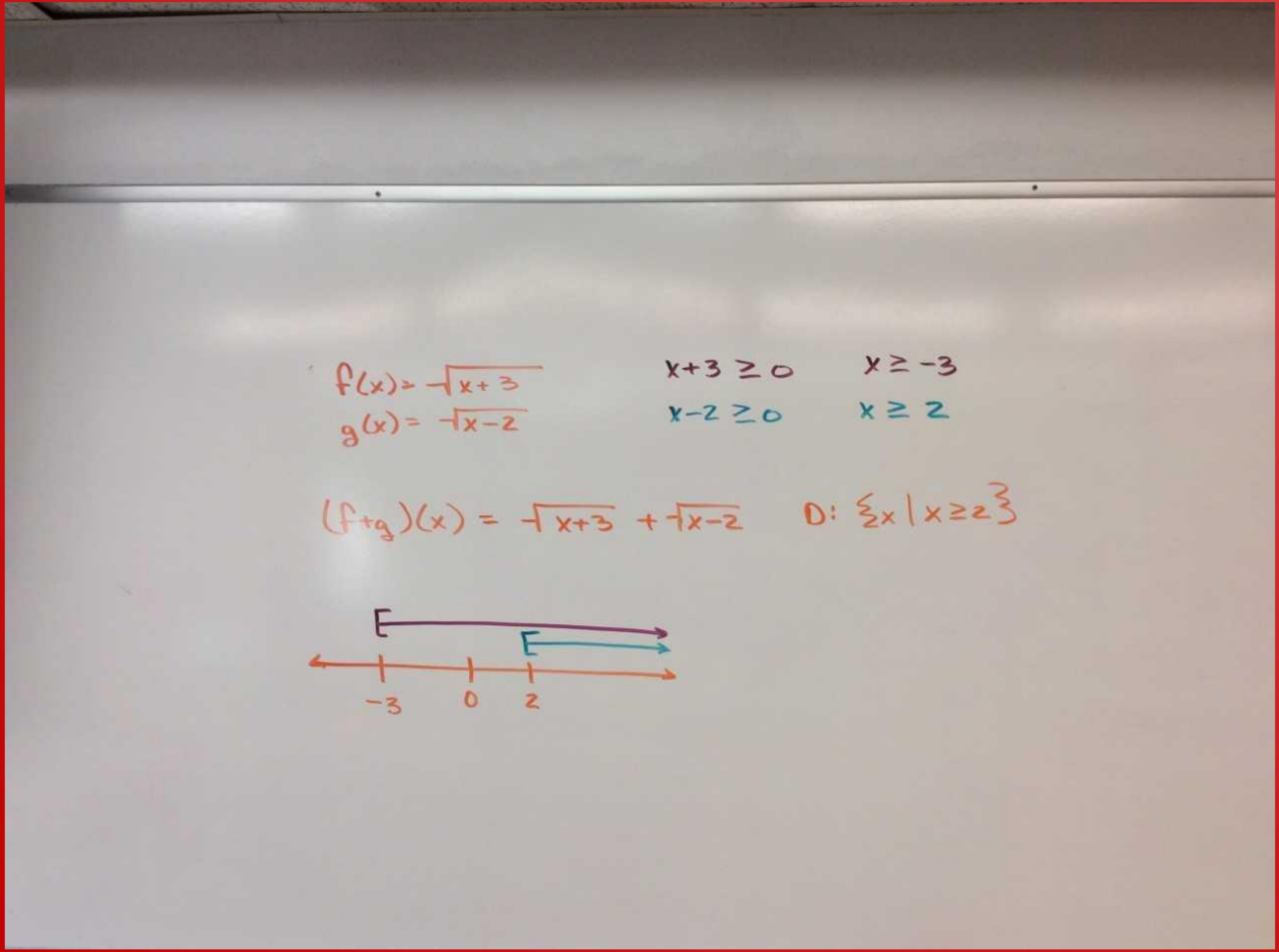

$f(x) = x^2 - 7x$ D: $\mathbb{R}$
$g(x) = \frac{3x+2}{x^2-2x-3}$ D: $\{x \mid x \neq 3, -1\}$ $x^2 - 2x - 3 = 0$
$(x+1)(x-3) = 0$
$h(x) = \sqrt{3x+12}$ D: $\{x \mid x \geq -4\}$ $3x + 12 \geq 0$
$k(x) = \frac{3x+2}{\sqrt{14-2x}}$ $(-\infty, 7)$ $14 - 2x > 0$

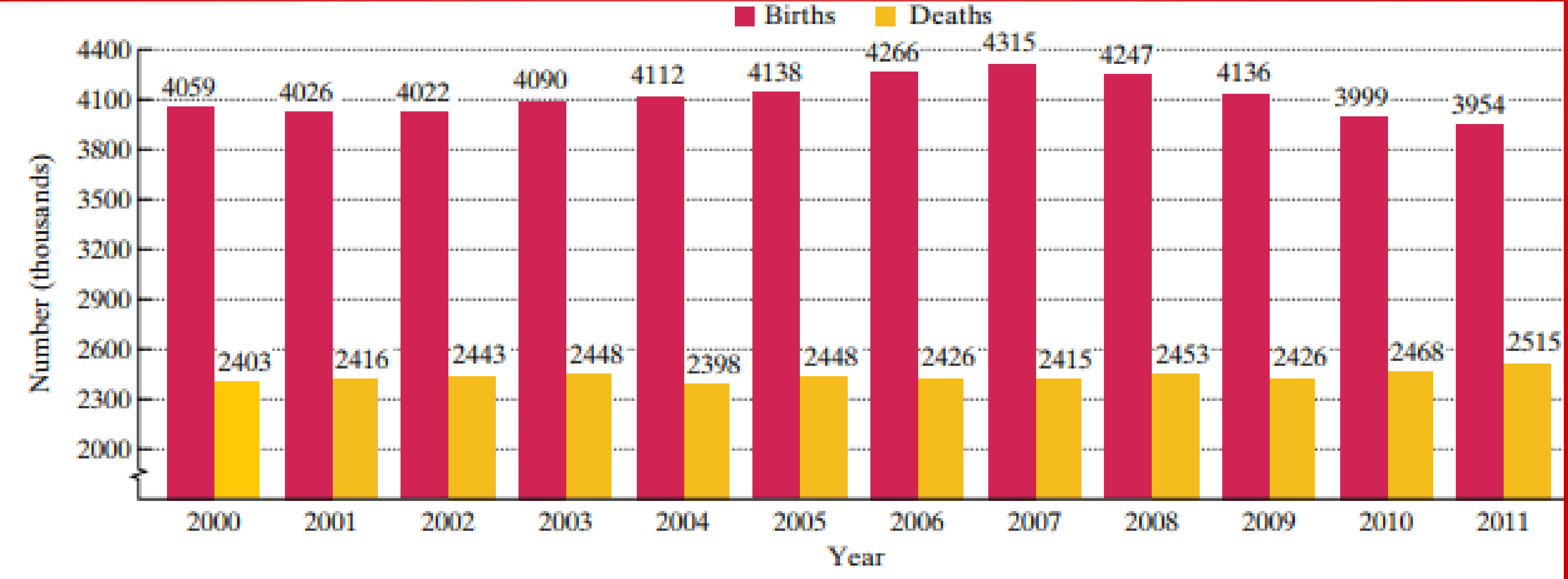
The Algebra of Functions: Sum, Difference, Product, and Quotient of Functions

Let f and g be two functions. The **sum** $f + g$, the **difference** $f - g$, the **product** fg, and the **quotient** $\frac{f}{g}$ are functions whose domains are the set of all real numbers common to the domains of f and g $(D_f \cap D_g)$, defined as follows:

1. Sum: $(f + g)(x) = f(x) + g(x)$
2. Difference: $(f - g)(x) = f(x) - g(x)$
3. Product: $(fg)(x) = f(x) \cdot g(x)$
4. Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0$.


f(x) = 2x g(x) = x − 1
(f+g)(x) = f(x) + g(x)
= 2x + (x−1)
= 3x − 1
x² + 3x − 3
(f−g)(x) = f(x) − g(x)
= 2x − (x−1)
= x + 1
−x² + x + 1
(fg)(x) = f(x)g(x)
= (2x)(x−1)
= 2x² − 2x
2x³ + x² − 5x + 2
(f/g)(x) = f(x)/g(x)
= 2x/(x−1) D: {x | x ≠ 1}
(2x−1)/((x+2)(x−1)) D: {x | x ≠ −2, 1}
f(x) = 2x − 1
g(x) = x² + x − 2
2x³ + 2x² − 4x
− x² − x + 2


f(x) = √x+3
g(x) = √x−2
x+3 ≥ 0
x−2 ≥ 0
x ≥ −3
x ≥ 2
(f+g)(x) = √x+3 + √x−2
D: {x | x ≥ 2}
−3
0
2



Births
Deaths
4400
4100
3800
3500
3200
2900
2600
2300
2000
Number (thousands)
4059
4026
4022
4090
4112
4138
4266
4315
4247
4136
3999
3954
2403
2416
2443
2448
2398
2448
2426
2415
2453
2426
2468
2515
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
Year

FIGURE 2.54

The Composition of Functions

The **composition of the function f with g** is denoted by $f \circ g$ and is defined by the equation

$$(f \circ g)(x) = f(g(x)).$$

The **domain of the composite function $f \circ g$** is the set of all x such that

1. x is in the domain of g and
2. $g(x)$ is in the domain of f.

Excluding Values from the Domain of $(f \circ g)(x) = f(g(x))$

The following values must be excluded from the input x:

- If x is not in the domain of g, it must not be in the domain of $f \circ g$.
- Any x for which $g(x)$ is not in the domain of f must not be in the domain of $f \circ g$.