BOARD NOTES

25 SEPTEMBER 2018

CC ALGEBRA
CHAPTER 2 FUNCTIONS & GRAPHS

- SECTION 2.6 - COMBINATIONS OF FUNCTIONS; COMPOSITE FUNCTIONS

Objectives:

- Find the domain of a function
- Combine functions using the algebra of functions, specifying the domains
- Form composite functions
- Determine domain for composite functions
- Write functions as compositions

Finding a Function's Domain

If a function f does not model data or verbal conditions, its domain is the largest set of real numbers for which the value of $f(x)$ is a real number. Exclude from a function's domain real numbers that cause division by zero and real numbers that result in an even root, such as a square root, of a negative number.

The Algebra of Functions: Sum, Difference, Product, and Quotient of Functions

Let f and g be two functions. The **sum** $f + g$, the **difference** $f - g$, the **product** fg, and the **quotient** $\frac{f}{g}$ are functions whose domains are the set of all real numbers common to the domains of f and g $(D_f \cap D_g)$, defined as follows:

1. **Sum:** $(f + g)(x) = f(x) + g(x)$
2. **Difference:** $(f - g)(x) = f(x) - g(x)$
3. **Product:** $(fg)(x) = f(x) \cdot g(x)$
4. **Quotient:** $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0$.

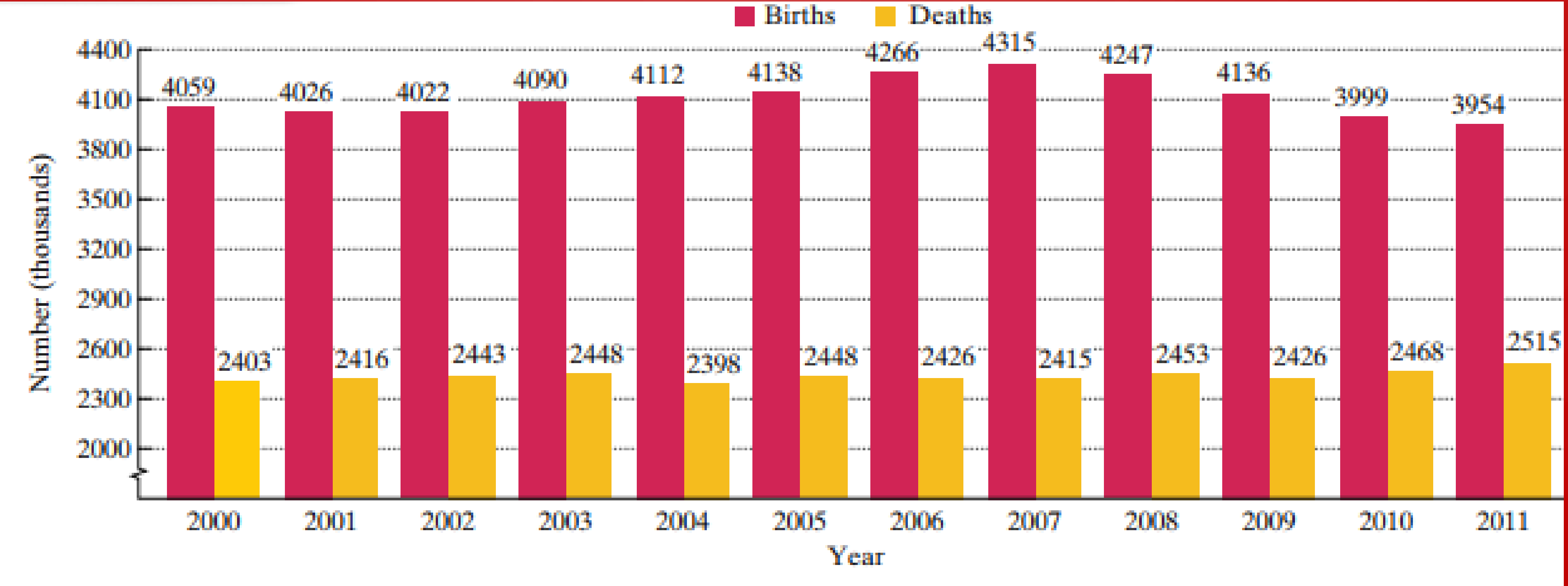

Births
Deaths
4400
4100
3800
3500
3200
2900
2600
2300
2000
Number (thousands)
4059
2403
4026
2416
4022
2443
4090
2448
4112
2398
4138
2448
4266
2426
4315
2415
4247
2453
4136
2426
3999
2468
3954
2515
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
Year

FIGURE 2.54

The Composition of Functions

The **composition of the function f with g** is denoted by $f \circ g$ and is defined by the equation

$$(f \circ g)(x) = f(g(x)).$$

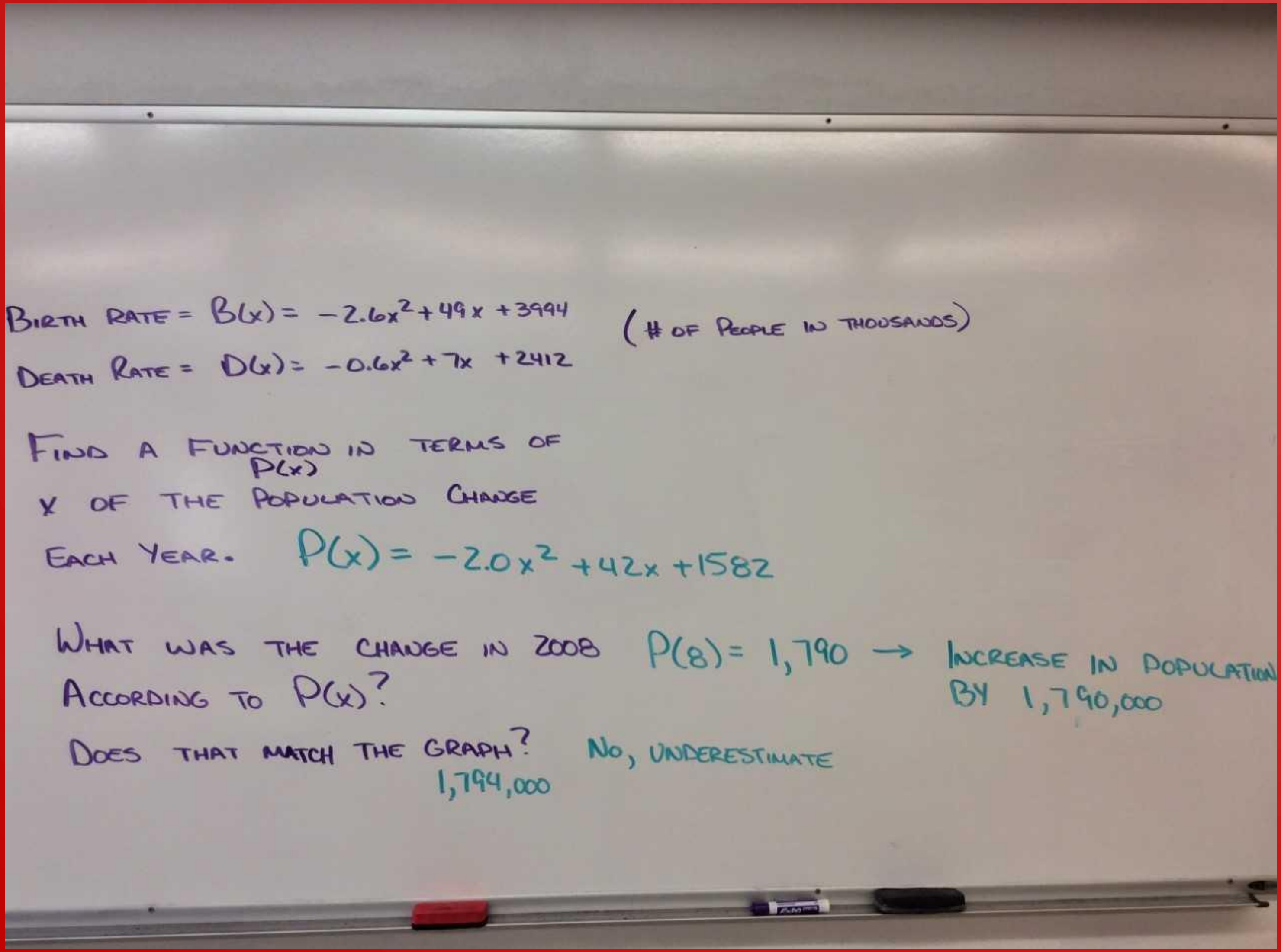
The **domain of the composite function $f \circ g$** is the set of all x such that

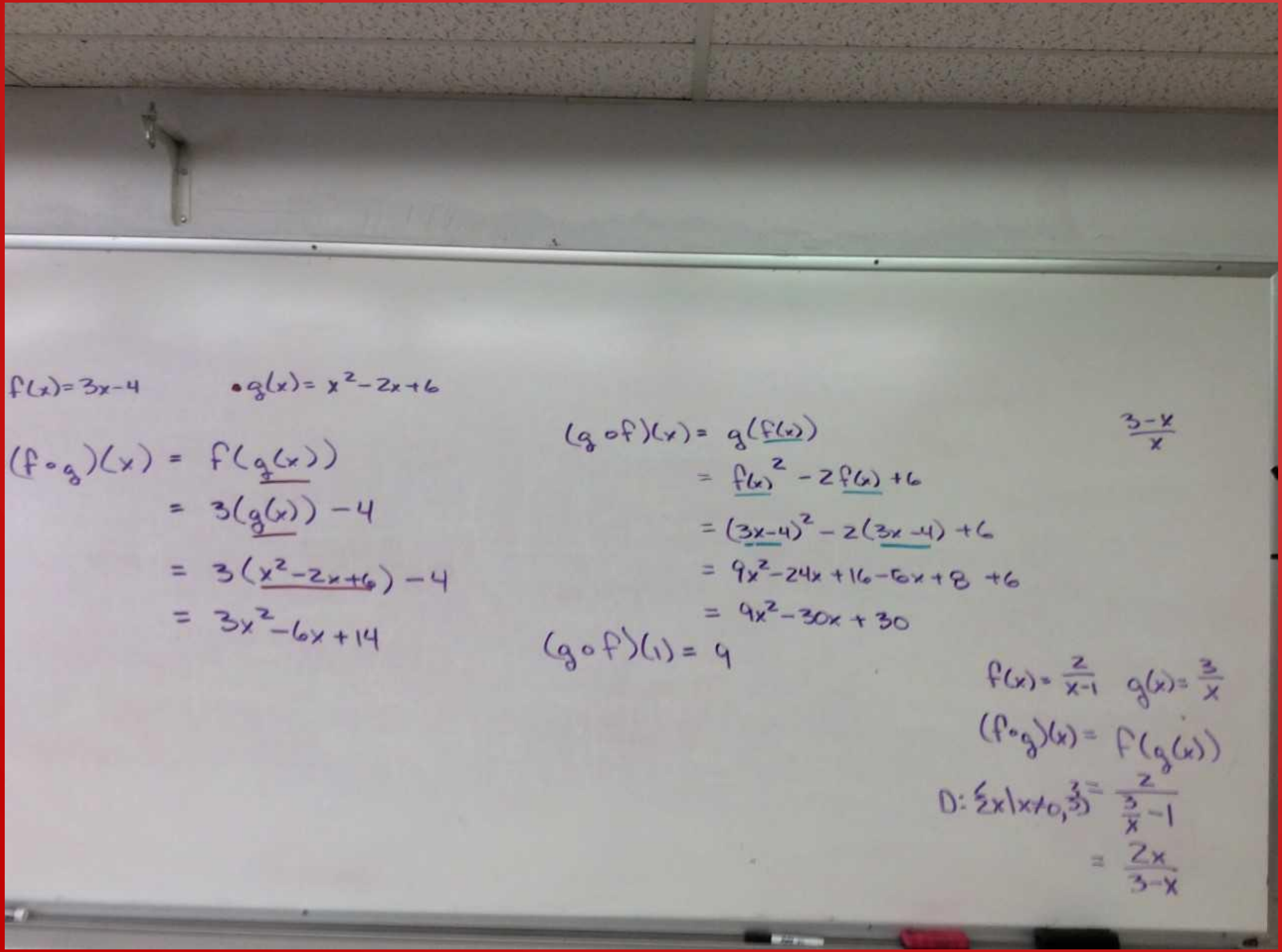
1. x is in the domain of g and
2. $g(x)$ is in the domain of f.

Excluding Values from the Domain of $(f \circ g)(x) = f(g(x))$

The following values must be excluded from the input x:

- If x is not in the domain of g, it must not be in the domain of $f \circ g$.
- Any x for which $g(x)$ is not in the domain of f must not be in the domain of $f \circ g$.


BIRTH RATE = B(x) = −2.6x² + 49x + 3994
(# OF PEOPLE IN THOUSANDS)
DEATH RATE = D(x) = −0.6x² + 7x + 2412
FIND A FUNCTION IN TERMS OF x OF THE POPULATION CHANGE EACH YEAR.
P(x)
P(x) = −2.0x² + 42x + 1582
WHAT WAS THE CHANGE IN 2008 ACCORDING TO P(x)?
P(8) = 1,790 → INCREASE IN POPULATION BY 1,790,000
DOES THAT MATCH THE GRAPH?
1,794,000
NO, UNDERESTIMATE



FRANK PHILLIPS COLLEGE Plainsmen

BORGER INDEPENDENT SCHOOL DISTRICT 1926

CC ALGEBRA
CHAPTER 2 FUNCTIONS & GRAPHS

- SECTION 2.7 - INVERSE FUNCTIONS

Objectives:

- Verify the inverse of functions
- Find the inverse of a function
- Use the horizontal line test to determine if a function has an inverse
- Graph the original and the inverse of a function

Definition of the Inverse of a Function

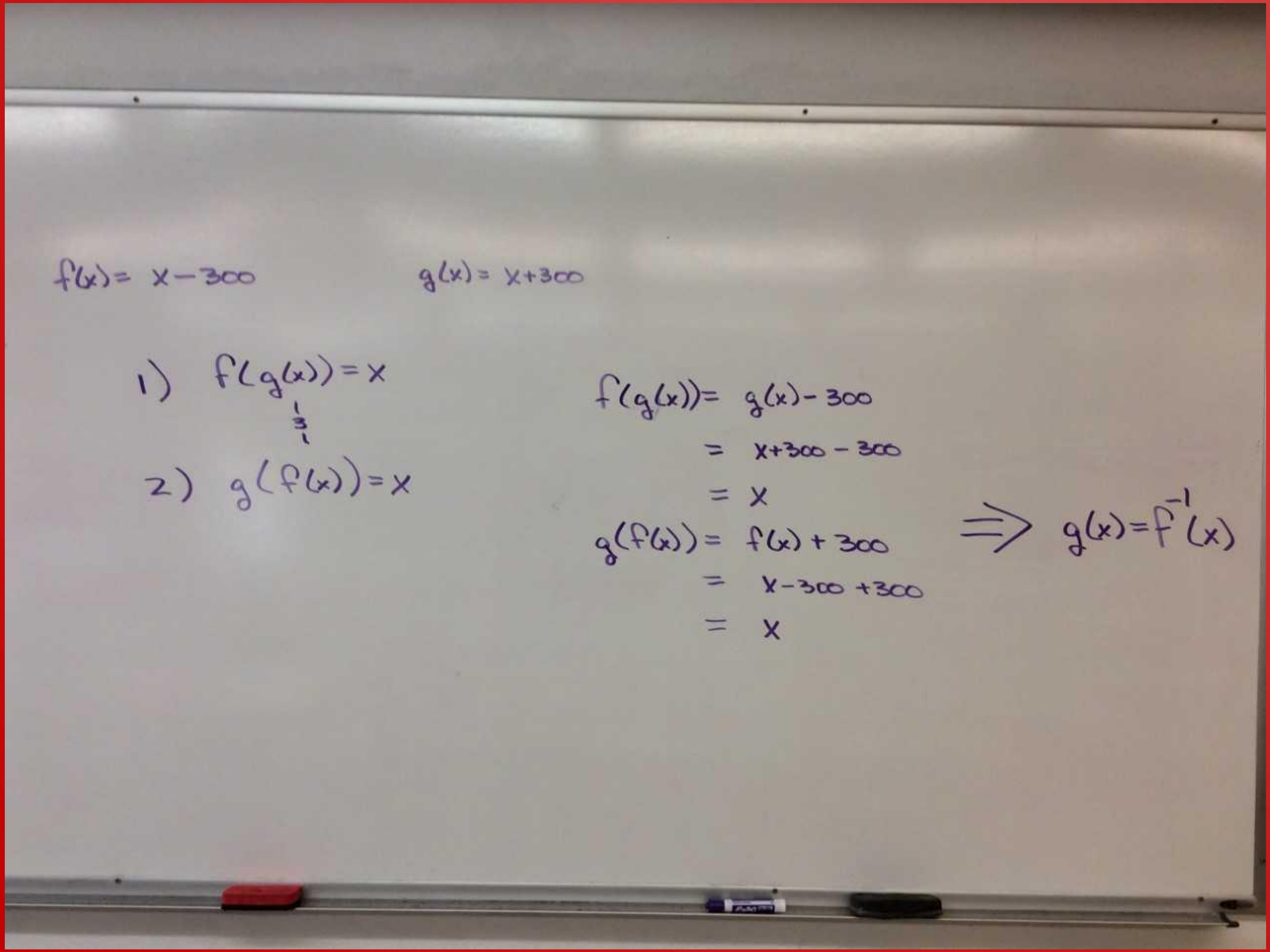
Let f and g be two functions such that

$$f(g(x)) = x \qquad \text{for every } x \text{ in the domain of } g$$

and

$$g(f(x)) = x \qquad \text{for every } x \text{ in the domain of } f.$$

The function g is the **inverse of the function** f and is denoted by f^{-1} (read "f-inverse"). Thus, $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$. The domain of f is equal to the range of f^{-1}, and vice versa.



f(x)= x−300
g(x)= x+300
1) f(g(x))=x
2) g(f(x))=x
f(g(x))= g(x)-300
= x+300 - 300
= x
g(f(x))= f(x) + 300
= x-300 +300
= x
⇒ g(x)=f⁻¹(x)

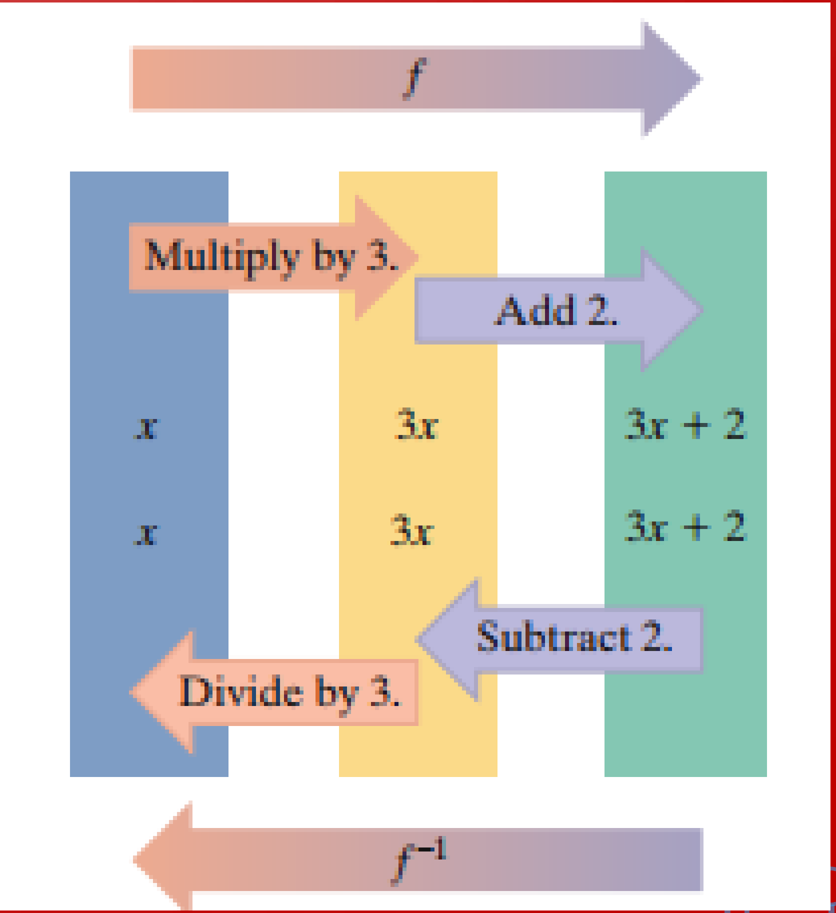
Finding the Inverse of a Function

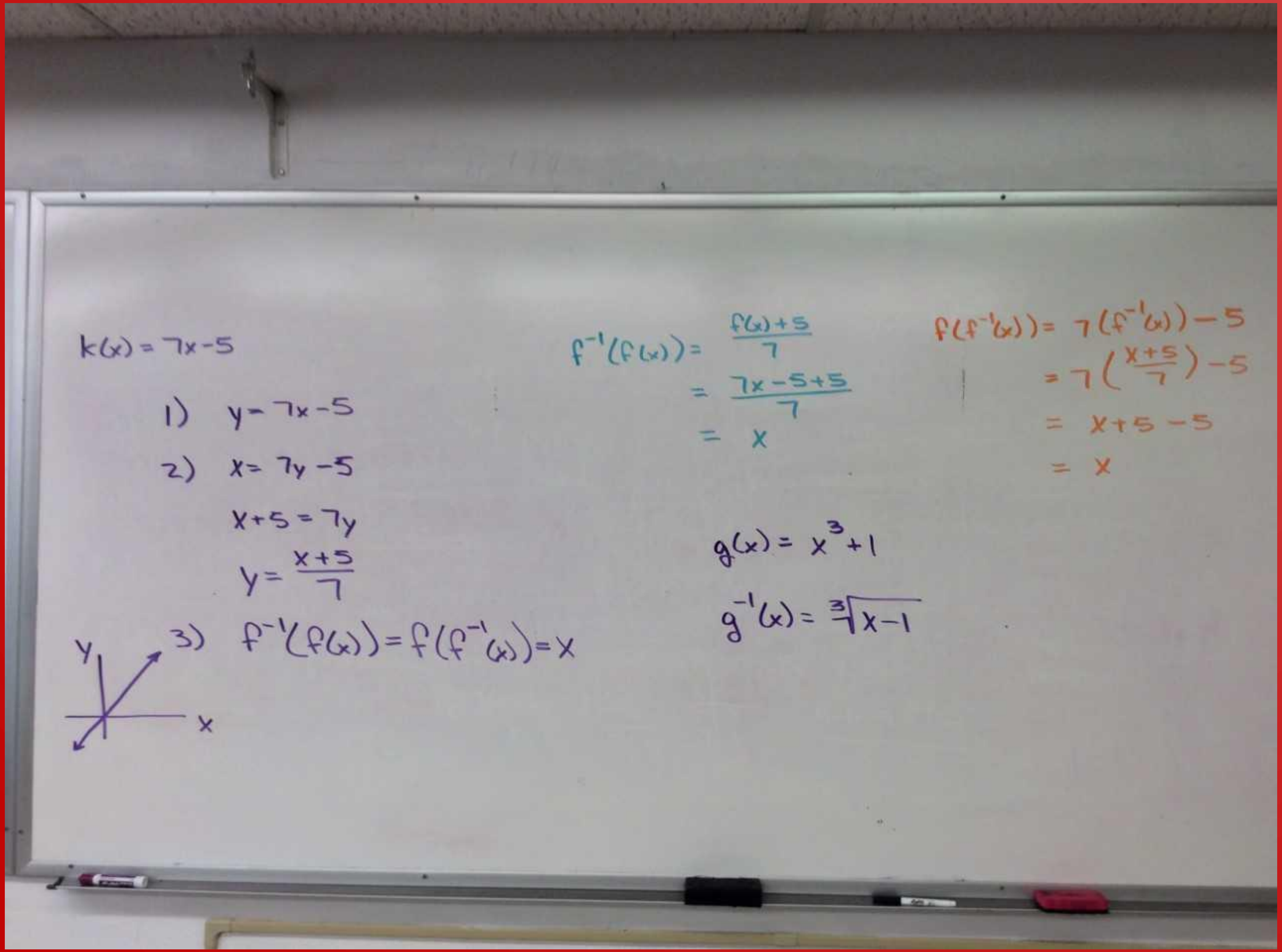
The equation for the inverse of a function f can be found as follows:

1. Replace $f(x)$ with y in the equation for $f(x)$.
2. Interchange x and y.
3. Solve for y. If this equation does not define y as a function of x, the function f does not have an inverse function and this procedure ends. If this equation does define y as a function of x, the function f has an inverse function.
4. If f has an inverse function, replace y in step 3 by $f^{-1}(x)$. We can verify our result by showing that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

The Horizontal Line Test for Inverse Functions

A function f has an inverse that is a function, f^{-1}, if there is no horizontal line that intersects the graph of the function f at more than one point.




k(x) = 7x − 5
1) y = 7x − 5
2) x = 7y − 5
x + 5 = 7y
y = (x + 5)/7
3) f⁻¹(f(x)) = f(f⁻¹(x)) = x
y
x
f⁻¹(f(x)) = (f(x) + 5)/7
= (7x − 5 + 5)/7
= x
f(f⁻¹(x)) = 7(f⁻¹(x)) − 5
= 7((x + 5)/7) − 5
= x + 5 − 5
= x
g(x) = x³ + 1
g⁻¹(x) = ∛(x − 1)

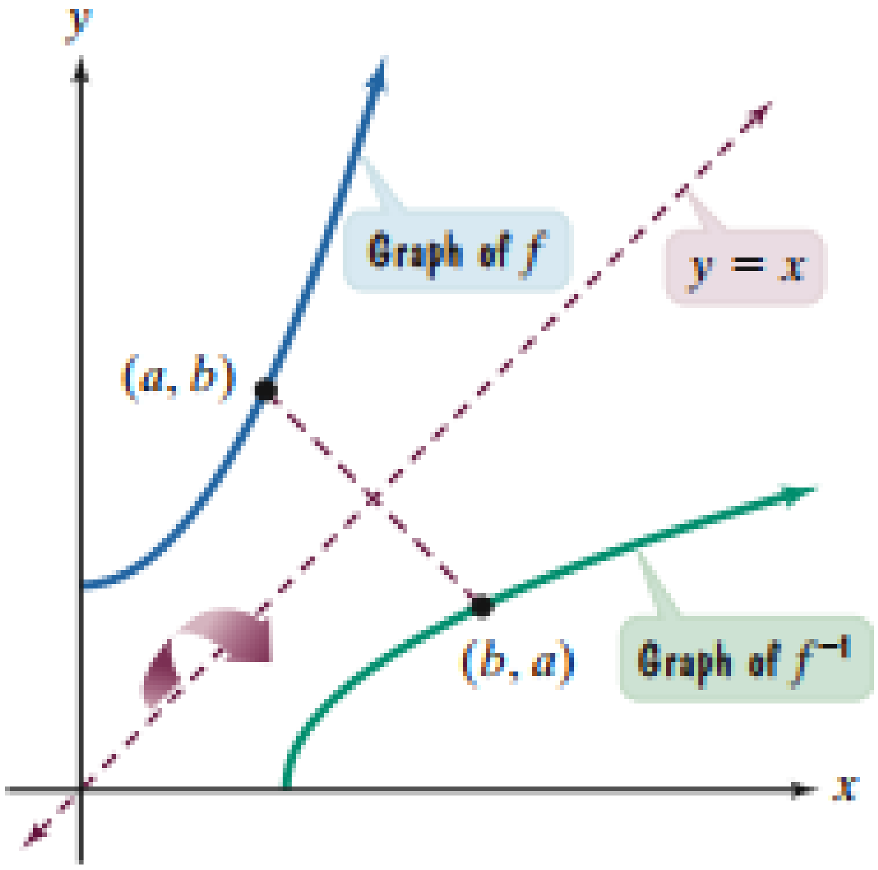


FIGURE 2.60 The graph of f^{-1} is a reflection of the graph of f about $y = x$.