BOARD NOTES

26 SEPTEMBER 2018

CC PRECALCULUS
CHAPTER 3 –
LINEAR AND QUADRATIC FUNCTIONS

- SECTION 3.3 - QUADRATIC FUNCTIONS AND THEIR PROPERTIES

Objectives:

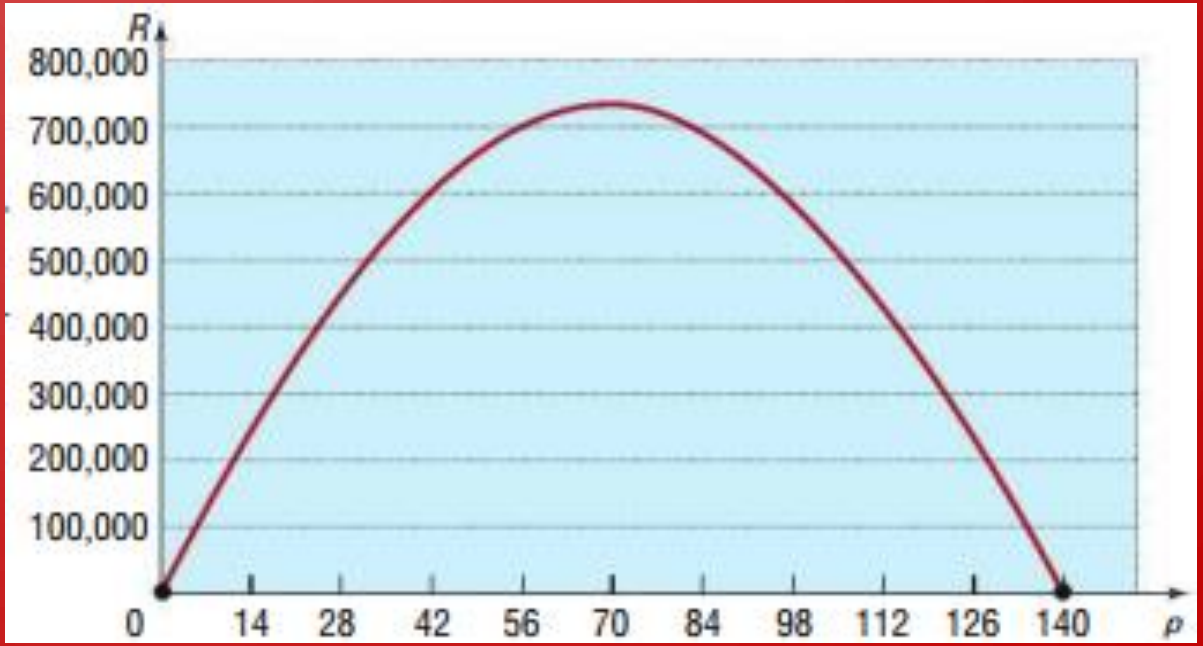
- Graph a quadratic function using transformations
- Identify the Vertex and Axis of Symmetry of a quadratic function
- Graph a quadratic using its vertex, axis of symmetry, and intercepts
- Find a quadratic function given its vertex and one other point
- Find the Maximum or Minimum value of a quadratic function

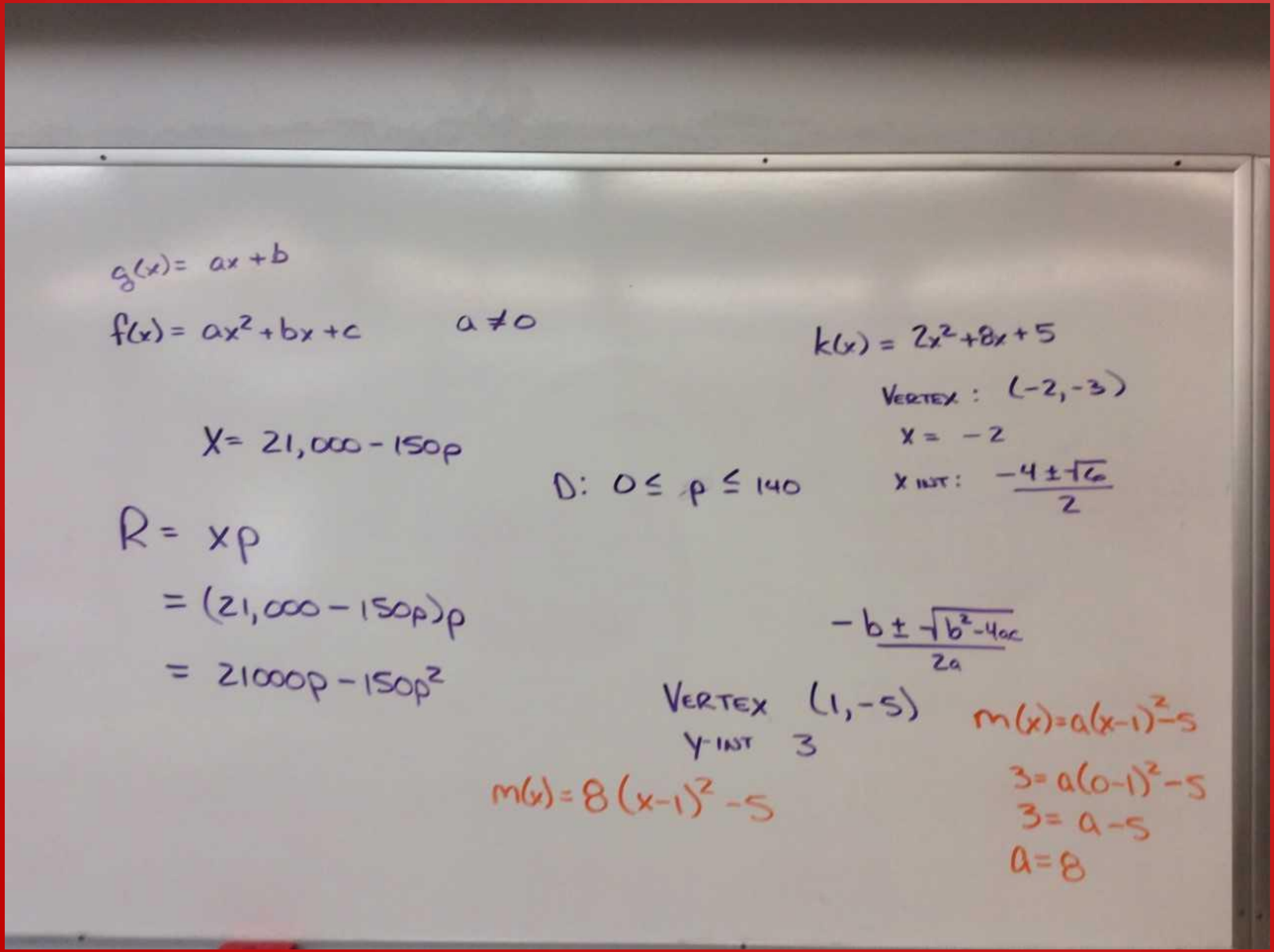
A **quadratic function** is a function of the form

$$f(x) = ax^2 + bx + c$$

where a, b, and c are real numbers and $a \neq 0$. The domain of a quadratic function is the set of all real numbers.

Price *p* per Calculator (in dollars)	Number of Calculators, *x*
60	12,000
65	11,250
70	10,500
75	9,750
80	9,000
85	8,250
90	7,500




g(x) = ax + b
f(x) = ax² + bx + c   a ≠ 0
k(x) = 2x² + 8x + 5
VERTEX : (-2, -3)
x = -2
x INT : (-4 ± √6)/2
X = 21,000 - 150p
D: 0 ≤ p ≤ 140
R = xp
= (21,000 - 150p)p
= 21000p - 150p²
(-b ± √(b² - 4ac))/2a
VERTEX (1, -5)
Y-INT 3
m(x) = a(x-1)² - 5
3 = a(0-1)² - 5
3 = a - 5
a = 8
m(x) = 8(x-1)² - 5

If $h = -\dfrac{b}{2a}$ and $k = \dfrac{4ac - b^2}{4a}$, then

$$f(x) = ax^2 + bx + c = a(x - h)^2 + k \qquad (1)$$

The graph of $f(x) = a(x - h)^2 + k$ is the parabola $y = ax^2$ shifted horizontally h units (replace x by $x - h$) and vertically k units (add k). As a result, the vertex is at (h, k), and the graph opens up if $a > 0$ and down if $a < 0$. The axis of symmetry is the vertical line $x = h$.

Properties of the Graph of a Quadratic Function

$$f(x) = ax^2 + bx + c \qquad a \neq 0$$

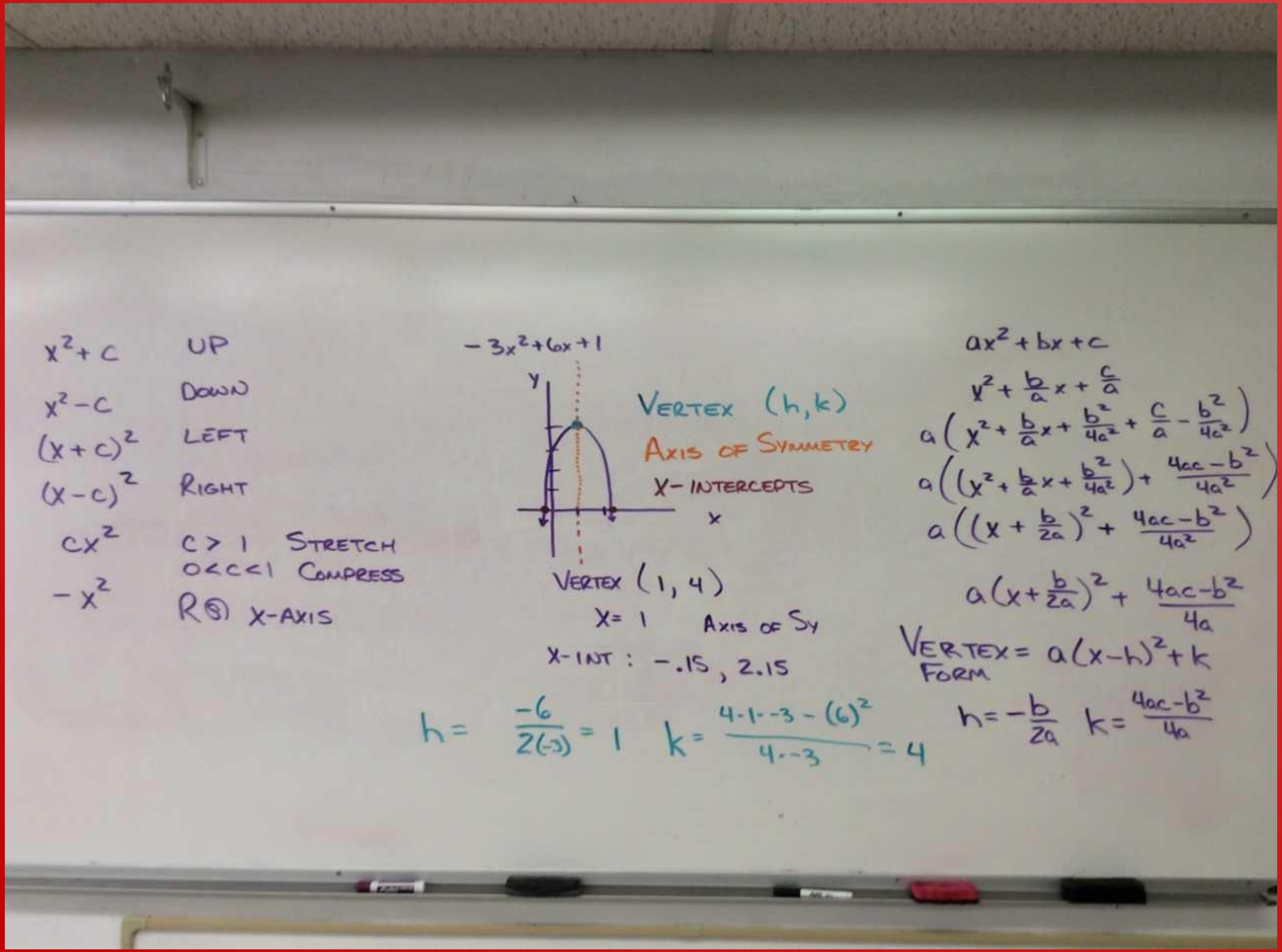
$$\text{Vertex} = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) \quad \text{Axis of symmetry: the vertical line } x = -\frac{b}{2a} \quad \textbf{(2)}$$

Parabola opens up if $a > 0$; the vertex is a minimum point.
Parabola opens down if $a < 0$; the vertex is a maximum point.

If the vertex (h, k) and one additional point on the graph of a quadratic function $f(x) = ax^2 + bx + c, a \neq 0$, are known, then

$$f(x) = a(x - h)^2 + k \tag{3}$$

can be used to obtain the quadratic function.


x² + c UP
x² − c DOWN
(x + c)² LEFT
(x − c)² RIGHT
cx² c > 1 STRETCH
0 < c < 1 COMPRESS
− x² R X-AXIS
−3x² + 6x + 1
VERTEX (h, k)
AXIS OF SYMMETRY
X- INTERCEPTS
VERTEX (1, 4)
X = 1 AXIS OF SY
X-INT : −.15 , 2.15
h = −6 / 2(−3) = 1
k = 4·1·−3 − (6)² / 4·−3 = 4
ax² + bx + c
x² + b/a x + c/a
a(x² + b/a x + b²/4a² + c/a − b²/4a²)
a((x² + b/a x + b²/4a²) + 4ac − b²/4a²)
a((x + b/2a)² + 4ac − b²/4a²)
a(x + b/2a)² + 4ac − b²/4a
VERTEX FORM = a(x − h)² + k
h = −b/2a
k = 4ac − b²/4a

The x-Intercepts of a Quadratic Function

1. If the discriminant $b^2 - 4ac > 0$, the graph of $f(x) = ax^2 + bx + c$ has two distinct x-intercepts so it crosses the x-axis in two places.
2. If the discriminant $b^2 - 4ac = 0$, the graph of $f(x) = ax^2 + bx + c$ has one x-intercept so it touches the x-axis at its vertex.
3. If the discriminant $b^2 - 4ac < 0$, the graph of $f(x) = ax^2 + bx + c$ has no x-intercepts so it does not cross or touch the x-axis.

SUMMARY

Steps for Graphing a Quadratic Function $f(x) = ax^2 + bx + c, a \neq 0$

Option 1

STEP 1: Complete the square in x to write the quadratic function in the form $f(x) = a(x - h)^2 + k$.

STEP 2: Graph the function in stages using transformations.

Option 2

STEP 1: Determine whether the parabola opens up $(a > 0)$ or down $(a < 0)$.

STEP 2: Determine the vertex $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

STEP 3: Determine the axis of symmetry, $x = -\frac{b}{2a}$.

STEP 4: Determine the y-intercept, $f(0)$, and the x-intercepts, if any.

(a) If $b^2 - 4ac > 0$, the graph of the quadratic function has two x-intercepts, which are found by solving the equation $ax^2 + bx + c = 0$.

(b) If $b^2 - 4ac = 0$, the vertex is the x-intercept.

(c) If $b^2 - 4ac < 0$, there are no x-intercepts.

STEP 5: Determine an additional point by using the y-intercept and the axis of symmetry.

STEP 6: Plot the points and draw the graph.