BOARD NOTES

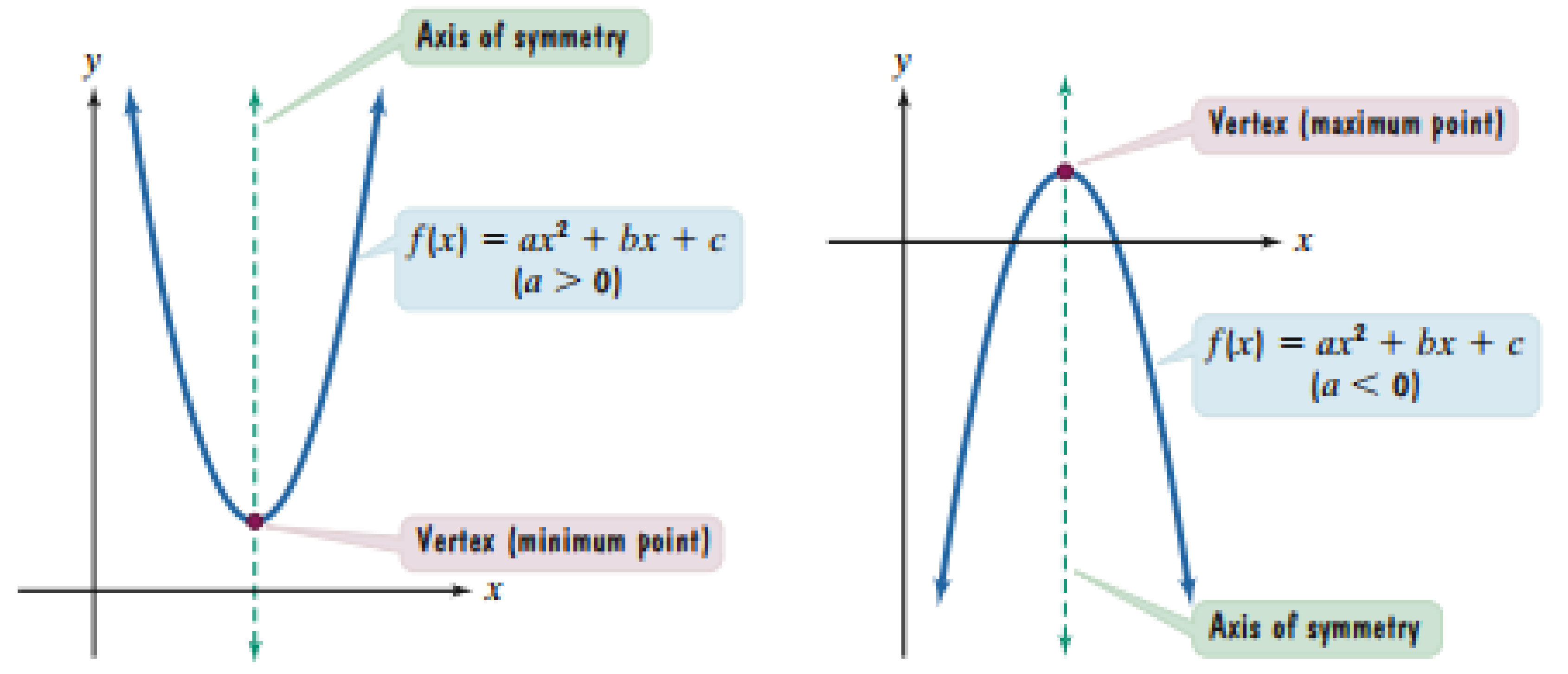
4 OCTOBER 2018

CC ALGEBRA
CHAPTER 3 –
POLYNOMIAL AND RATIONAL FUNCTIONS

- SECTION 3.1 - QUADRATIC FUNCTIONS

Objectives:

- Recognize the characteristics of parabolas
- Graph parabolas
- Determine a quadratic function's minimum or maximum value
- Solve problems involving a quadratic function's minimum or maximum value



$a > 0$: Parabola opens upward.

$a < 0$: Parabola opens downward.

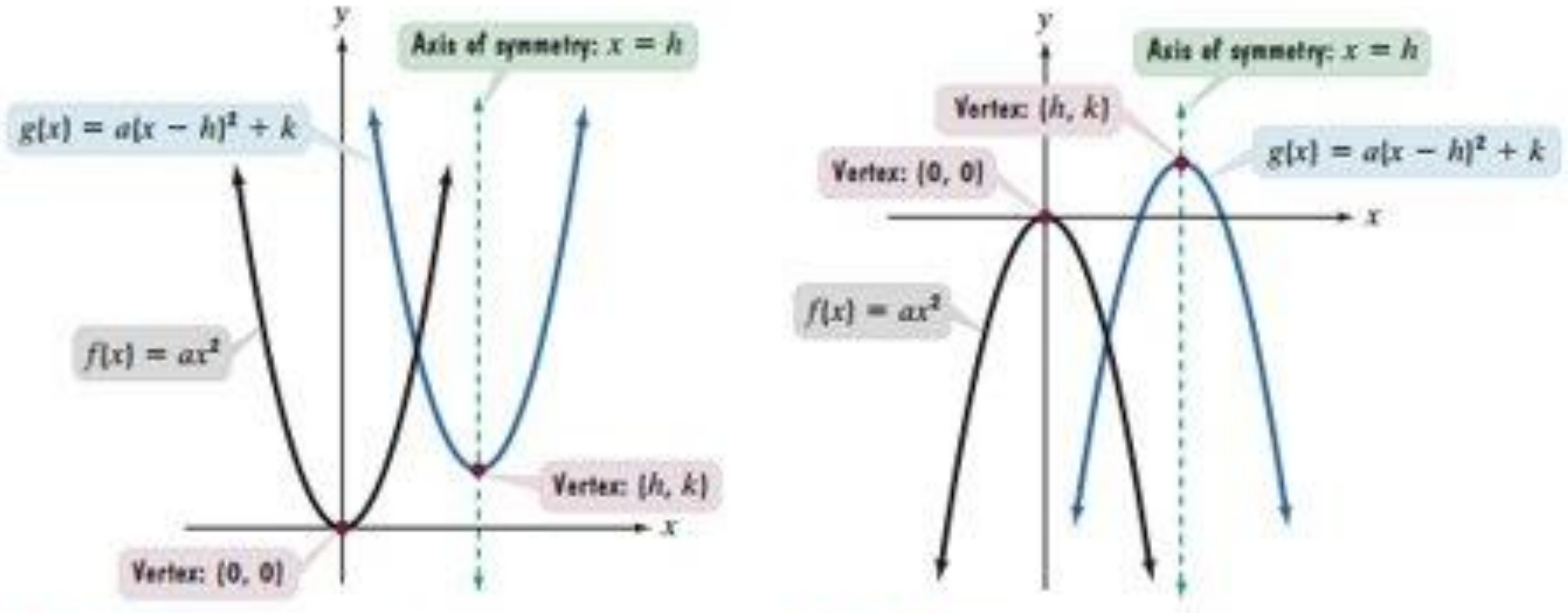


FIGURE 3.2(a) $a > 0$: Parabola opens upward.

FIGURE 3.2(b) $a < 0$: Parabola opens downward.

The Standard Form of a Quadratic Function

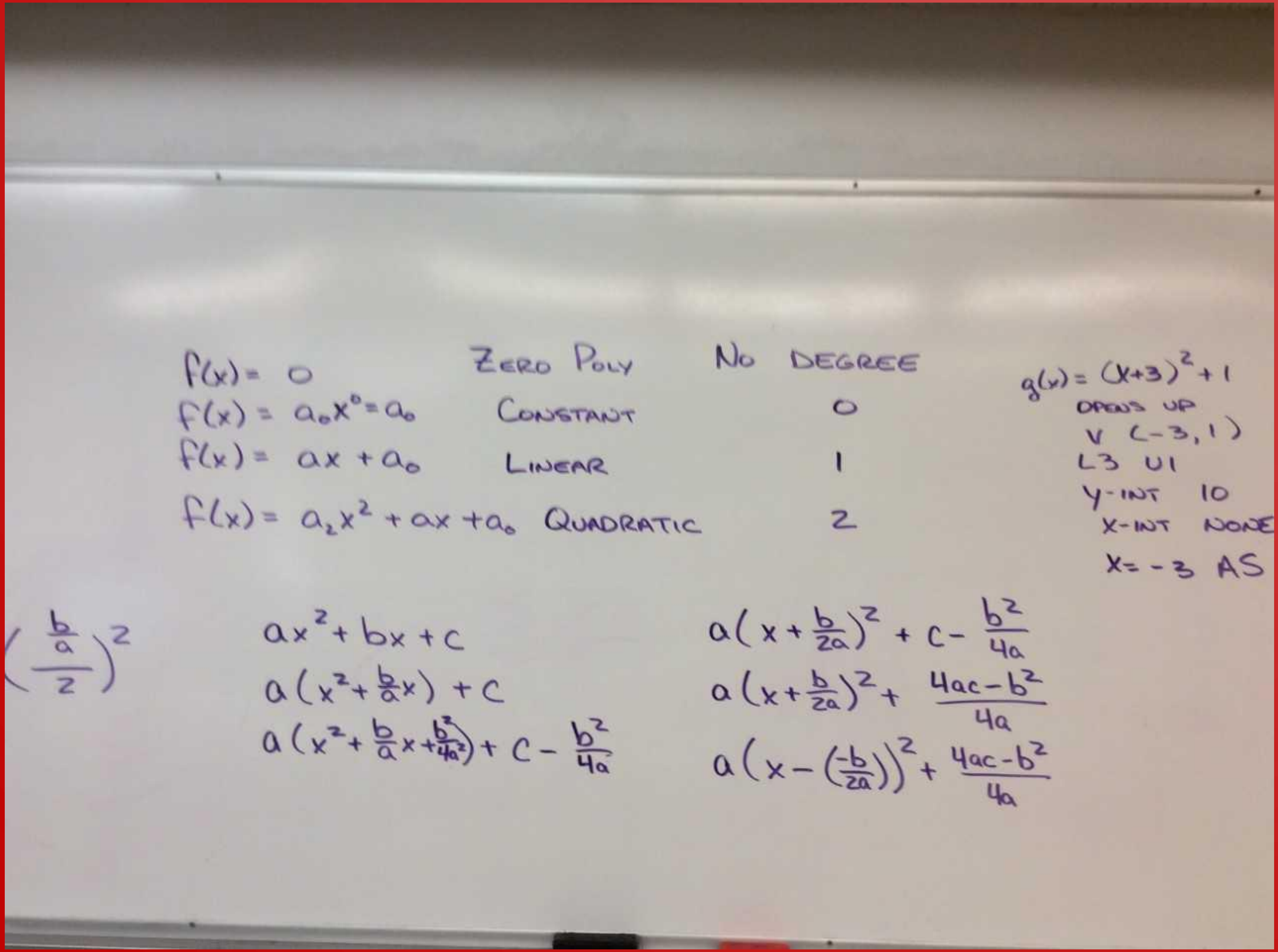
The quadratic function

$$f(x) = a(x - h)^2 + k, \qquad a \neq 0$$

is in **standard form.** The graph of f is a parabola whose vertex is the point (h, k). The parabola is symmetric with respect to the line $x = h$. If $a > 0$, the parabola opens upward; if $a < 0$, the parabola opens downward.

The Vertex of a Parabola Whose Equation Is $f(x) = ax^2 + bx + c$

Consider the parabola defined by the quadratic function $f(x) = ax^2 + bx + c$. The parabola's vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$. The x-coordinate is $-\frac{b}{2a}$. The y-coordinate is found by substituting the x-coordinate into the parabola's equation and evaluating the function at this value of x.


f(x) = 0 ZERO POLY NO DEGREE
f(x) = a₀x⁰ = a₀ CONSTANT 0
f(x) = ax + a₀ LINEAR 1
f(x) = a₂x² + ax + a₀ QUADRATIC 2
g(x) = (x+3)² + 1
OPENS UP
V (-3, 1)
L3 U1
Y-INT 10
X-INT NONE
X = -3 AS
(b/a / 2)²
ax² + bx + c
a(x² + b/a x) + c
a(x² + b/a x + b²/4a²) + c − b²/4a
a(x + b/2a)² + c − b²/4a
a(x + b/2a)² + (4ac − b²)/4a
a(x − (−b/2a))² + (4ac − b²)/4a

Graphing Quadratic Functions with Equations in Standard Form

To graph $f(x) = a(x - h)^2 + k$,

1. Determine whether the parabola opens upward or downward. If $a > 0$, it opens upward. If $a < 0$, it opens downward.
2. Determine the vertex of the parabola. The vertex is (h, k).
3. Find any x-intercepts by solving $f(x) = 0$. The function's real zeros are the x-intercepts.
4. Find the y-intercept by computing $f(0)$.
5. Plot the intercepts, the vertex, and additional points as necessary. Connect these points with a smooth curve that is shaped like a bowl or an inverted bowl.

Graphing Quadratic Functions with Equations in the Form $f(x) = ax^2 + bx + c$

To graph $f(x) = ax^2 + bx + c$,

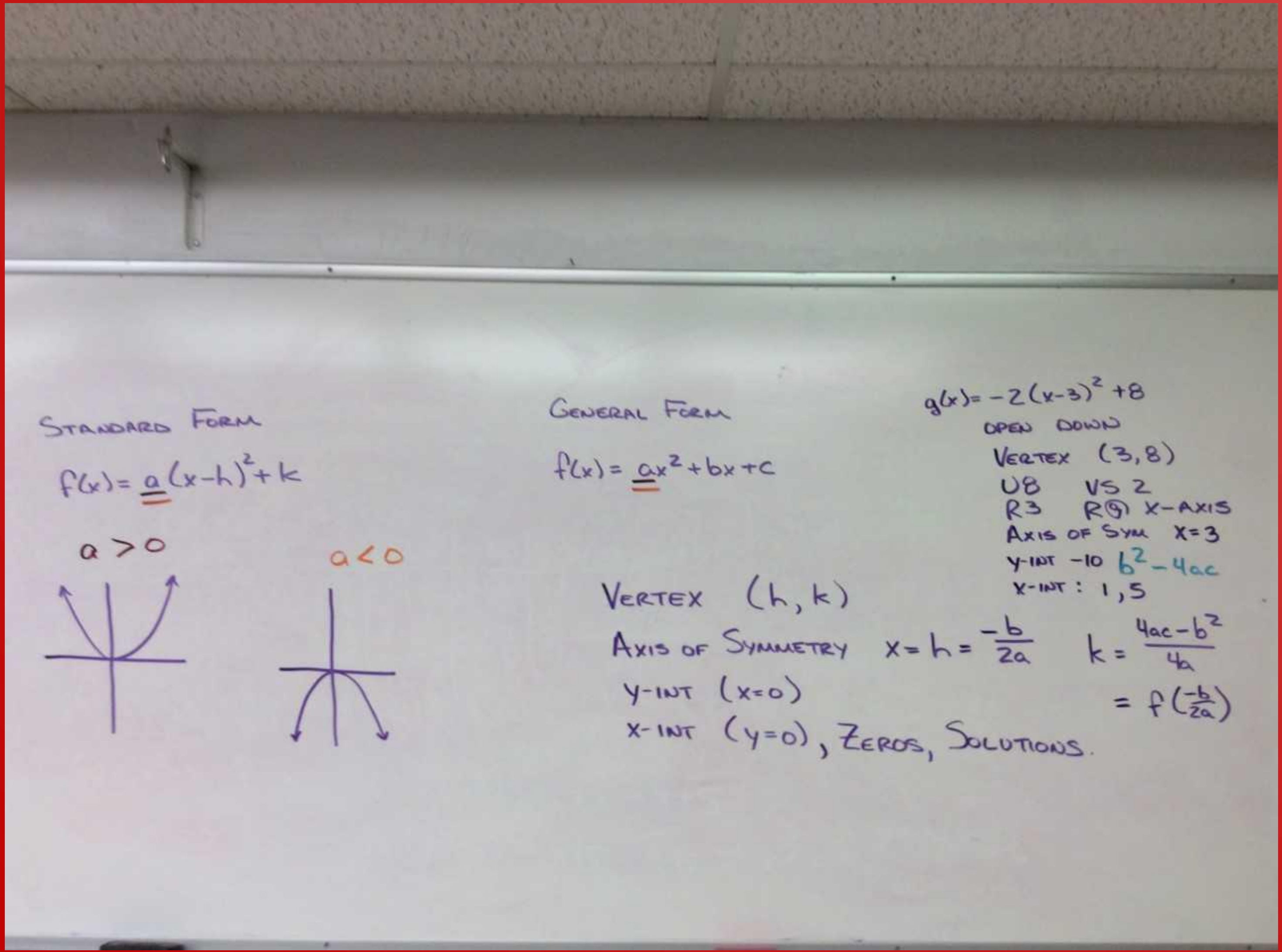
1. Determine whether the parabola opens upward or downward. If $a > 0$, it opens upward. If $a < 0$, it opens downward.
2. Determine the vertex of the parabola. The vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.
3. Find any x-intercepts by solving $f(x) = 0$. The real solutions of $ax^2 + bx + c = 0$ are the x-intercepts.
4. Find the y-intercept by computing $f(0)$. Because $f(0) = c$ (the constant term in the function's equation), the y-intercept is c and the parabola passes through $(0, c)$.
5. Plot the intercepts, the vertex, and additional points as necessary. Connect these points with a smooth curve.

Minimum and Maximum: Quadratic Functions

Consider the quadratic function $f(x) = ax^2 + bx + c$.

1. If $a > 0$, then f has a minimum that occurs at $x = -\frac{b}{2a}$. This minimum value is $f\left(-\frac{b}{2a}\right)$.
2. If $a < 0$, then f has a maximum that occurs at $x = -\frac{b}{2a}$. This maximum value is $f\left(-\frac{b}{2a}\right)$.

In each case, the value of x gives the location of the minimum or maximum value. The value of y, or $f\left(-\frac{b}{2a}\right)$, gives that minimum or maximum value.



STANDARD FORM
f(x) = a(x−h)²+k
a > 0
a < 0
GENERAL FORM
f(x) = ax²+bx+c
VERTEX (h, k)
AXIS OF SYMMETRY x = h = −b/2a
k = (4ac−b²)/4a
= f(−b/2a)
Y-INT (x=0)
X-INT (y=0), ZEROS, SOLUTIONS.
g(x) = −2(x−3)²+8
OPEN DOWN
VERTEX (3,8)
U8 VS 2
R3 R X-AXIS
AXIS OF SYM X=3
Y-INT −10 b²−4ac
X-INT : 1,5