BOARD NOTES

4 OCTOBER 2018

CC PRECALCULUS
CHAPTER 4 –
POLYNOMIAL AND RATIONAL FUNCTIONS

- SECTION 4.1 - POLYNOMIAL FUNCTIONS AND MODELS

Objectives:

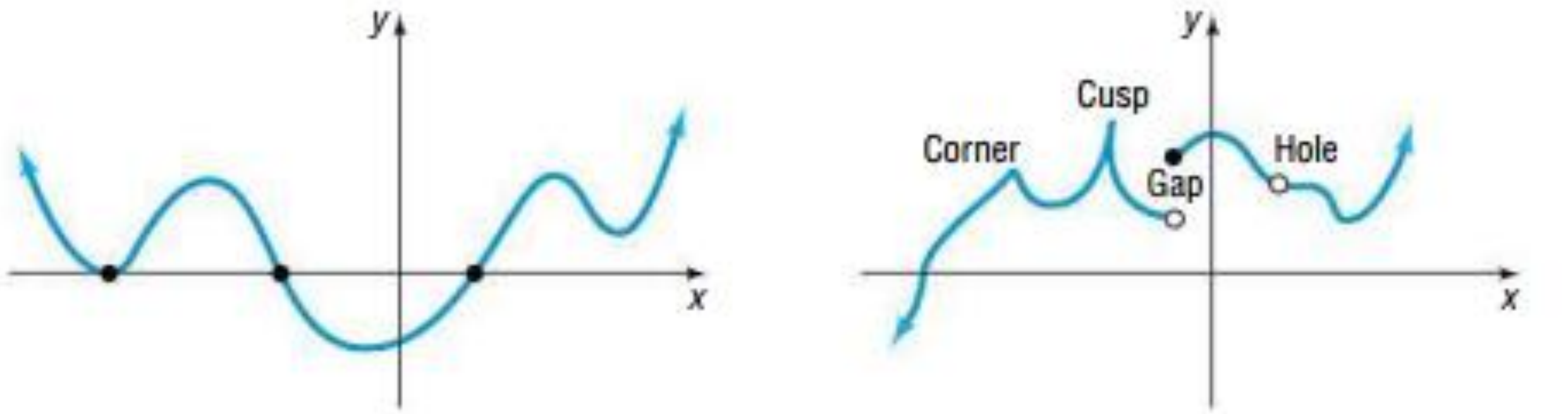
- Identify polynomial functions and their degree
- Graph polynomial functions using transformations
- Know properties of the graph of a polynomial function
- Analyze the graph of a polynomial function
- Build cubic models from data

A **polynomial function** in one variable is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \qquad \textbf{(1)}$$

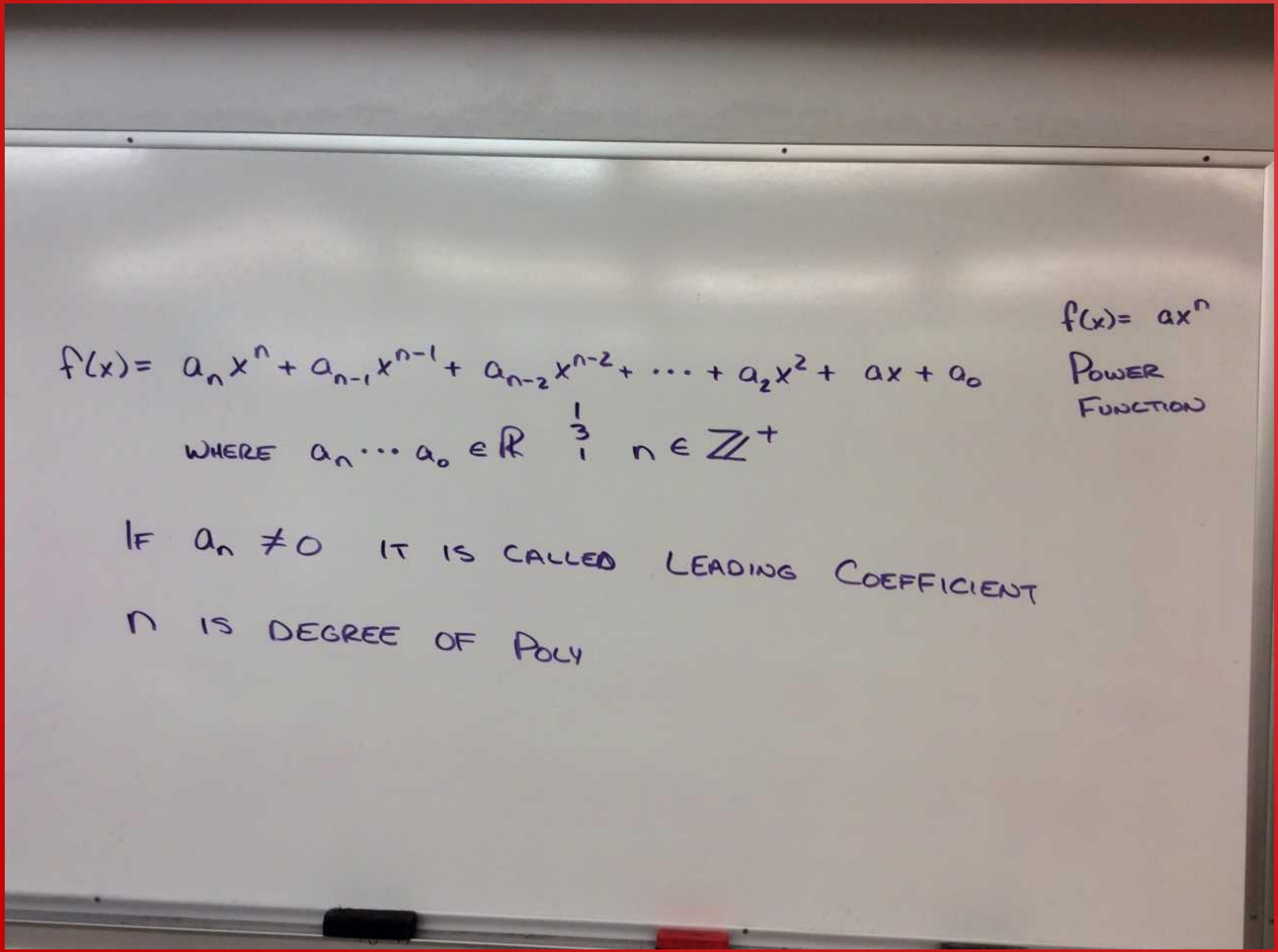
where $a_n, a_{n-1}, \ldots, a_1, a_0$ are constants, called the **coefficients** of the polynomial, $n \geq 0$ is an integer, and x is the variable. If $a_n \neq 0$, it is called the **leading coefficient**, and n is the **degree** of the polynomial.

The domain of a polynomial function is the set of all real numbers.



(a) Graph of a polynomial function: smooth, continuous

(b) Cannot be the graph of a polynomial function

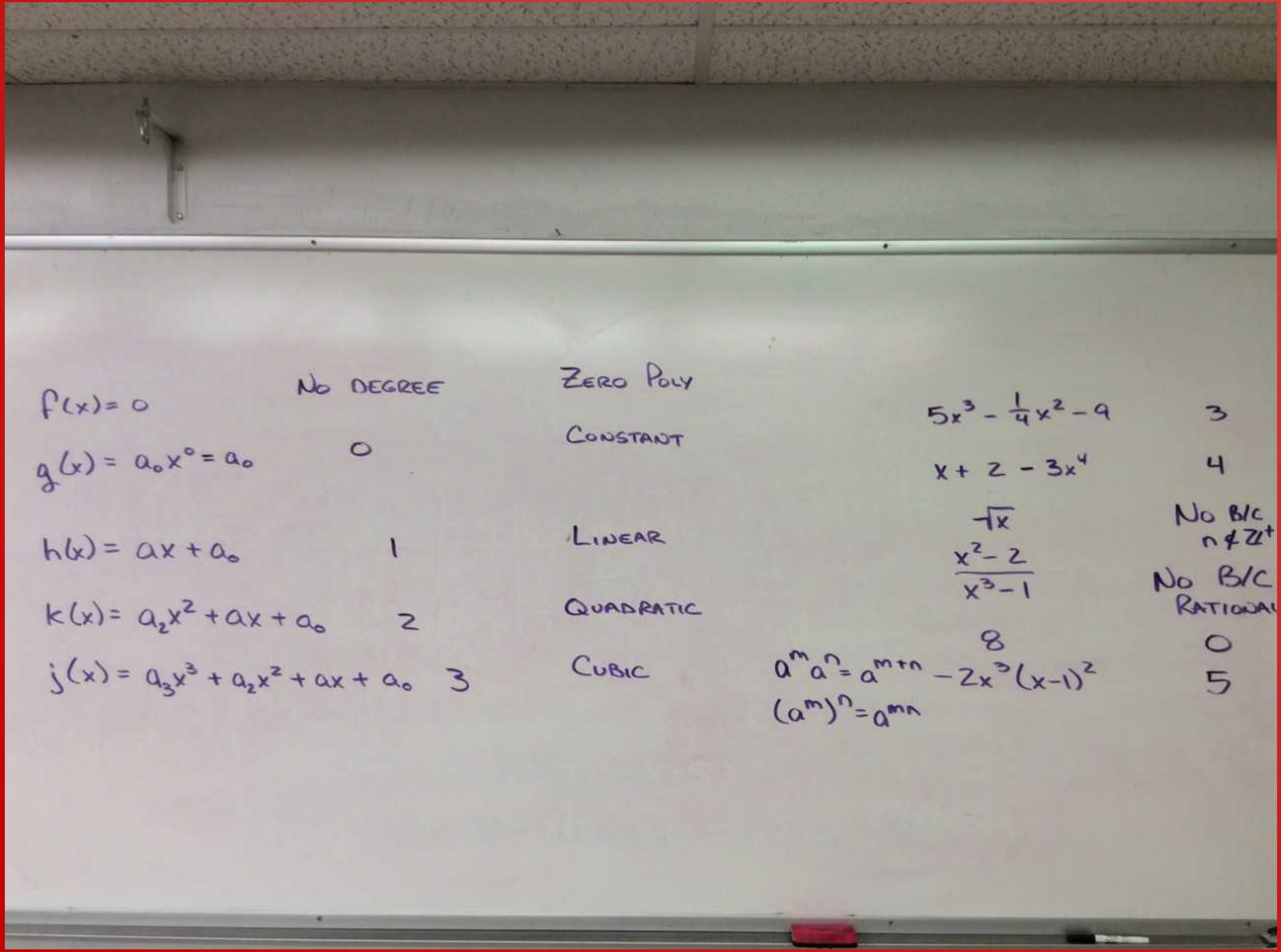

$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x^2 + ax + a_0$
WHERE $a_n \cdots a_0 \in \mathbb{R}$ $n \in \mathbb{Z}^+$
IF $a_n \neq 0$ IT IS CALLED LEADING COEFFICIENT
n IS DEGREE OF POLY
$f(x) = ax^n$
POWER FUNCTION

Degree	Form	Name	Graph
No degree	$f(x) = 0$	Zero function	The x-axis
0	$f(x) = a_0, \quad a_0 \neq 0$	Constant function	Horizontal line with y-intercept a_0
1	$f(x) = a_1x + a_0, \quad a_1 \neq 0$	Linear function	Nonvertical, nonhorizontal line with slope a_1 and y-intercept a_0
2	$f(x) = a_2x^2 + a_1x + a_0, \quad a_2 \neq 0$	Quadratic function	Parabola: graph opens up if $a_2 > 0$; graph opens down if $a_2 < 0$

A **power function of degree *n*** is a monomial function of the form

$$f(x) = ax^n \tag{2}$$

where a is a real number, $a \neq 0$, and $n > 0$ is an integer.



f(x) = 0
No DEGREE
ZERO POLY
g(x) = a₀x° = a₀
0
CONSTANT
h(x) = ax + a₀
1
LINEAR
k(x) = a₂x² + ax + a₀
2
QUADRATIC
j(x) = a₃x³ + a₂x² + ax + a₀
3
CUBIC
5x³ − ¼x² − 9
3
x + 2 − 3x⁴
4
√x
No B/C n ∉ ℤ⁺
(x² − 2)/(x³ − 1)
No B/C RATIONAL
8
0
aᵐaⁿ = aᵐ⁺ⁿ
−2x³(x−1)²
5
(aᵐ)ⁿ = aᵐⁿ

Properties of Power Functions, $f(x) = x^n$, n Is a Positive Even Integer

1. f is an even function, so its graph is symmetric with respect to the y-axis.
2. The domain is the set of all real numbers. The range is the set of nonnegative real numbers.
3. The graph always contains the points $(-1, 1)$, $(0, 0)$, and $(1, 1)$.
4. As the exponent n increases in magnitude, the graph is steeper when $x < -1$ or $x > 1$; but for x near the origin, the graph tends to flatten out and lie closer to the x-axis.

Properties of Power Functions, $f(x) = x^n$, n Is a Positive Odd Integer

1. f is an odd function, so its graph is symmetric with respect to the origin.
2. The domain and the range are the set of all real numbers.
3. The graph always contains the points $(-1, -1)$, $(0, 0)$, and $(1, 1)$.
4. As the exponent n increases in magnitude, the graph is steeper when $x < -1$ or $x > 1$; but for x near the origin, the graph tends to flatten out and lie closer to the x-axis.

FRANK PHILLIPS COLLEGE Plainsmen

BORGER INDEPENDENT SCHOOL DISTRICT 1926

If f is a function and r is a real number for which $f(r) = 0$, then r is called a **real zero** of f.

As a consequence of this definition, the following statements are equivalent.

1. r is a real zero of a polynomial function f.
2. r is an x-intercept of the graph of f.
3. $x - r$ is a factor of f.
4. r is a solution to the equation $f(x) = 0$.

So the real zeros of a polynomial function are the x-intercepts of its graph, and they are found by solving the equation $f(x) = 0$.

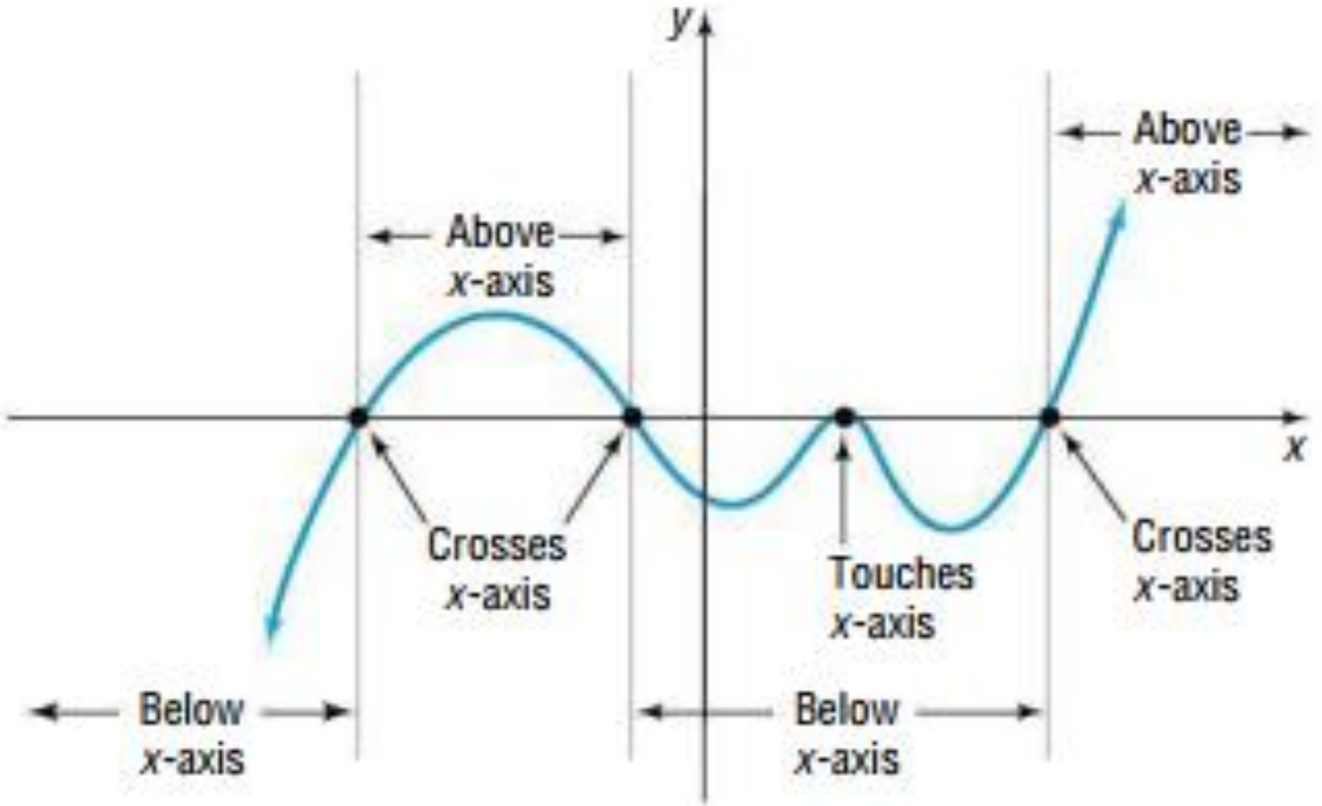


Figure 9 Graph of a polynomial function