BOARD NOTES

9 OCTOBER 2018

CC PRECALCULUS
CHAPTER 4 –
POLYNOMIAL AND RATIONAL FUNCTIONS

- SECTION 4.1 - POLYNOMIAL FUNCTIONS AND MODELS

Objectives:

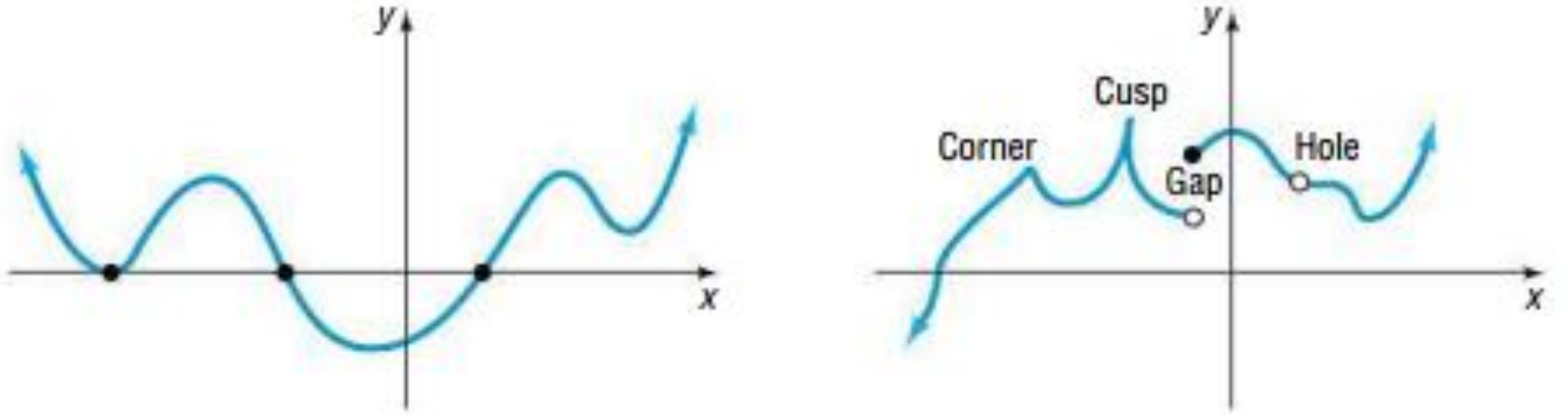
- Identify polynomial functions and their degree
- Graph polynomial functions using transformations
- Know properties of the graph of a polynomial function
- Analyze the graph of a polynomial function
- Build cubic models from data

A **polynomial function** in one variable is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \qquad \textbf{(1)}$$

where $a_n, a_{n-1}, \ldots, a_1, a_0$ are constants, called the **coefficients** of the polynomial, $n \geq 0$ is an integer, and x is the variable. If $a_n \neq 0$, it is called the **leading coefficient**, and n is the **degree** of the polynomial.

The domain of a polynomial function is the set of all real numbers.



(a) Graph of a polynomial function: smooth, continuous

(b) Cannot be the graph of a polynomial function

Degree	Form	Name	Graph
No degree	$f(x) = 0$	Zero function	The *x*-axis
0	$f(x) = a_0, \quad a_0 \neq 0$	Constant function	Horizontal line with *y*-intercept a_0
1	$f(x) = a_1x + a_0, \quad a_1 \neq 0$	Linear function	Nonvertical, nonhorizontal line with slope a_1 and *y*-intercept a_0
2	$f(x) = a_2x^2 + a_1x + a_0, \quad a_2 \neq 0$	Quadratic function	Parabola: graph opens up if $a_2 > 0$; graph opens down if $a_2 < 0$

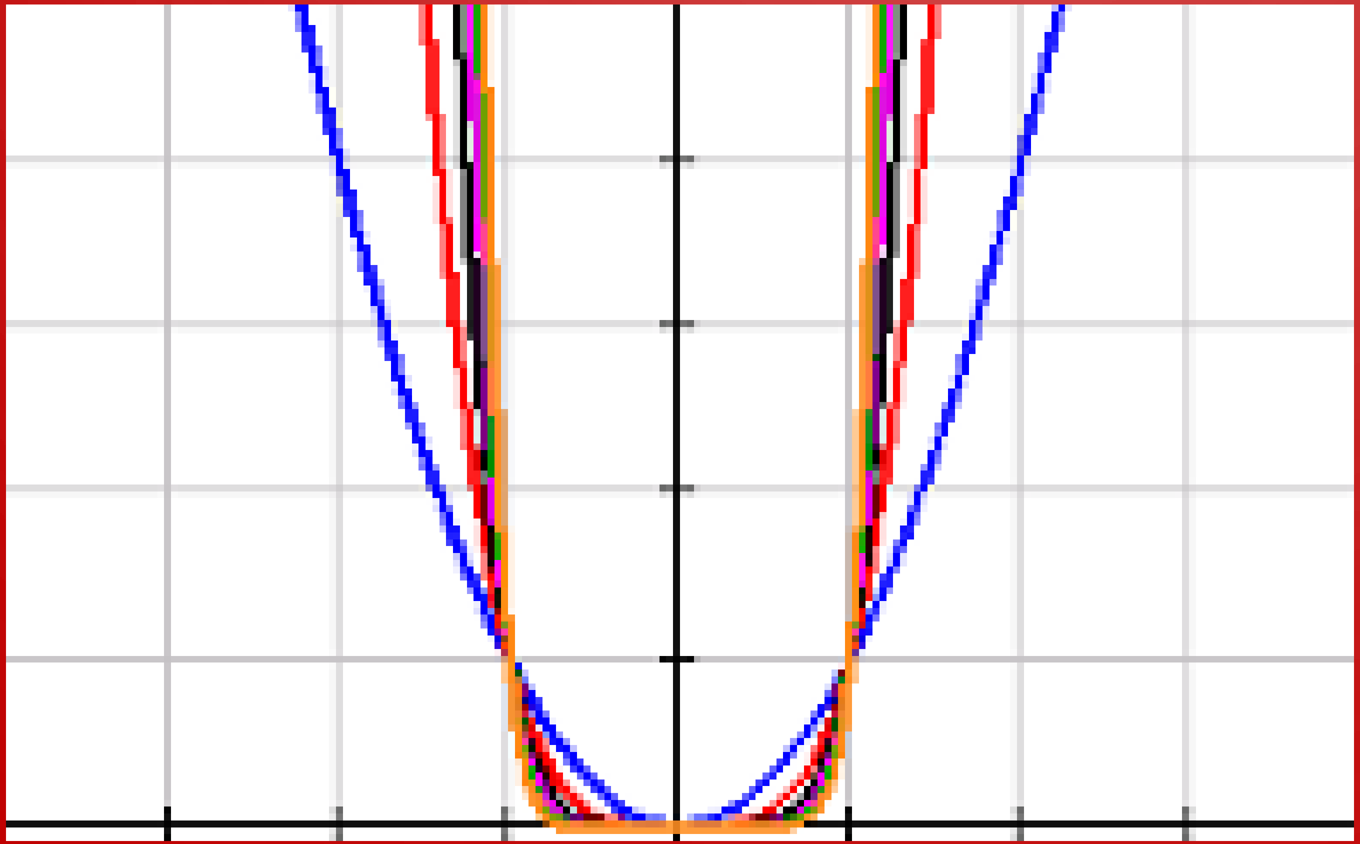
A **power function of degree *n*** is a monomial function of the form

$$f(x) = ax^n \tag{2}$$

where a is a real number, $a \neq 0$, and $n > 0$ is an integer.

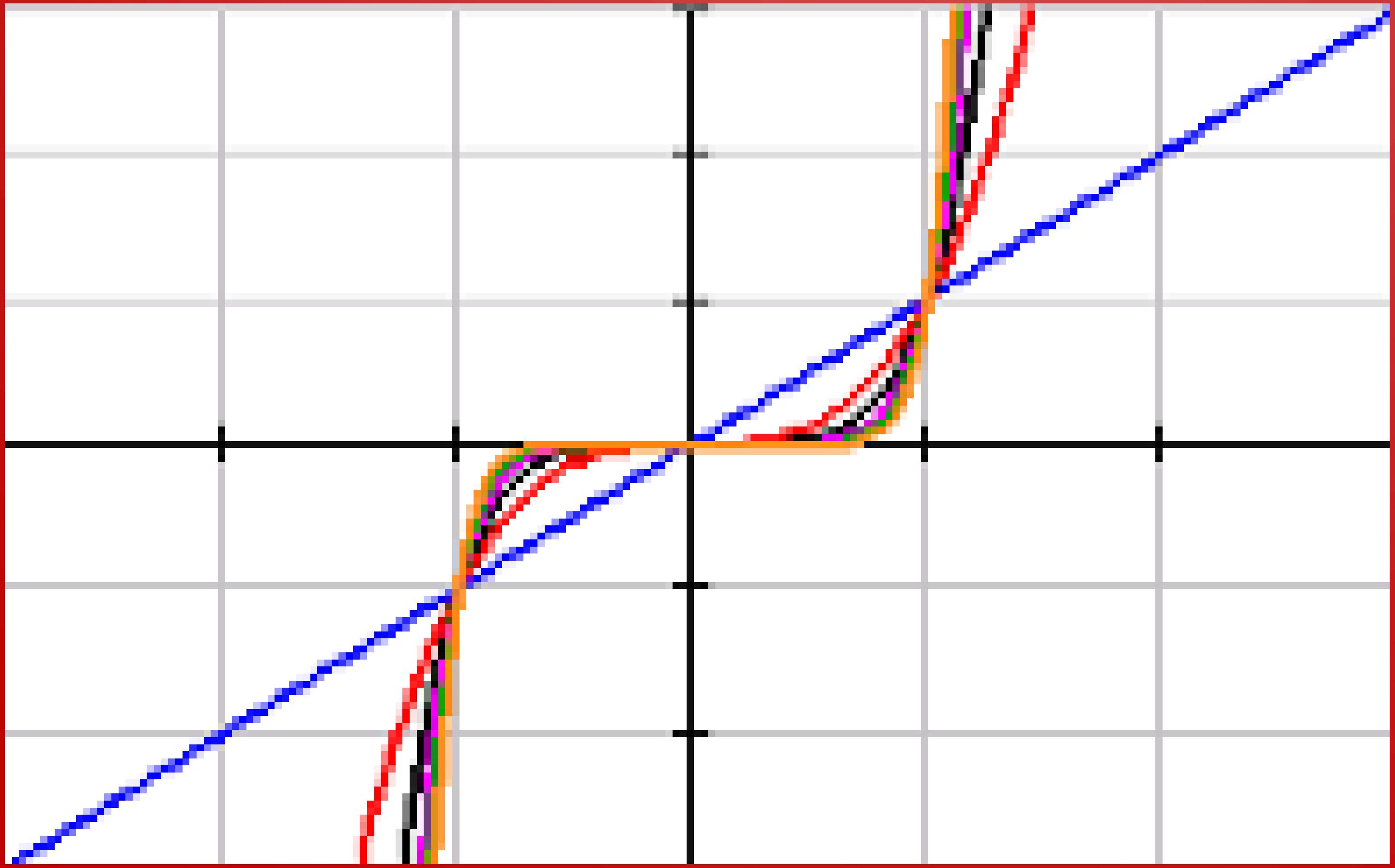
Properties of Power Functions, $f(x) = x^n$, n Is a Positive Even Integer

1. f is an even function, so its graph is symmetric with respect to the y-axis.
2. The domain is the set of all real numbers. The range is the set of nonnegative real numbers.
3. The graph always contains the points $(-1, 1)$, $(0, 0)$, and $(1, 1)$.
4. As the exponent n increases in magnitude, the graph is steeper when $x < -1$ or $x > 1$; but for x near the origin, the graph tends to flatten out and lie closer to the x-axis.



Properties of Power Functions, $f(x) = x^n$, n Is a Positive Odd Integer

1. f is an odd function, so its graph is symmetric with respect to the origin.
2. The domain and the range are the set of all real numbers.
3. The graph always contains the points $(-1, -1)$, $(0, 0)$, and $(1, 1)$.
4. As the exponent n increases in magnitude, the graph is steeper when $x < -1$ or $x > 1$; but for x near the origin, the graph tends to flatten out and lie closer to the x-axis.



If f is a function and r is a real number for which $f(r) = 0$, then r is called a **real zero** of f.

1. r is a real zero of a polynomial function f.
2. r is an x-intercept of the graph of f.
3. $x - r$ is a factor of f.
4. r is a solution to the equation $f(x) = 0$.

If $(x - r)^m$ is a factor of a polynomial f and $(x - r)^{m+1}$ is not a factor of f, then r is called a **zero of multiplicity *m* of *f***.*

If *r* Is a Zero of Even Multiplicity

Numerically: The sign of $f(x)$ does not change from one side to the other side of r.

Graphically: The graph of f **touches** the x-axis at r.

If *r* Is a Zero of Odd Multiplicity

Numerically: The sign of $f(x)$ changes from one side to the other side of r.

Graphically: The graph of f **crosses** the x-axis at r.

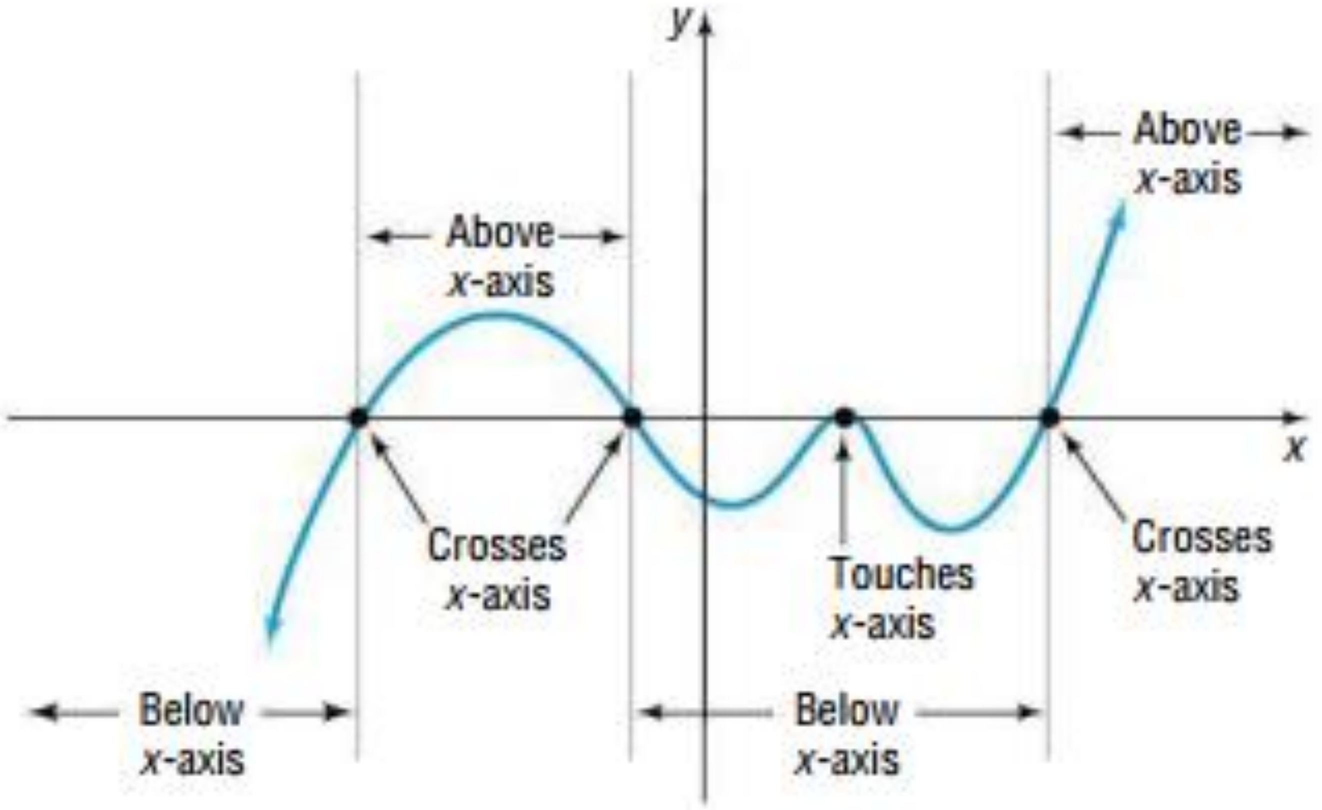


Figure 9 Graph of a polynomial function

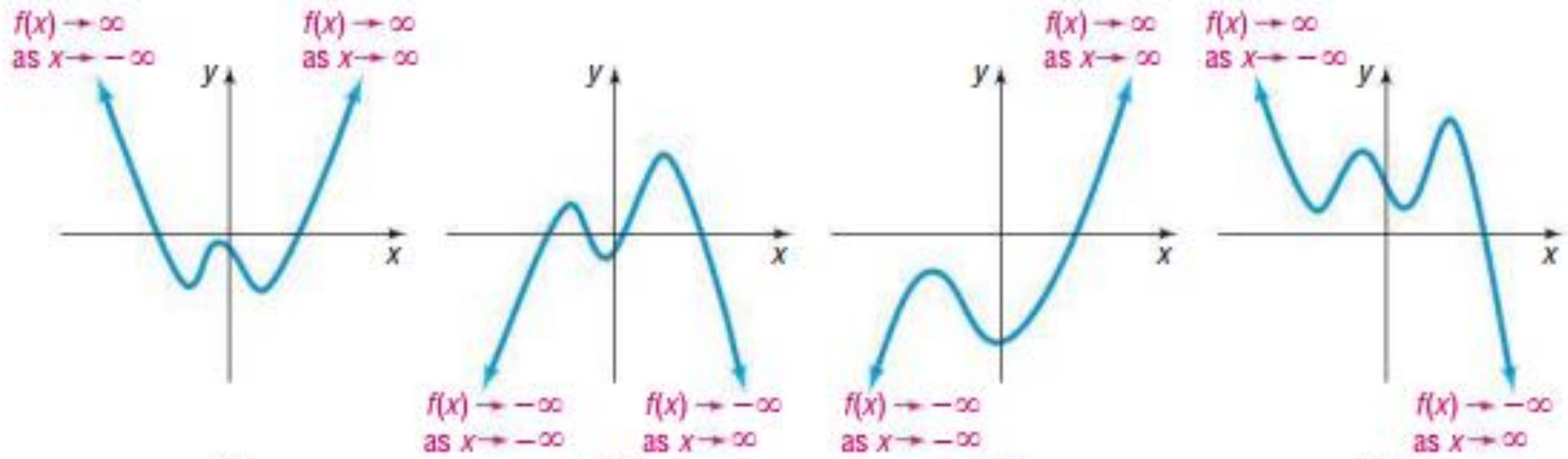
End Behavior

For large values of x, either positive or negative, the graph of the polynomial function

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \quad a_n \neq 0$$

resembles the graph of the power function

$$y = a_n x^n$$



(a) $n \geq 2$ even; $a_n > 0$

(b) $n \geq 2$ even; $a_n < 0$

(c) $n \geq 3$ odd; $a_n > 0$

(d) $n \geq 3$ odd; $a_n < 0$

Graph of a Polynomial Function $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \quad a_n \neq 0$

Degree of the polynomial function f: n

y-intercept: $f(0) = a_0$

Graph is smooth and continuous.

Maximum number of turning points: $n - 1$

At a zero of even multiplicity: The graph of f touches the x-axis.

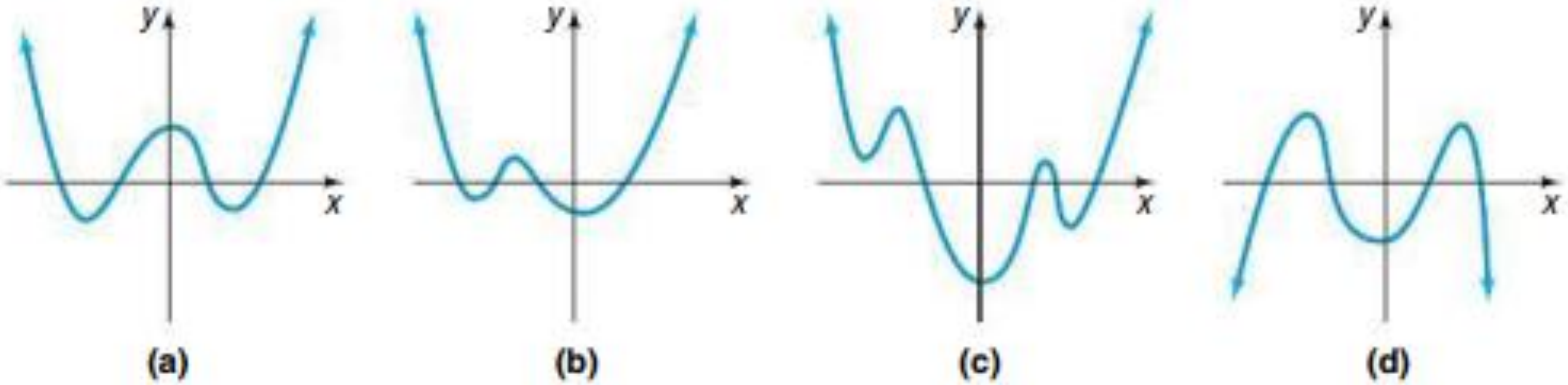
At a zero of odd multiplicity: The graph of f crosses the x-axis.

Between zeros, the graph of f is either above or below the x-axis.

End behavior: For large $|x|$, the graph of f behaves like the graph of $y = a_n x^n$.

$$f(x) = x^4 + ax^3 + bx^2 - 5x - 6$$

where $a > 0, b > 0$?



Analyzing the Graph of a Polynomial Function

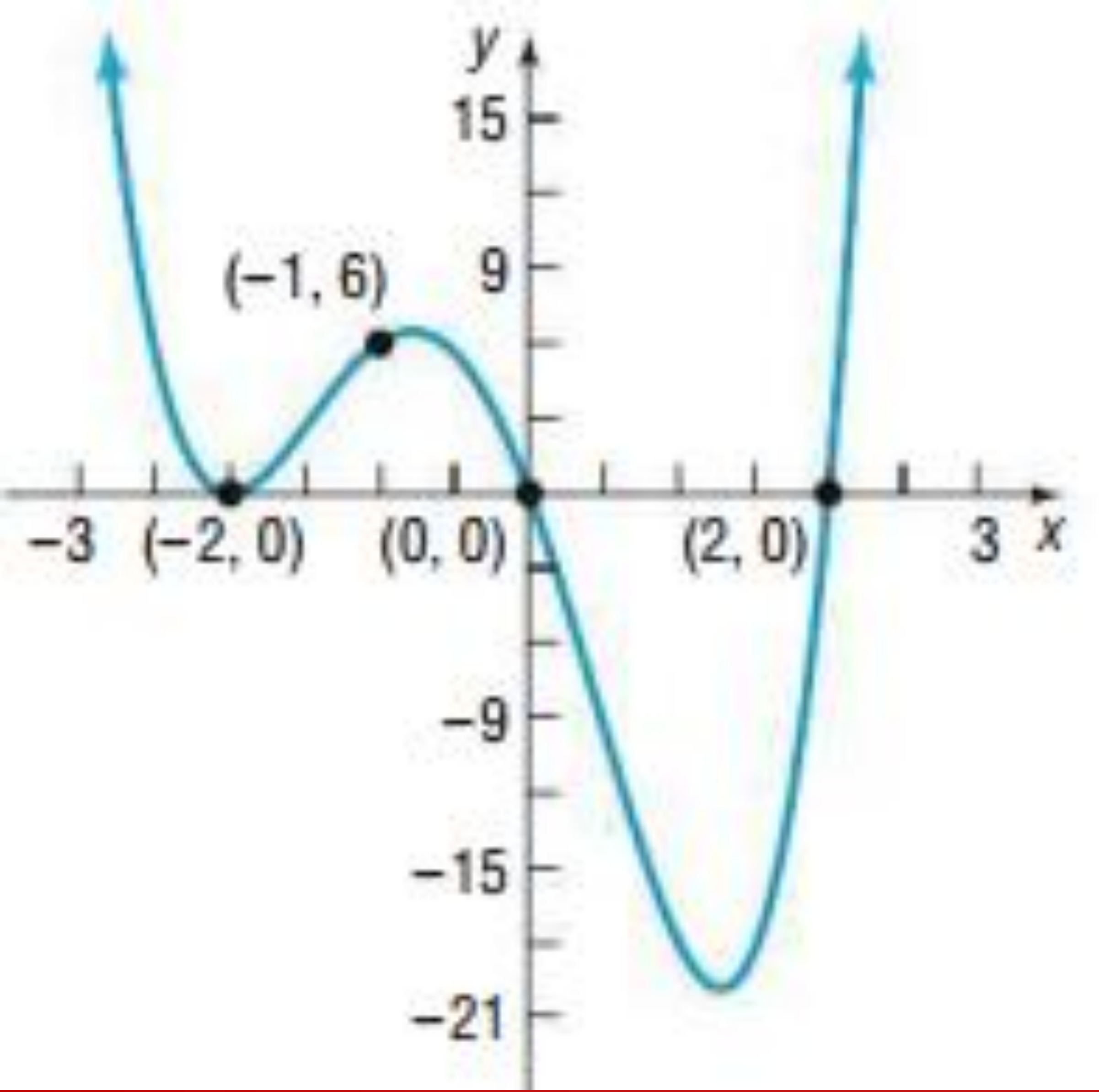
Step 1: Determine the end behavior of the graph of the function.

Step 2: Find the *x*- and *y*-intercepts of the graph of the function.

Step 3: Determine the zeros of the function and their multiplicity. Use this information to determine whether the graph crosses or touches the *x*-axis at each *x*-intercept.

Step 4: Determine the maximum number of turning points on the graph of the function.

Step 5: Use the information in Steps 1 through 4 to draw a complete graph of the function. To help establish the *y*-axis scale, find additional points on the graph on each side of any *x*-intercept.



y
15
9
(−1, 6)
−3
(−2, 0)
(0, 0)
(2, 0)
3
x
−9
−15
−21

Using a Graphing Utility to Analyze the Graph of a Polynomial Function

Step 1: Determine the end behavior of the graph of the function.

Step 2: Graph the function using a graphing utility.

Step 3: Use a graphing utility to approximate the x- and y-intercepts of the graph.

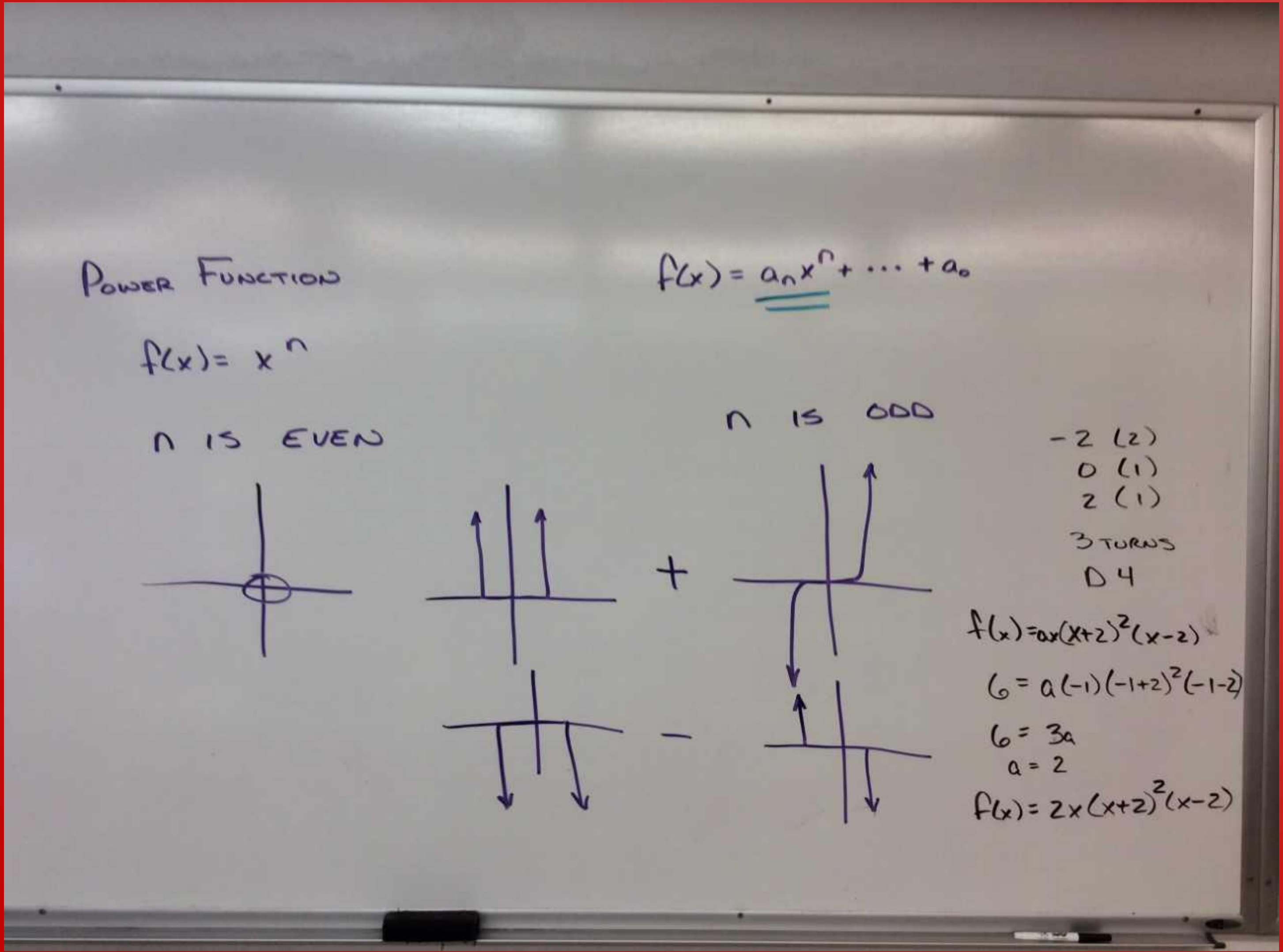
Step 4: Use a graphing utility to create a TABLE to find points on the graph around each x-intercept.

Step 5: Approximate the turning points of the graph.

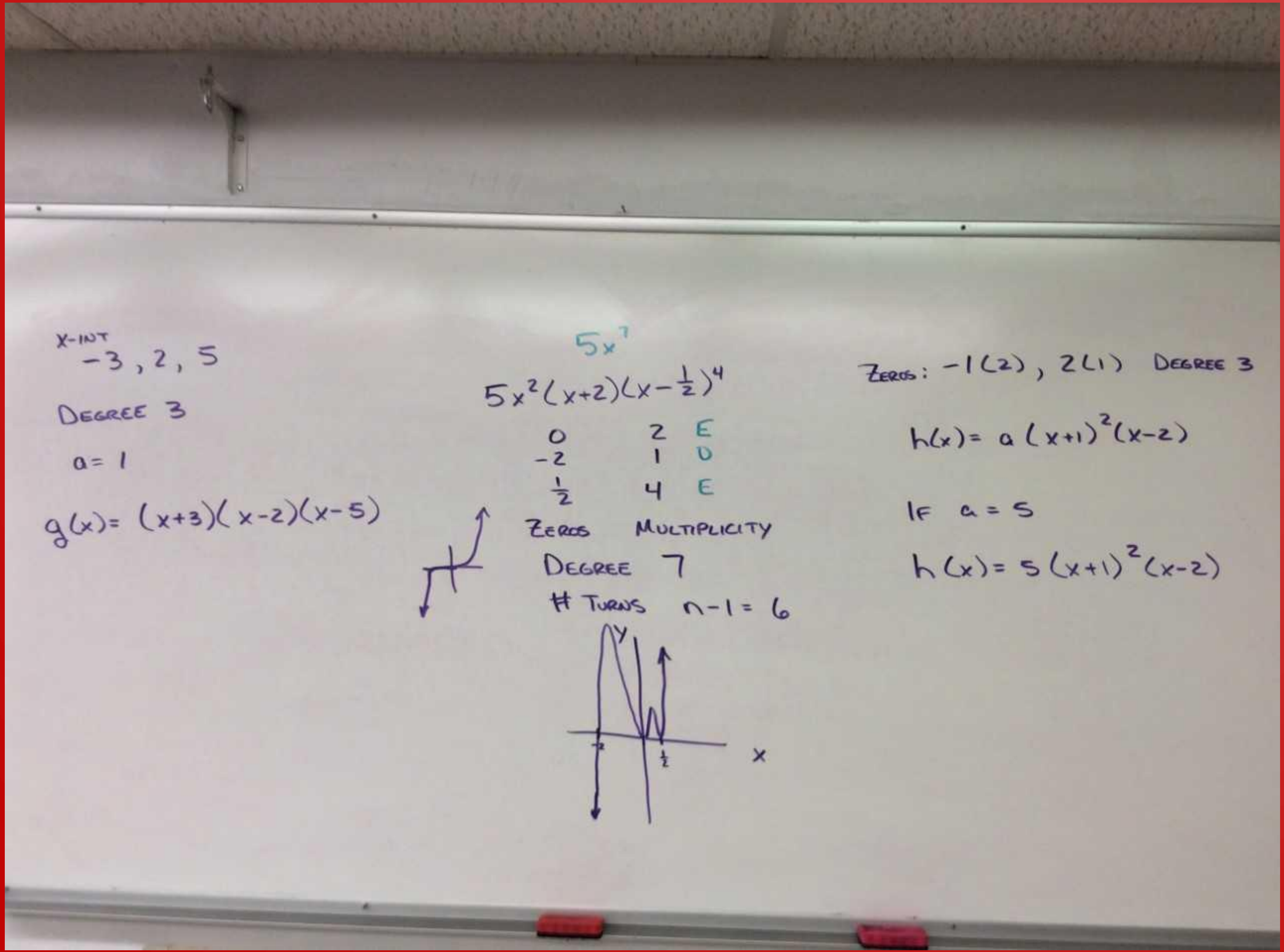
Step 6: Use the information in Steps 1 through 5 to draw a complete graph of the function by hand.

Step 7: Find the domain and the range of the function.

Step 8: Use the graph to determine where the function is increasing and where it is decreasing.



POWER FUNCTION
f(x) = anx^n + ... + a0
f(x) = x^n
n IS EVEN
n IS ODD
+
-
-2 (2)
0 (1)
2 (1)
3 TURNS
D 4
f(x)=ax(x+2)^2(x-2)
6 = a(-1)(-1+2)^2(-1-2)
6 = 3a
a = 2
f(x) = 2x(x+2)^2(x-2)


X-INT
-3, 2, 5
DEGREE 3
a = 1
g(x) = (x+3)(x-2)(x-5)
5x^7
5x^2(x+2)(x-1/2)^4
0 2 E
-2 1 O
1/2 4 E
ZEROS MULTIPLICITY
DEGREE 7
TURNS n-1 = 6
y
x
ZEROS: -1(2), 2(1) DEGREE 3
h(x) = a(x+1)^2(x-2)
IF a = 5
h(x) = 5(x+1)^2(x-2)