BOARD NOTES

9 OCTOBER 2018

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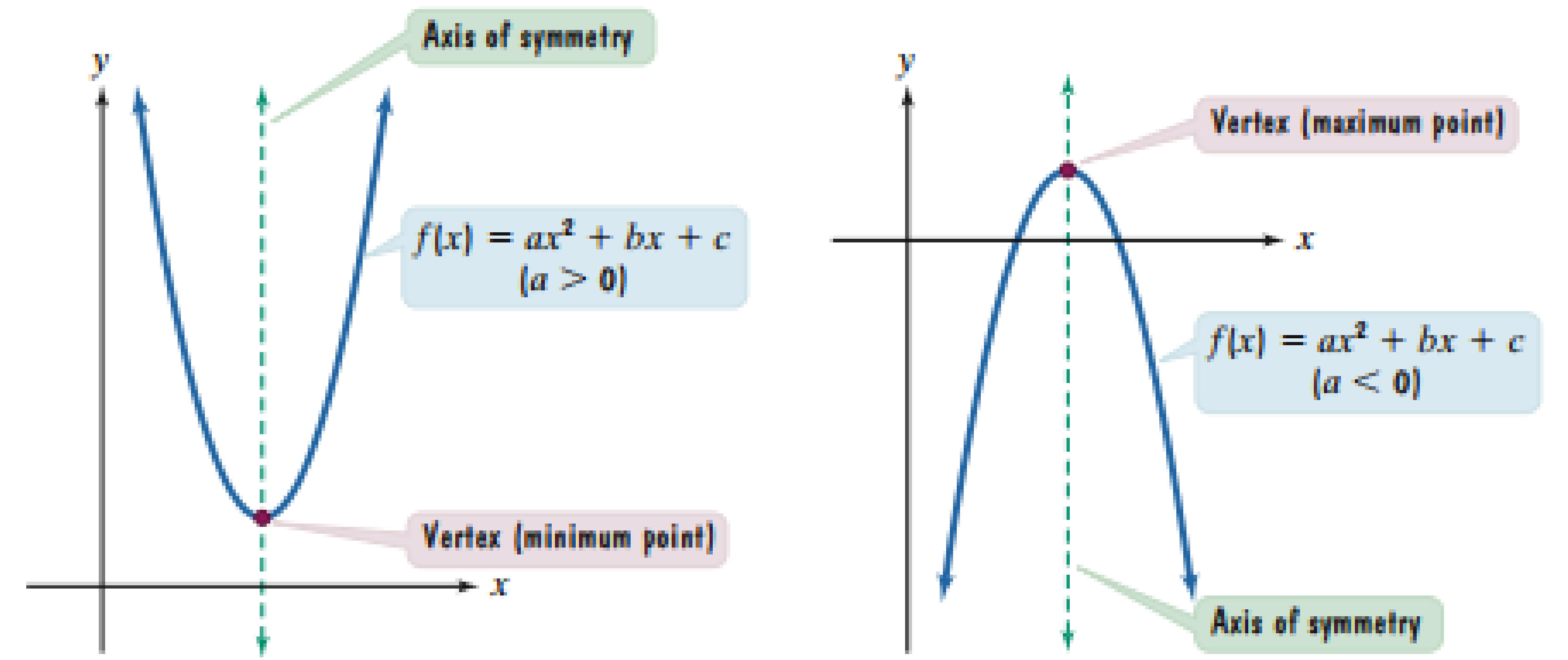
CHAPTER 3 –
POLYNOMIAL AND RATIONAL FUNCTIONS

- SECTION 3.1 - QUADRATIC FUNCTIONS

Objectives:

- Recognize the characteristics of parabolas
- Graph parabolas
- Determine a quadratic function's minimum or maximum value
- Solve problems involving a quadratic function's minimum or maximum value



$a > 0$: Parabola opens upward. $a < 0$: Parabola opens downward.

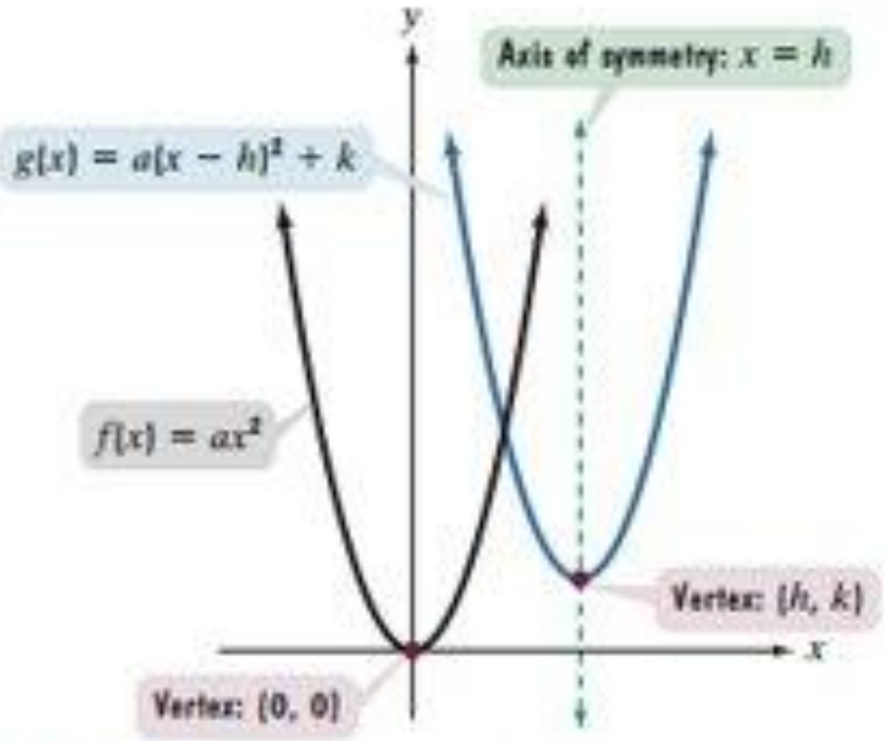


FIGURE 3.2(a) $a > 0$: Parabola opens upward.

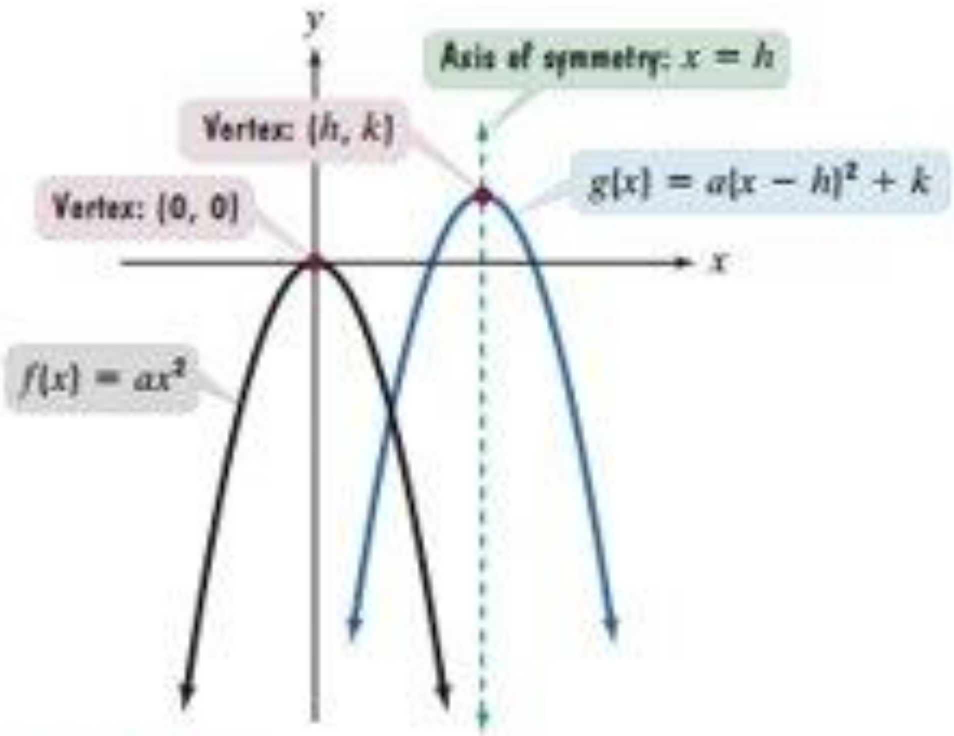


FIGURE 3.2(b) $a < 0$: Parabola opens downward.

The Standard Form of a Quadratic Function

The quadratic function

$$f(x) = a(x - h)^2 + k, \qquad a \neq 0$$

is in **standard form.** The graph of f is a parabola whose vertex is the point (h, k). The parabola is symmetric with respect to the line $x = h$. If $a > 0$, the parabola opens upward; if $a < 0$, the parabola opens downward.

The Vertex of a Parabola Whose Equation Is $f(x) = ax^2 + bx + c$

Consider the parabola defined by the quadratic function $f(x) = ax^2 + bx + c$. The parabola's vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$. The x-coordinate is $-\frac{b}{2a}$. The y-coordinate is found by substituting the x-coordinate into the parabola's equation and evaluating the function at this value of x.

Graphing Quadratic Functions with Equations in Standard Form

To graph $f(x) = a(x - h)^2 + k$,

1. Determine whether the parabola opens upward or downward. If $a > 0$, it opens upward. If $a < 0$, it opens downward.
2. Determine the vertex of the parabola. The vertex is (h, k).
3. Find any x-intercepts by solving $f(x) = 0$. The function's real zeros are the x-intercepts.
4. Find the y-intercept by computing $f(0)$.
5. Plot the intercepts, the vertex, and additional points as necessary. Connect these points with a smooth curve that is shaped like a bowl or an inverted bowl.

Graphing Quadratic Functions with Equations in the Form $f(x) = ax^2 + bx + c$

To graph $f(x) = ax^2 + bx + c$,

1. Determine whether the parabola opens upward or downward. If $a > 0$, it opens upward. If $a < 0$, it opens downward.
2. Determine the vertex of the parabola. The vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.
3. Find any x-intercepts by solving $f(x) = 0$. The real solutions of $ax^2 + bx + c = 0$ are the x-intercepts.
4. Find the y-intercept by computing $f(0)$. Because $f(0) = c$ (the constant term in the function's equation), the y-intercept is c and the parabola passes through $(0, c)$.
5. Plot the intercepts, the vertex, and additional points as necessary. Connect these points with a smooth curve.

Minimum and Maximum: Quadratic Functions

Consider the quadratic function $f(x) = ax^2 + bx + c$.

1. If $a > 0$, then f has a minimum that occurs at $x = -\frac{b}{2a}$. This minimum value is $f\left(-\frac{b}{2a}\right)$.
2. If $a < 0$, then f has a maximum that occurs at $x = -\frac{b}{2a}$. This maximum value is $f\left(-\frac{b}{2a}\right)$.

In each case, the value of x gives the location of the minimum or maximum value. The value of y, or $f\left(-\frac{b}{2a}\right)$, gives that minimum or maximum value.

EXAMPLE 5 The Parabolic Path of a Punted Football

Figure 3.7 shows that when a football is kicked, the nearest defensive player is 6 feet from the point of impact with the kicker's foot. The height of the punted football, $f(x)$, in feet, can be modeled by

$$f(x) = -0.01x^2 + 1.18x + 2,$$

where x is the ball's horizontal distance, in feet, from the point of impact with the kicker's foot.

a. What is the maximum height of the punt and how far from the point of impact does this occur?

b. How far must the nearest defensive player, who is 6 feet from the kicker's point of impact, reach to block the punt?

c. If the ball is not blocked by the defensive player, how far down the field will it go before hitting the ground?

d. Graph the function that models the football's parabolic path.

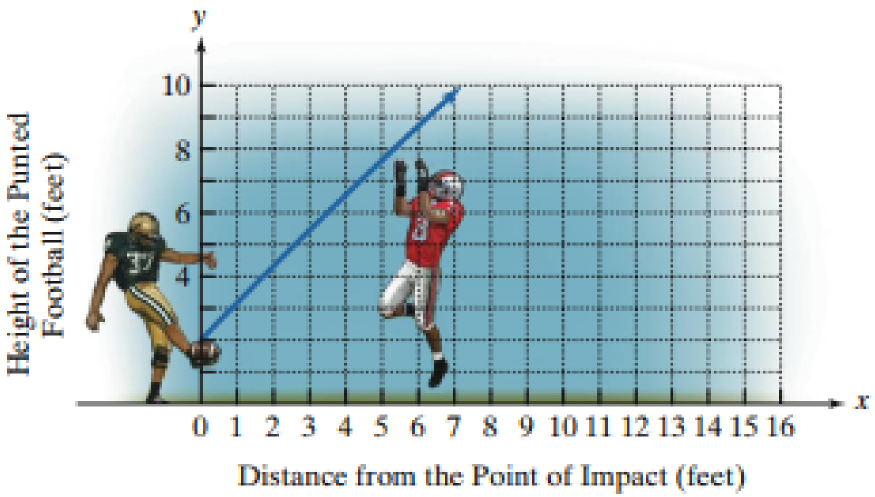
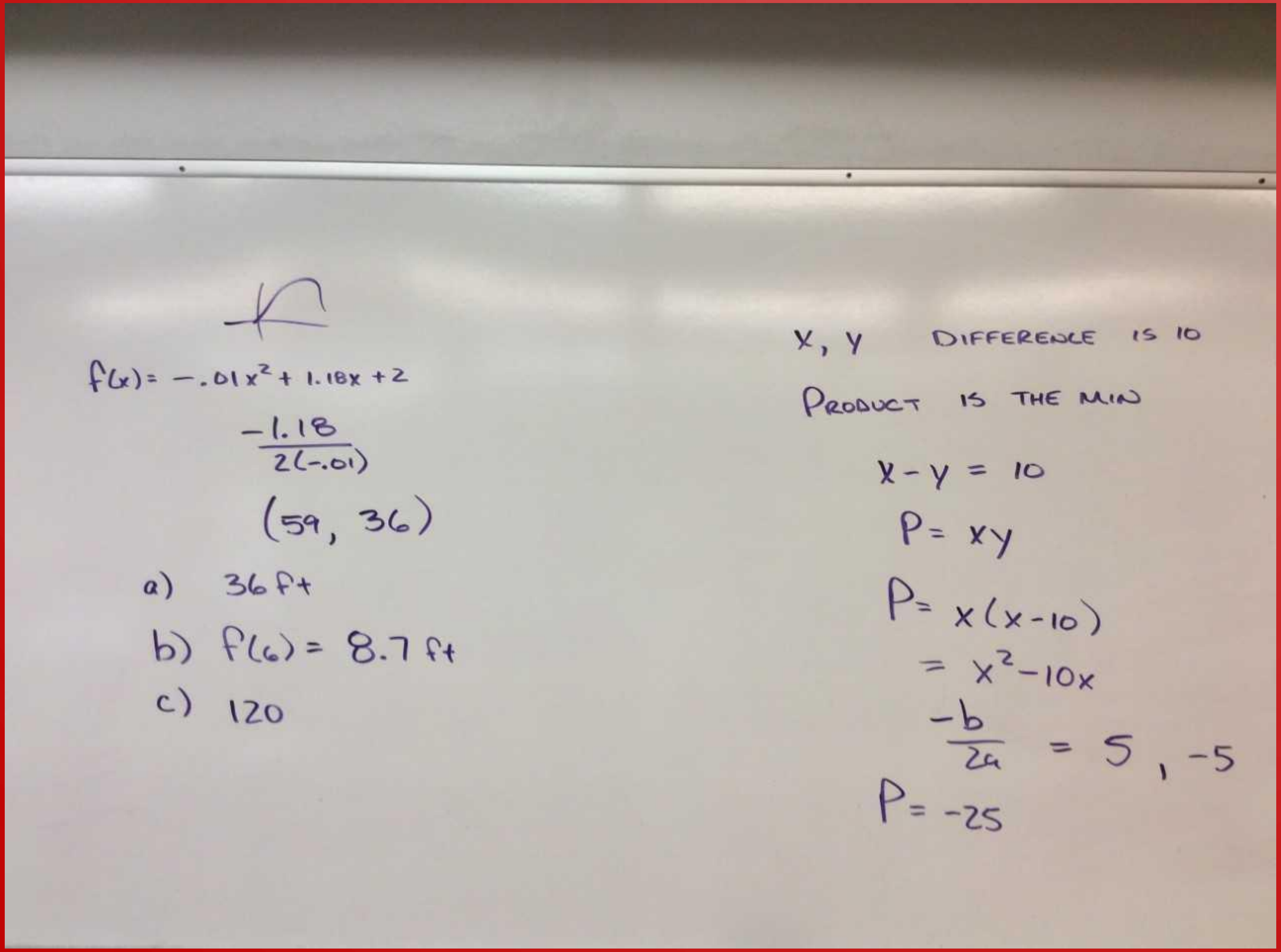


FIGURE 3.7


f(x) = -.01x² + 1.18x + 2
-1.18 / 2(-.01)
(59, 36)
a) 36 ft
b) f(6) = 8.7 ft
c) 120
x, y DIFFERENCE IS 10
PRODUCT IS THE MIN
x - y = 10
P = xy
P = x(x-10)
= x² - 10x
-b / 2a = 5, -5
P = -25

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CHAPTER 3 –
POLYNOMIAL AND RATIONAL FUNCTIONS

- SECTION 3.2 - POLYNOMIAL FUNCTIONS AND THEIR GRAPHS

Objectives:

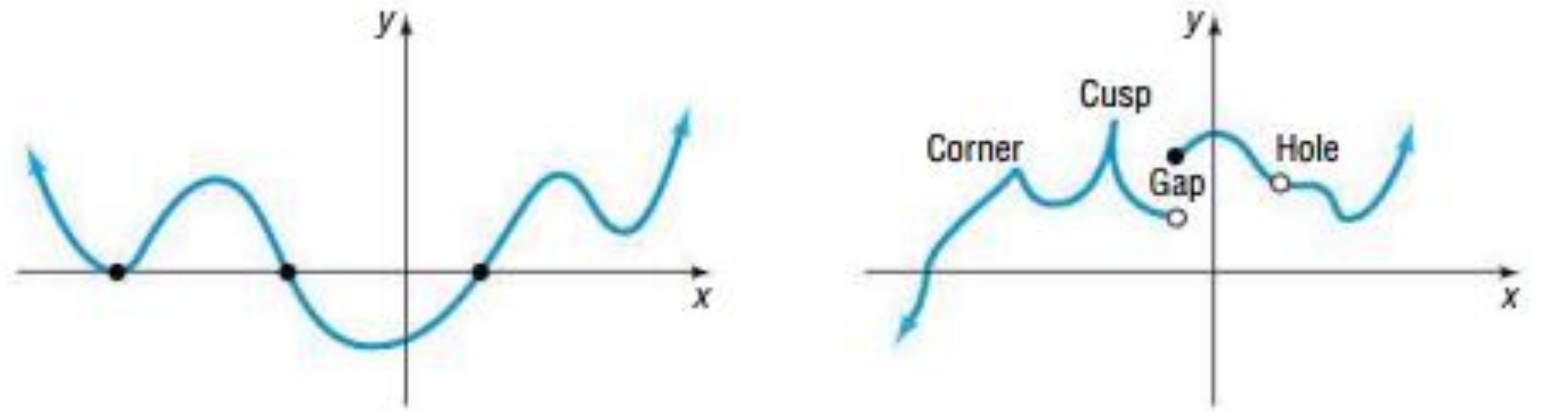
- Identitfy polynomial functions
- Recognize characteristics of polynomial functions
- Determine the end behavior
- Use factoring to find zeros of polynomial functions
- Identitfy zeros and their multiplicities
- Use the Intermediate Value Theorem
- Understand the relationship between degree and turning points
- Graph polynomial functions

A **polynomial function** in one variable is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \qquad \textbf{(1)}$$

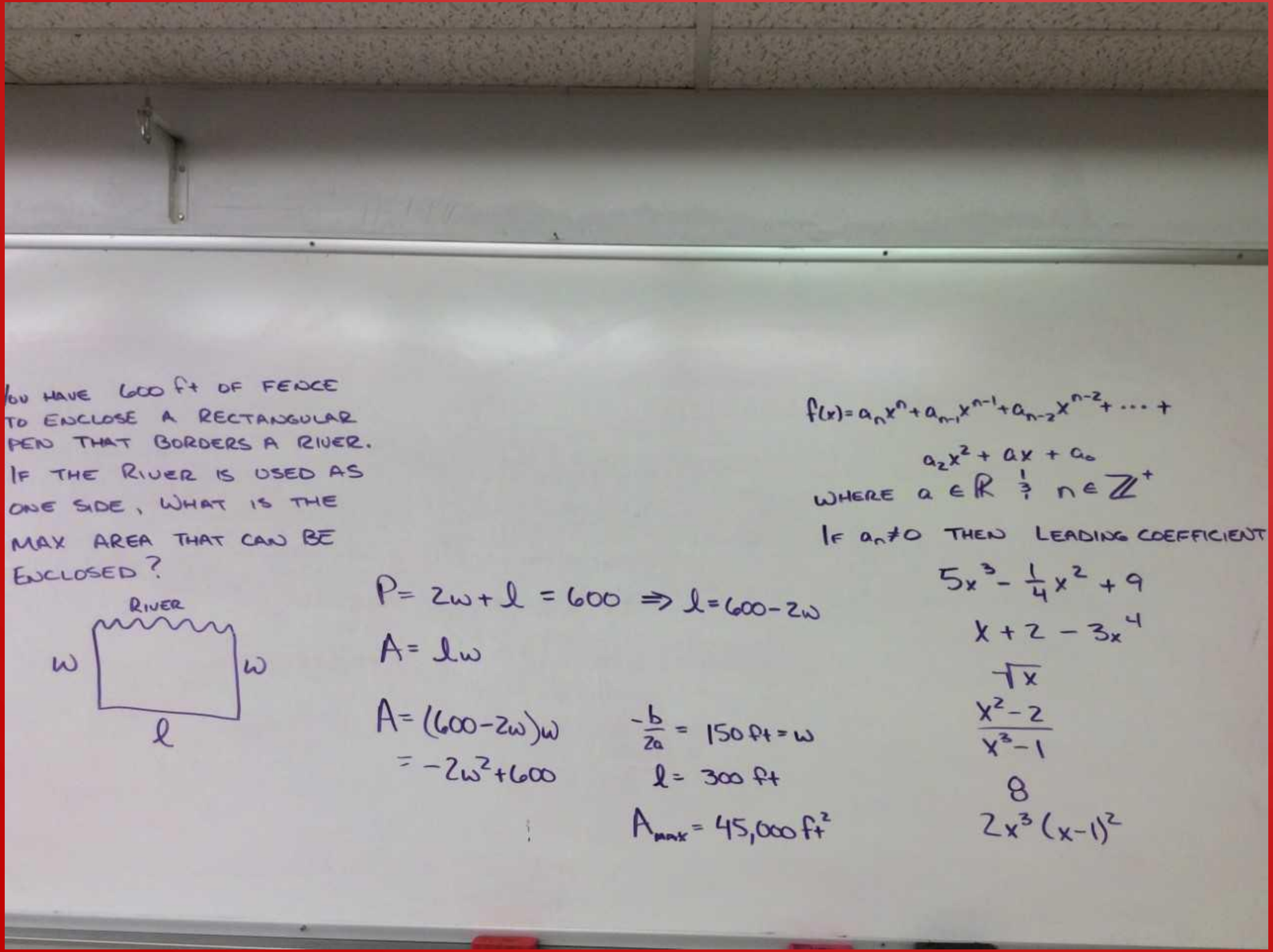
where $a_n, a_{n-1}, \ldots, a_1, a_0$ are constants, called the **coefficients** of the polynomial, $n \geq 0$ is an integer, and x is the variable. If $a_n \neq 0$, it is called the **leading coefficient**, and n is the **degree** of the polynomial.

The domain of a polynomial function is the set of all real numbers.



(a) Graph of a polynomial function: smooth, continuous

(b) Cannot be the graph of a polynomial function


You have 600 ft of fence to enclose a rectangular pen that borders a river. If the river is used as one side, what is the max area that can be enclosed?
River
w
w
l
P = 2w + l = 600 ⇒ l = 600 − 2w
A = lw
A = (600 − 2w)w
= −2w² + 600
−b/2a = 150 ft = w
l = 300 ft
A_max = 45,000 ft²
f(x) = aₙxⁿ + aₙ₋₁xⁿ⁻¹ + aₙ₋₂xⁿ⁻² + … +
a₂x² + ax + a₀
where a ∈ ℝ ; n ∈ ℤ⁺
If aₙ ≠ 0 then leading coefficient
5x³ − ¼x² + 9
x + 2 − 3x⁴
√x
(x² − 2)/(x³ − 1)
8
2x³(x − 1)²

If f is a function and r is a real number for which $f(r) = 0$, then r is called a **real zero** of f.

1. r is a real zero of a polynomial function f.
2. r is an x-intercept of the graph of f.
3. $x - r$ is a factor of f.
4. r is a solution to the equation $f(x) = 0$.

If $(x - r)^m$ is a factor of a polynomial f and $(x - r)^{m+1}$ is not a factor of f, then r is called a **zero of multiplicity m of f**.*

If r Is a Zero of Even Multiplicity

Numerically: The sign of $f(x)$ does not change from one side to the other side of r.

Graphically: The graph of f **touches** the x-axis at r.

If r Is a Zero of Odd Multiplicity

Numerically: The sign of $f(x)$ changes from one side to the other side of r.

Graphically: The graph of f **crosses** the x-axis at r.

Graphing a Polynomial Function

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0,\ a_n \neq 0$$

1. Use the Leading Coefficient Test to determine the graph's end behavior.
2. Find x-intercepts by setting $f(x) = 0$ and solving the resulting polynomial equation. If there is an x-intercept at r as a result of $(x - r)^k$ in the complete factorization of $f(x)$, then
   - **a.** If k is even, the graph touches the x-axis at r and turns around.
   - **b.** If k is odd, the graph crosses the x-axis at r.
   - **c.** If $k > 1$, the graph flattens out near $(r, 0)$.
3. Find the y-intercept by computing $f(0)$.
4. Use symmetry, if applicable, to help draw the graph:
   - **a.** y-axis symmetry: $f(-x) = f(x)$
   - **b.** Origin symmetry: $f(-x) = -f(x)$.
5. Use the fact that the maximum number of turning points of the graph is $n - 1$, where n is the degree of the polynomial function, to check whether it is drawn correctly.