BOARD NOTES

9 OCTOBER 2018

CC ALGEBRA
CHAPTER 3 –
POLYNOMIAL AND RATIONAL FUNCTIONS

- SECTION 3.2 - POLYNOMIAL FUNCTIONS AND THEIR GRAPHS

Objectives:

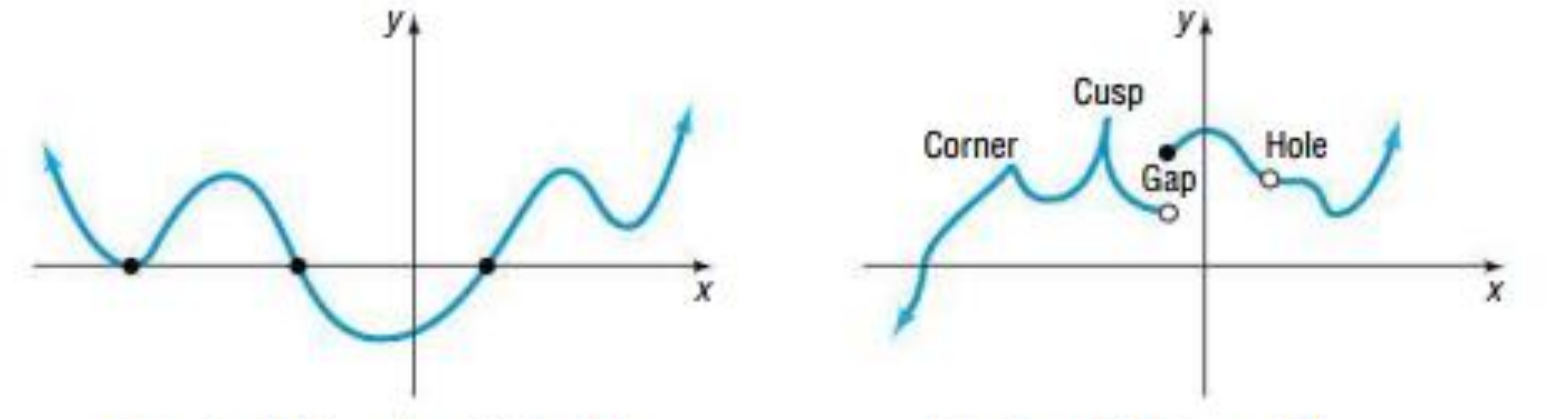
- Identitfy polynomial functions
- Recognize characteristics of polynomial functions
- Determine the end behavior
- Use factoring to find zeros of polynomial functions
- Identitfy zeros and their multiplicities
- Use the Intermediate Value Theorem
- Understand the relationship between degree and turning points
- Graph polynomial functions

A **polynomial function** in one variable is a function of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \qquad (1)$$

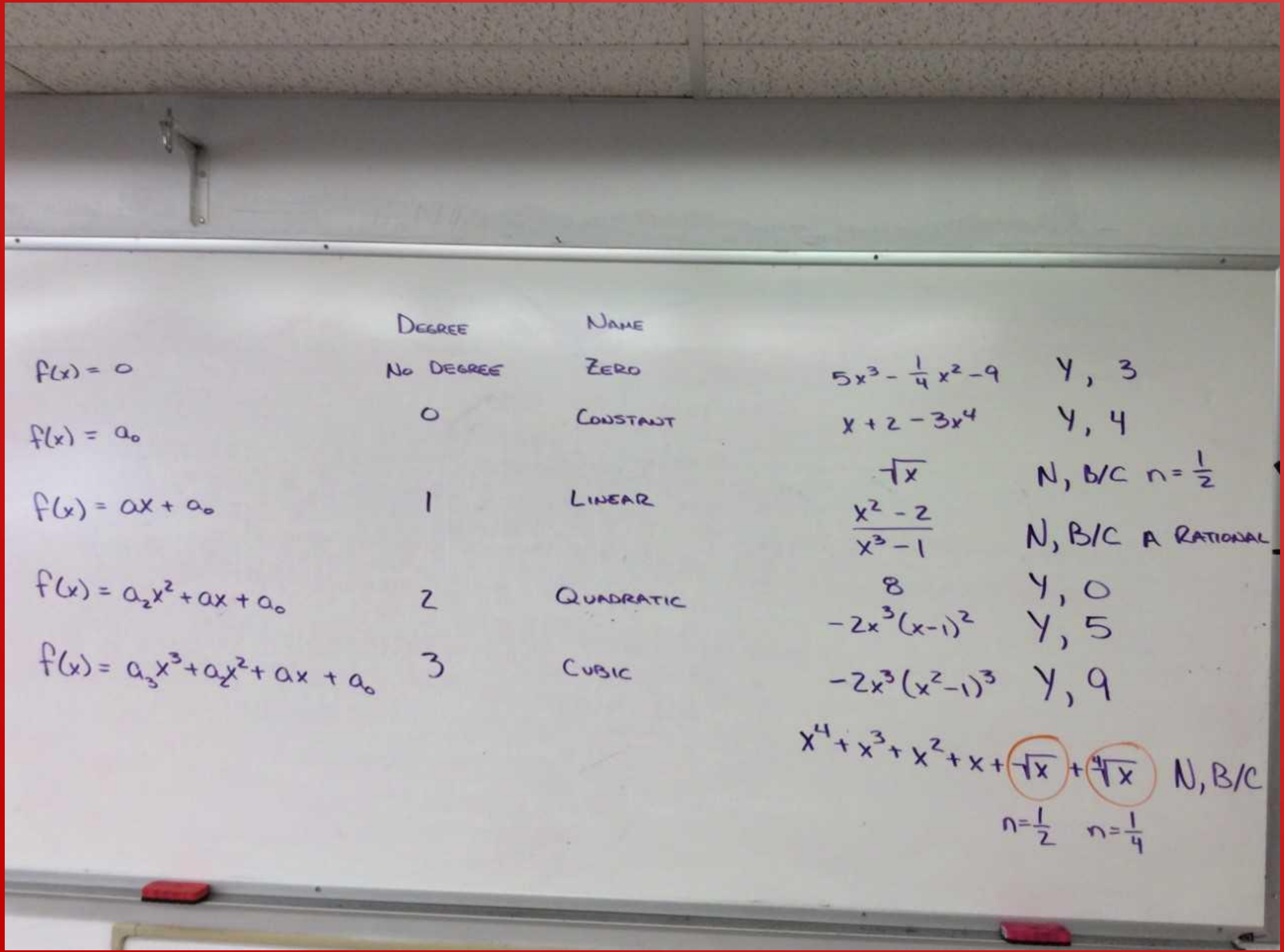
where $a_n, a_{n-1}, \ldots, a_1, a_0$ are constants, called the **coefficients** of the polynomial, $n \geq 0$ is an integer, and x is the variable. If $a_n \neq 0$, it is called the **leading coefficient**, and n is the **degree** of the polynomial.

The domain of a polynomial function is the set of all real numbers.



(a) Graph of a polynomial function: smooth, continuous

(b) Cannot be the graph of a polynomial function


DEGREE
NAME
f(x) = 0
NO DEGREE
ZERO
f(x) = a₀
0
CONSTANT
f(x) = ax + a₀
1
LINEAR
f(x) = a₂x² + ax + a₀
2
QUADRATIC
f(x) = a₃x³ + a₂x² + ax + a₀
3
CUBIC
5x³ - 1/4 x² - 9
Y, 3
x + 2 - 3x⁴
Y, 4
√x
N, B/C n = 1/2
(x² - 2)/(x³ - 1)
N, B/C A RATIONAL
8
Y, 0
-2x³(x-1)²
Y, 5
-2x³(x²-1)³
Y, 9
x⁴ + x³ + x² + x + √x + ⁴√x
N, B/C
n = 1/2
n = 1/4

If f is a function and r is a real number for which $f(r) = 0$, then r is called a **real zero** of f.

1. r is a real zero of a polynomial function f.
2. r is an x-intercept of the graph of f.
3. $x - r$ is a factor of f.
4. r is a solution to the equation $f(x) = 0$.

If $(x - r)^m$ is a factor of a polynomial f and $(x - r)^{m+1}$ is not a factor of f, then r is called a **zero of multiplicity *m* of *f*.***

If *r* Is a Zero of Even Multiplicity

Numerically: The sign of $f(x)$ does not change from one side to the other side of r.

Graphically: The graph of f **touches** the x-axis at r.

If *r* Is a Zero of Odd Multiplicity

Numerically: The sign of $f(x)$ changes from one side to the other side of r.

Graphically: The graph of f **crosses** the x-axis at r.

FRANK PHILLIPS COLLEGE Plainsmen

BORGER INDEPENDENT SCHOOL DISTRICT 1926

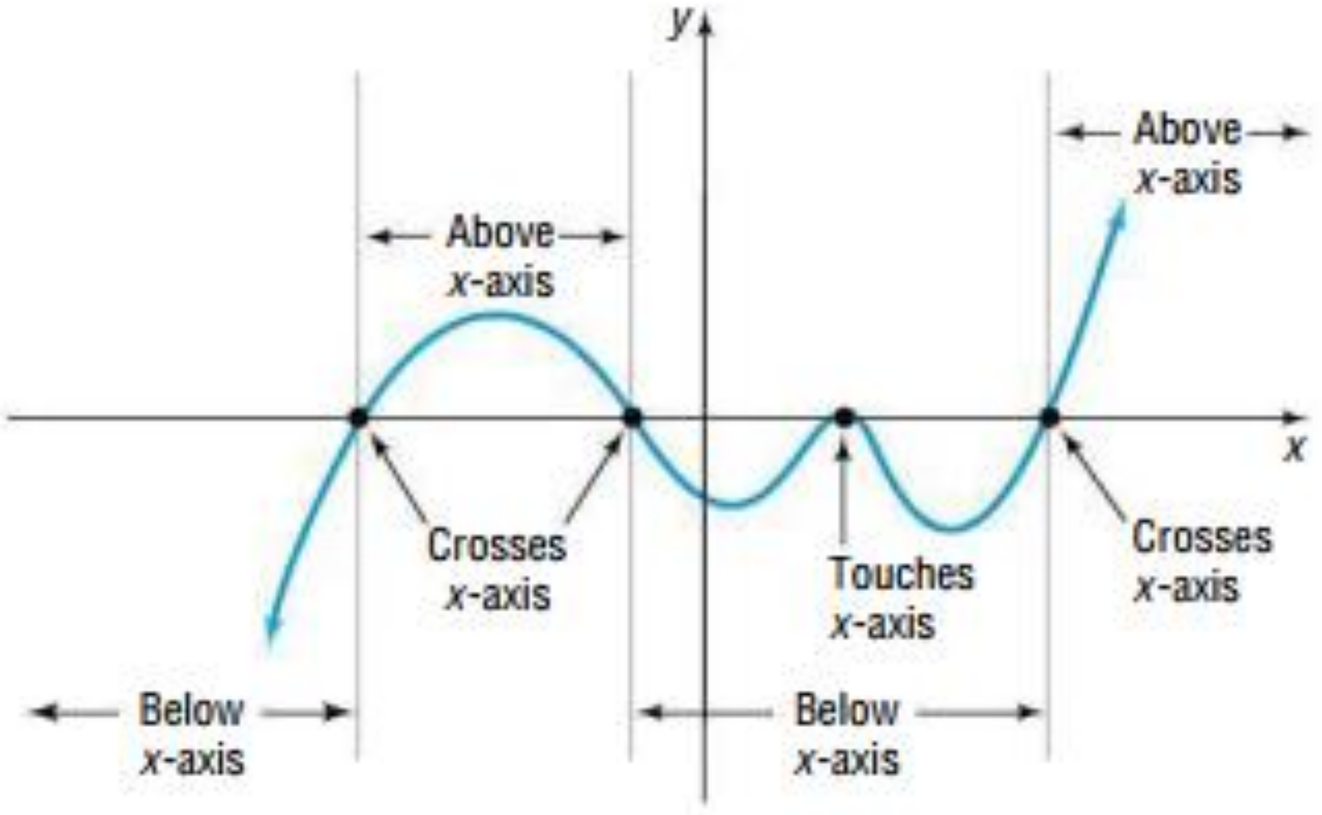


Figure 9 Graph of a polynomial function

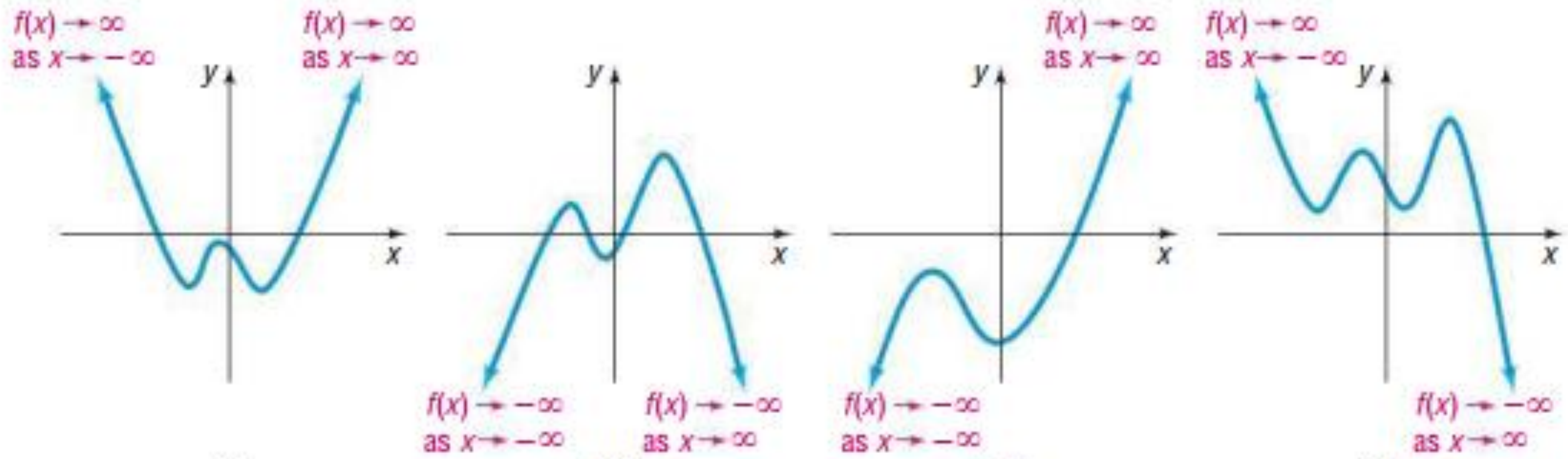
End Behavior

For large values of x, either positive or negative, the graph of the polynomial function

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \quad a_n \neq 0$$

resembles the graph of the power function

$$y = a_n x^n$$



(a) $n \geq 2$ even; $a_n > 0$

(b) $n \geq 2$ even; $a_n < 0$

(c) $n \geq 3$ odd; $a_n > 0$

(d) $n \geq 3$ odd; $a_n < 0$

Graphing a Polynomial Function

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0,\ a_n \neq 0$$

1. Use the Leading Coefficient Test to determine the graph's end behavior.
2. Find x-intercepts by setting $f(x) = 0$ and solving the resulting polynomial equation. If there is an x-intercept at r as a result of $(x - r)^k$ in the complete factorization of $f(x)$, then
   a. If k is even, the graph touches the x-axis at r and turns around.
   b. If k is odd, the graph crosses the x-axis at r.
   c. If $k > 1$, the graph flattens out near $(r, 0)$.
3. Find the y-intercept by computing $f(0)$.
4. Use symmetry, if applicable, to help draw the graph:
   a. y-axis symmetry: $f(-x) = f(x)$
   b. Origin symmetry: $f(-x) = -f(x)$.
5. Use the fact that the maximum number of turning points of the graph is $n - 1$, where n is the degree of the polynomial function, to check whether it is drawn correctly.