BOARD NOTES

9 OCTOBER 2018

BORGER INDEPENDENT SCHOOL DISTRICT

1926

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CHAPTER 3 –
POLYNOMIAL AND RATIONAL FUNCTIONS

- SECTION 3.2 - POLYNOMIAL FUNCTIONS AND THEIR GRAPHS

Objectives:

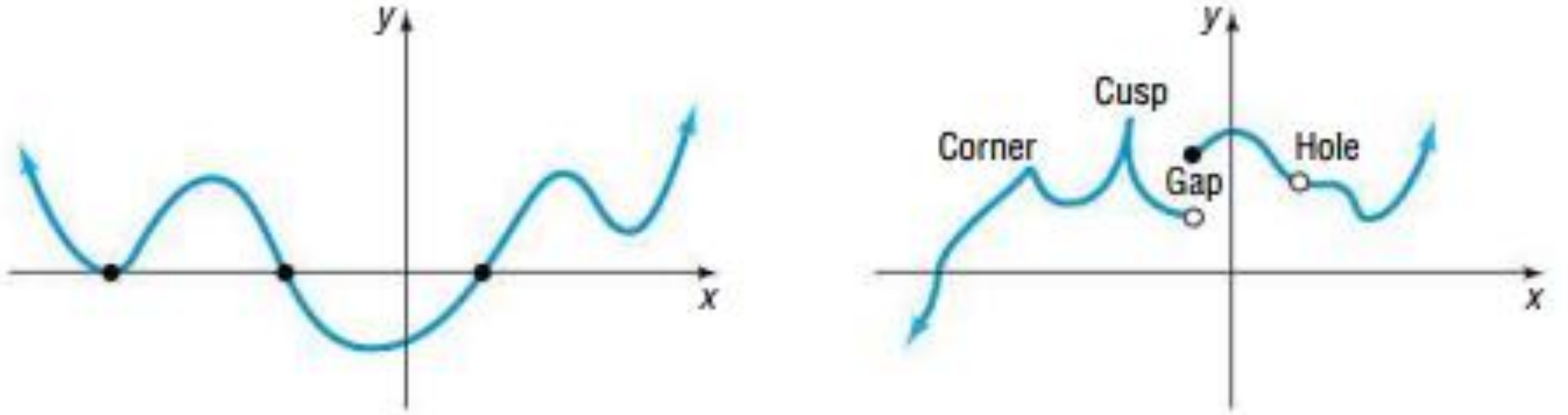
- Identitfy polynomial functions
- Recognize characteristics of polynomial functions
- Determine the end behavior
- Use factoring to find zeros of polynomial functions
- Identitfy zeros and their multiplicities
- Use the Intermediate Value Theorem
- Understand the relationship between degree and turning points
- Graph polynomial functions

A **polynomial function** in one variable is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \qquad \textbf{(1)}$$

where $a_n, a_{n-1}, \ldots, a_1, a_0$ are constants, called the **coefficients** of the polynomial, $n \geq 0$ is an integer, and x is the variable. If $a_n \neq 0$, it is called the **leading coefficient**, and n is the **degree** of the polynomial.

The domain of a polynomial function is the set of all real numbers.



(a) Graph of a polynomial function: smooth, continuous

(b) Cannot be the graph of a polynomial function

If f is a function and r is a real number for which $f(r) = 0$, then r is called a **real zero** of f.

1. r is a real zero of a polynomial function f.
2. r is an x-intercept of the graph of f.
3. $x - r$ is a factor of f.
4. r is a solution to the equation $f(x) = 0$.

If $(x - r)^m$ is a factor of a polynomial f and $(x - r)^{m+1}$ is not a factor of f, then r is called a **zero of multiplicity m of f**.*

If r Is a Zero of Even Multiplicity

Numerically: The sign of $f(x)$ does not change from one side to the other side of r.

Graphically: The graph of f **touches** the x-axis at r.

If r Is a Zero of Odd Multiplicity

Numerically: The sign of $f(x)$ changes from one side to the other side of r.

Graphically: The graph of f **crosses** the x-axis at r.

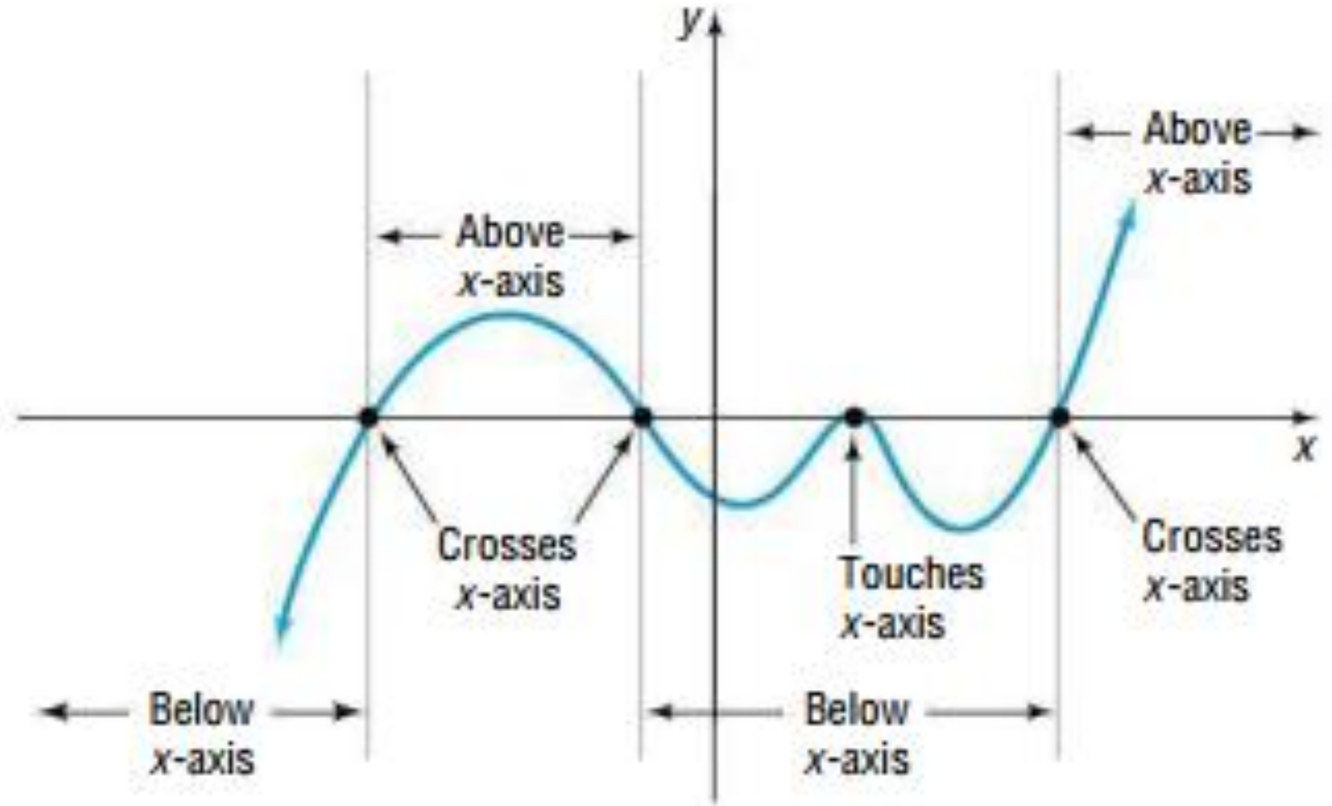


Figure 9 Graph of a polynomial function

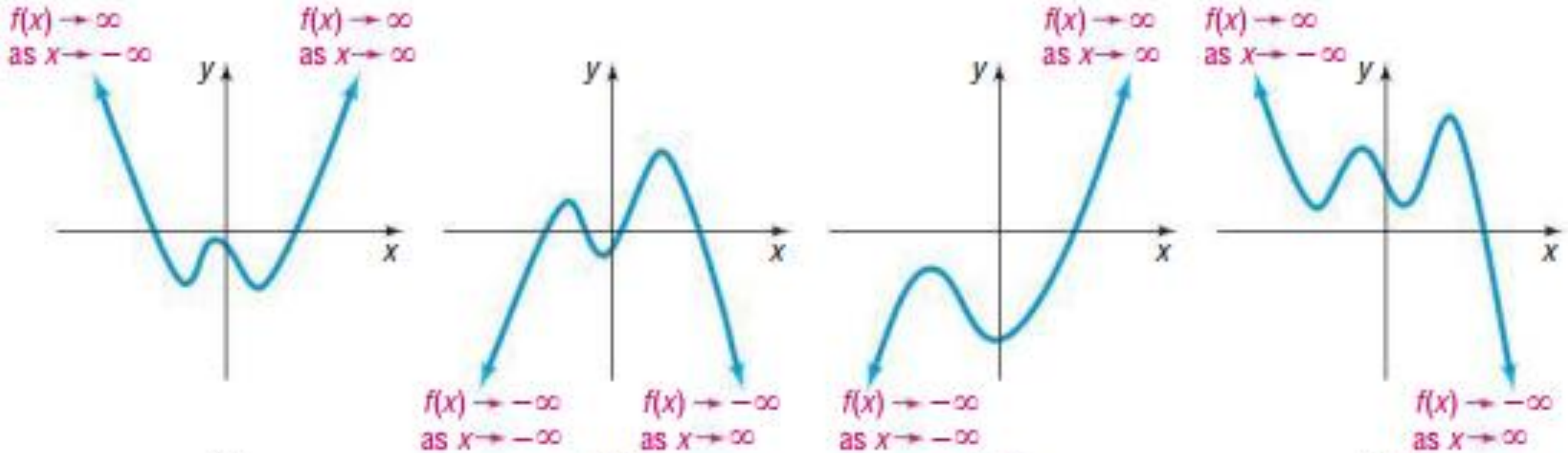
End Behavior

For large values of x, either positive or negative, the graph of the polynomial function

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \quad a_n \neq 0$$

resembles the graph of the power function

$$y = a_n x^n$$

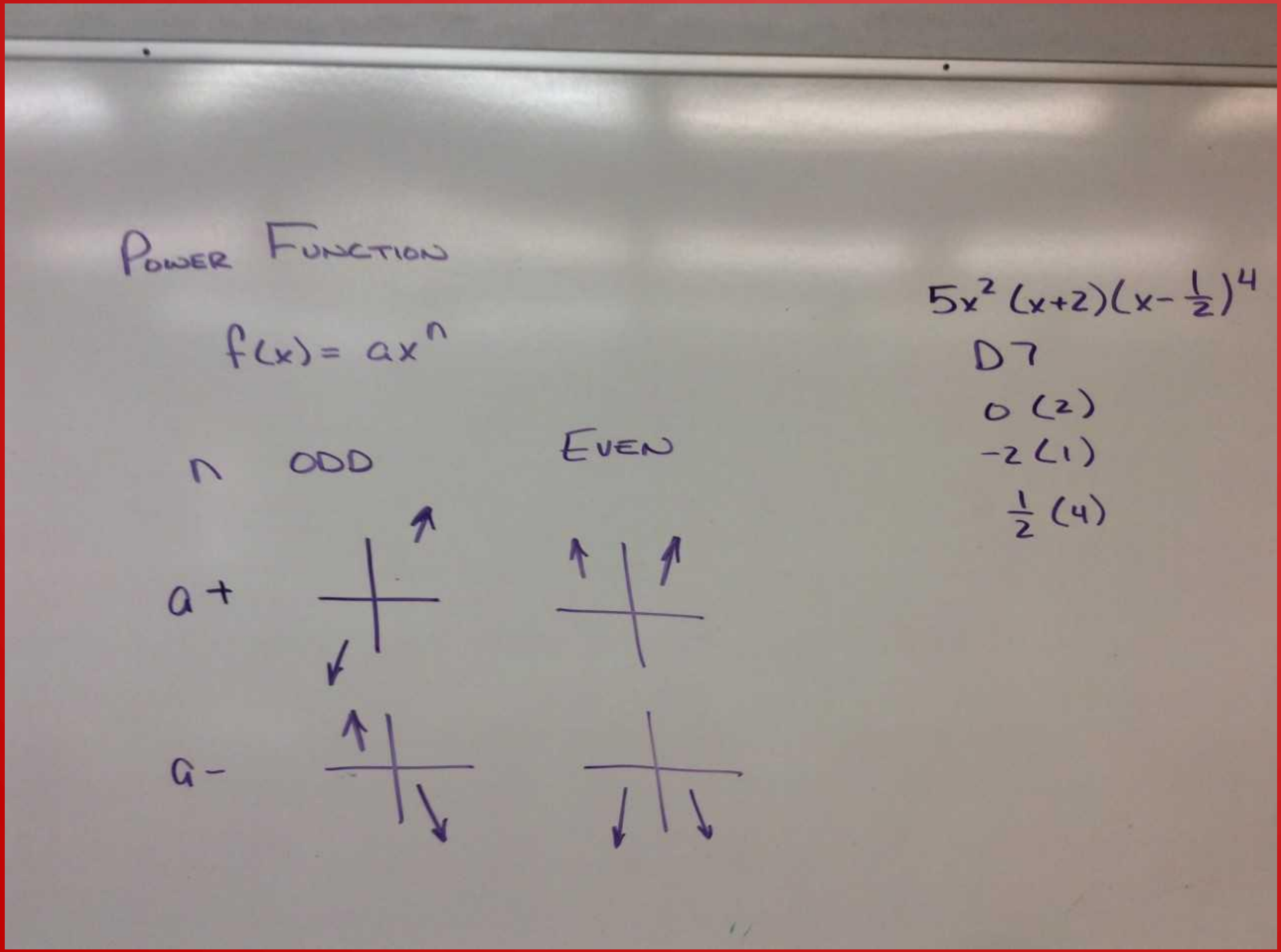


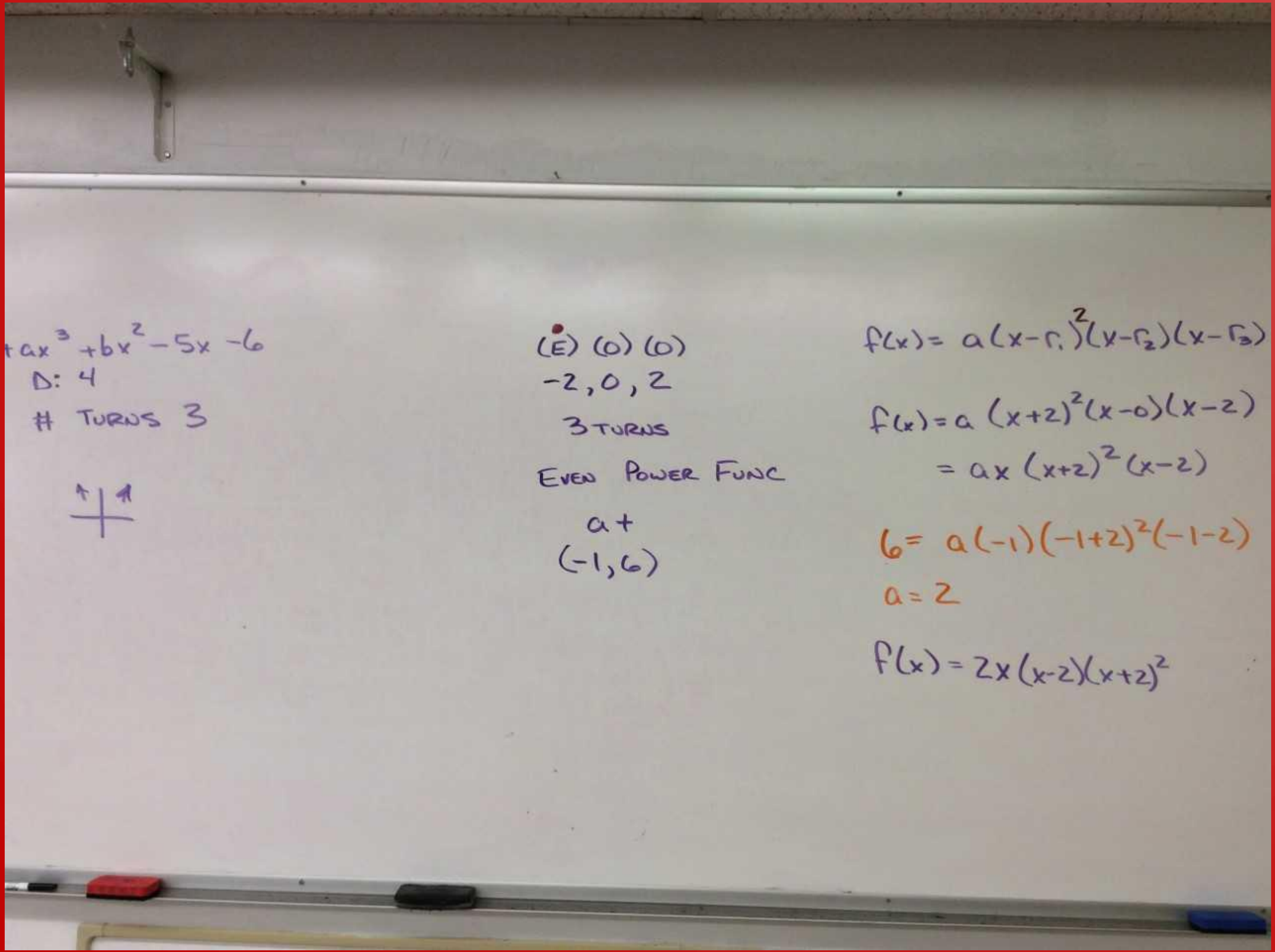
(a) $n \geq 2$ even; $a_n > 0$

(b) $n \geq 2$ even; $a_n < 0$

(c) $n \geq 3$ odd; $a_n > 0$

(d) $n \geq 3$ odd; $a_n < 0$


Power Function
f(x)= ax^n
n ODD EVEN
a+
a-
5x^2 (x+2)(x-1/2)^4
D7
0 (2)
-2 (1)
1/2 (4)



FRANK PHILLIPS COLLEGE Plainsmen

BORGER INDEPENDENT SCHOOL DISTRICT 1926

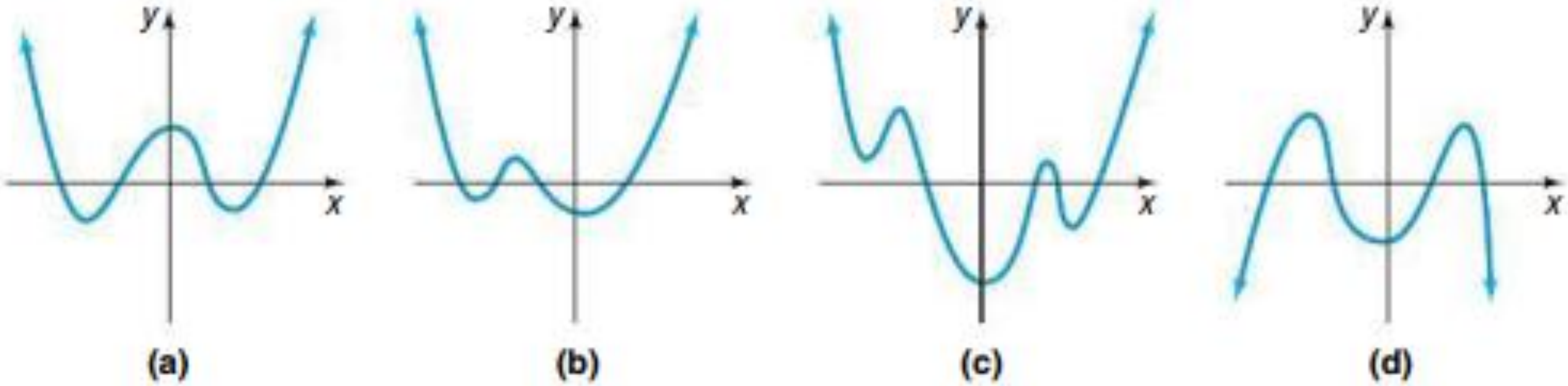
Graphing a Polynomial Function

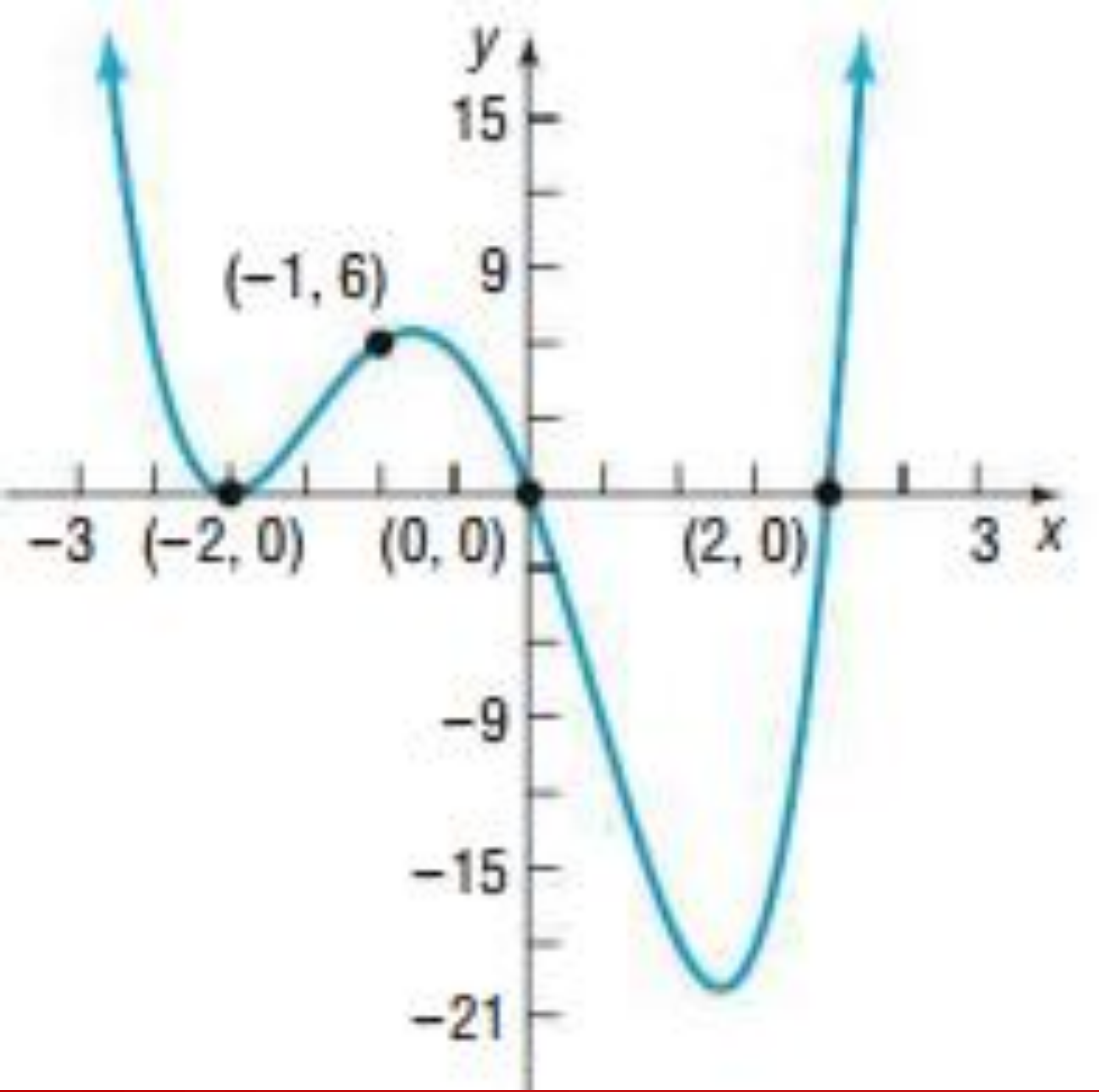
$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0, a_n \neq 0$$

1. Use the Leading Coefficient Test to determine the graph's end behavior.
2. Find x-intercepts by setting $f(x) = 0$ and solving the resulting polynomial equation. If there is an x-intercept at r as a result of $(x - r)^k$ in the complete factorization of $f(x)$, then
   - **a.** If k is even, the graph touches the x-axis at r and turns around.
   - **b.** If k is odd, the graph crosses the x-axis at r.
   - **c.** If $k > 1$, the graph flattens out near $(r, 0)$.
3. Find the y-intercept by computing $f(0)$.
4. Use symmetry, if applicable, to help draw the graph:
   - **a.** y-axis symmetry: $f(-x) = f(x)$
   - **b.** Origin symmetry: $f(-x) = -f(x)$.
5. Use the fact that the maximum number of turning points of the graph is $n - 1$, where n is the degree of the polynomial function, to check whether it is drawn correctly.

$$f(x) = x^4 + ax^3 + bx^2 - 5x - 6$$

where $a > 0, b > 0$?




y
15
9
(−1, 6)
−3
(−2, 0)
(0, 0)
(2, 0)
3
x
−9
−15
−21

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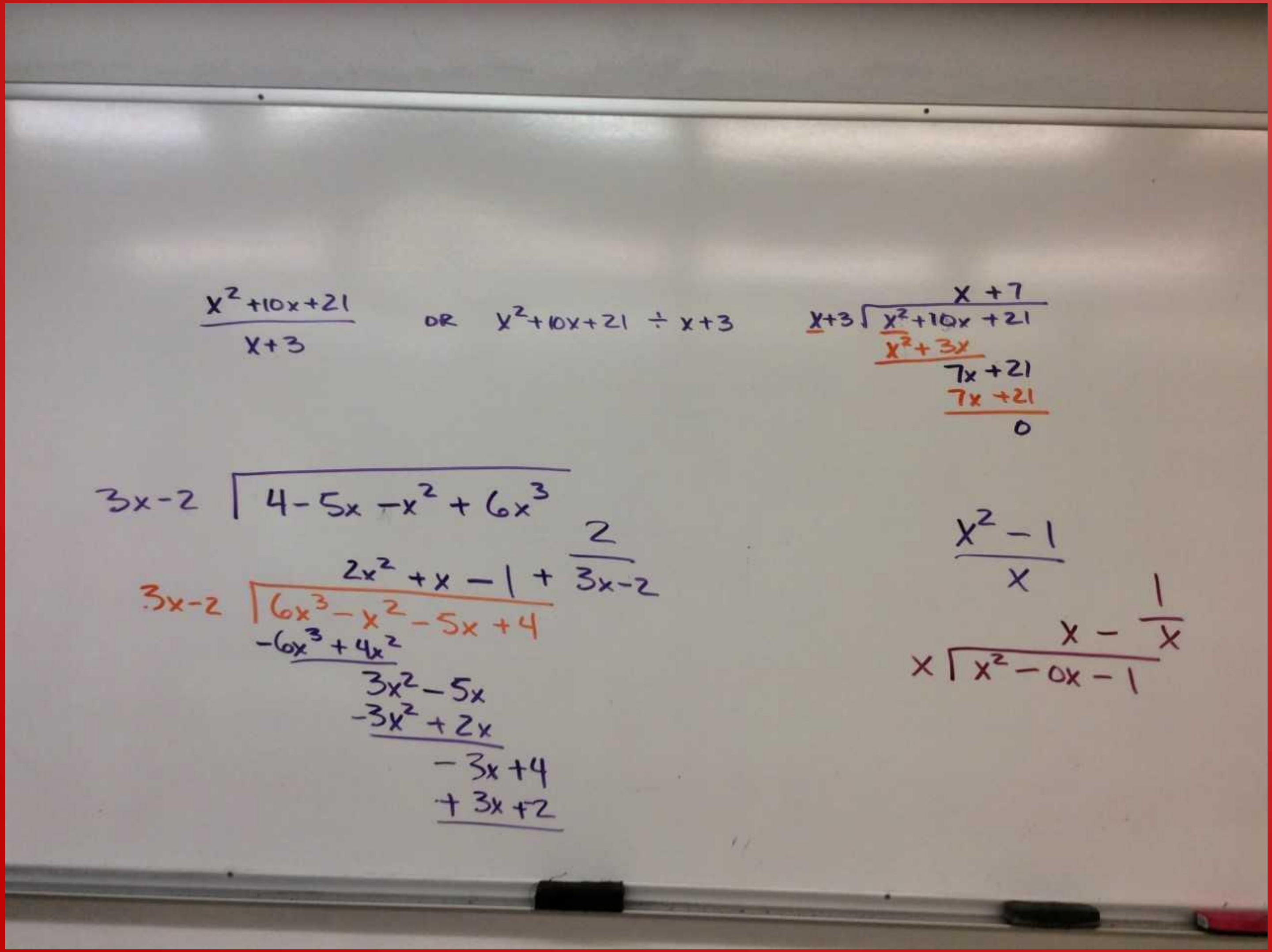
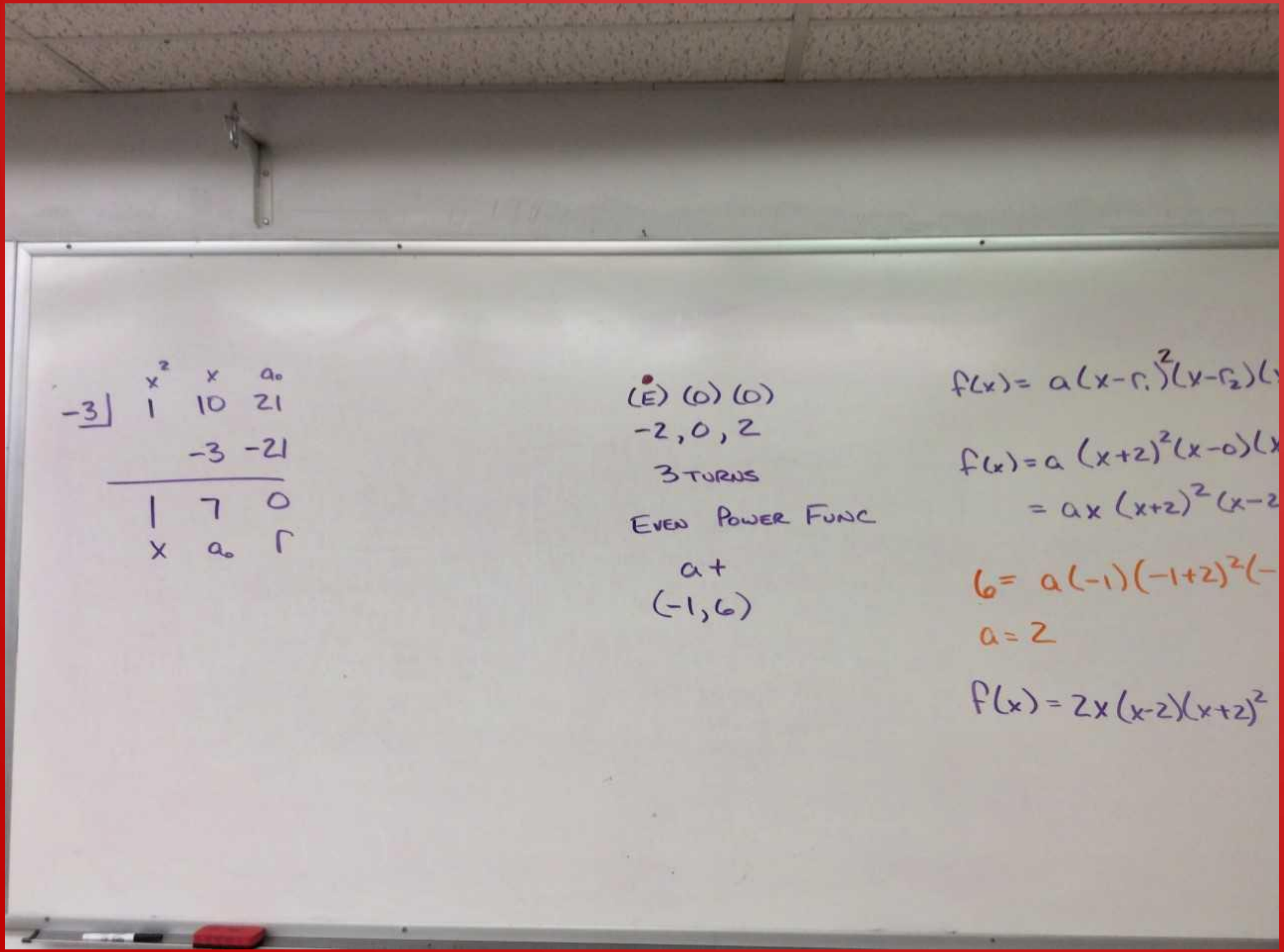
CHAPTER 3 –
POLYNOMIAL AND RATIONAL FUNCTIONS

- SECTION 3.3 - DIVIDING POLYNOLIALS; REMAINDER AND FACTOR THEOREM

Objectives:

- Use long division to divide polynomials
- Use synthetic division to divide polynomials
- Evaluate a polynomial using the remainder theorem
- Use the factor theorem to solve a polynomial equation


x²+10x+21 / x+3
OR
x²+10x+21 ÷ x+3
x +7
x+3 | x²+10x +21
x²+3x
7x +21
7x +21
0
3x-2 | 4-5x -x² + 6x³
2x² + x − 1 + 2/(3x-2)
3x-2 | 6x³ - x² - 5x + 4
-6x³ + 4x²
3x² - 5x
-3x² + 2x
- 3x +4
+ 3x +2
x² - 1 / x
x − 1/x
x | x² - 0x - 1


x² x a₀
-3 | 1 10 21
-3 -21
1 7 0
x a₀ r
(E) (O) (O)
-2, 0, 2
3 TURNS
EVEN POWER FUNC
a+
(-1, 6)
f(x) = a(x-r₁)²(x-r₂)(
f(x) = a(x+2)²(x-0)(x
= ax(x+2)²(x-2
6 = a(-1)(-1+2)²(-
a = 2
f(x) = 2x(x-2)(x+2)²