BOARD NOTES

9 OCTOBER 2018

CC PRECALCULUS
CHAPTER 4 –
POLYNOMIAL AND RATIONAL FUNCTIONS

- SECTION 4.2 - PROPERTIES OF RATIONAL FUNCTIONS

Objectives:

- Find the domain of a rational function
- Find the vertical asymptotes of a rational function
- Find the horizontal or oblique (slant) asymptote of a rational function

A **rational function** is a function of the form

$$R(x) = \frac{p(x)}{q(x)}$$

where p and q are polynomial functions and q is not the zero polynomial. The domain of a rational function is the set of all real numbers except those for which the denominator q is 0.

Let R denote a function.

If, as $x \to -\infty$ or as $x \to \infty$, the values of $R(x)$ approach some fixed number L, then the line $y = L$ is a **horizontal asymptote** of the graph of R. [Refer to Figures 27(a) and (b) on page 192.]

If, as x approaches some number c, the values $|R(x)| \to \infty$ [that is, $R(x) \to -\infty$ or $R(x) \to \infty$], then the line $x = c$ is a **vertical asymptote** of the graph of R. [Refer to Figures 27(c) and (d).]

Locating Vertical Asymptotes

A rational function $R(x) = \frac{p(x)}{q(x)}$, *in lowest terms*, will have a vertical asymptote $x = r$ if r is a real zero of the *denominator* q. That is, if $x - r$ is a factor of the denominator q of a rational function $R(x) = \frac{p(x)}{q(x)}$, in lowest terms, R will have the vertical asymptote $x = r$.

Finding a Horizontal or Oblique Asymptote of a Rational Function

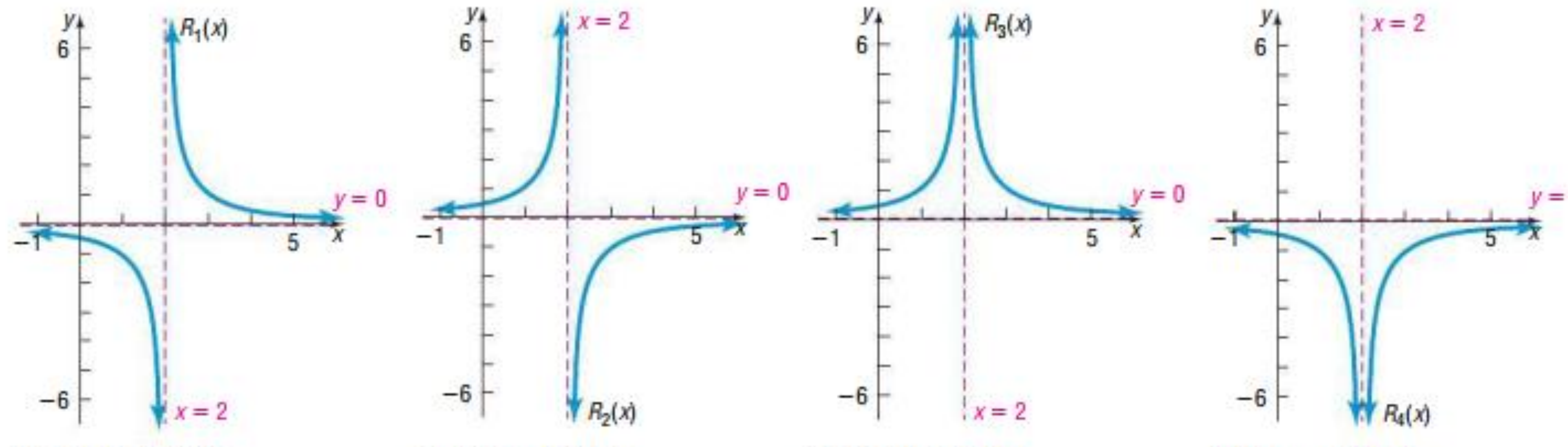
Consider the rational function

$$R(x) = \frac{p(x)}{q(x)} = \frac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_1 x + b_0}$$

in which the degree of the numerator is n and the degree of the denominator is m.

1. If $n < m$ (the degree of the numerator is less than the degree of the denominator), the line $y = 0$ is a horizontal asymptote.
2. If $n = m$ (the degree of the numerator equals the degree of the denominator), the line $y = \frac{a_n}{b_m}$ is a horizontal asymptote. (That is, the horizontal asymptote equals the ratio of the leading coefficients.)
3. If $n = m + 1$ (the degree of the numerator is one more than the degree of the denominator), the line $y = ax + b$ is an oblique asymptote, which is the quotient found using long division.
4. If $n \geq m + 2$ (the degree of the numerator is two or more greater than the degree of the denominator), there are no horizontal or oblique asymptotes. The end behavior of the graph will resemble the power function $y = \frac{a_n}{b_m}x^{n-m}$.

Note: A rational function will never have both a horizontal asymptote and an oblique asymptote. A rational function may have neither a horizontal nor an oblique asymptote.

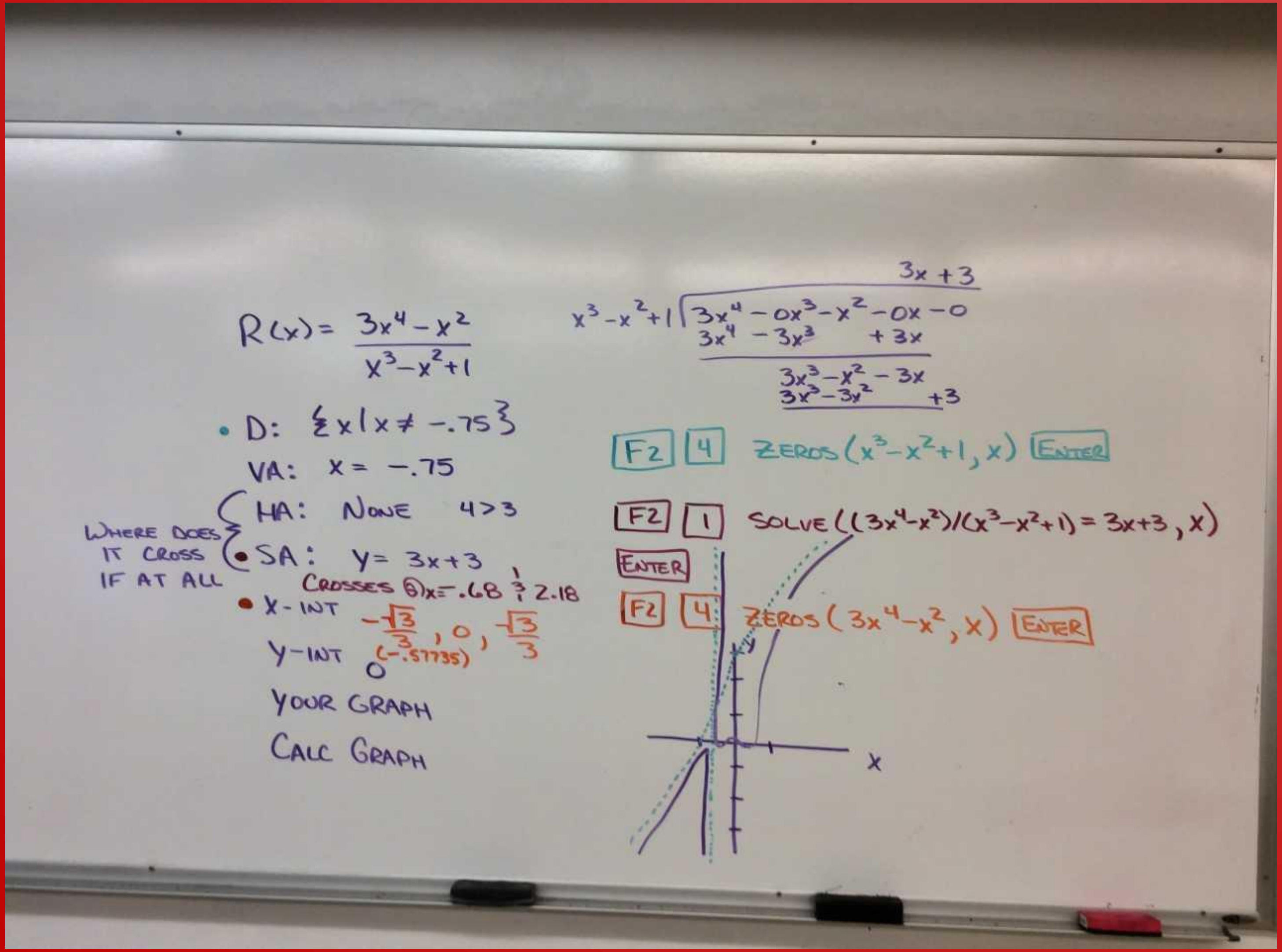


(a) Odd multiplicity
$\lim_{x \to 2^-} R_1(x) = -\infty$
$\lim_{x \to 2^+} R_1(x) = \infty$

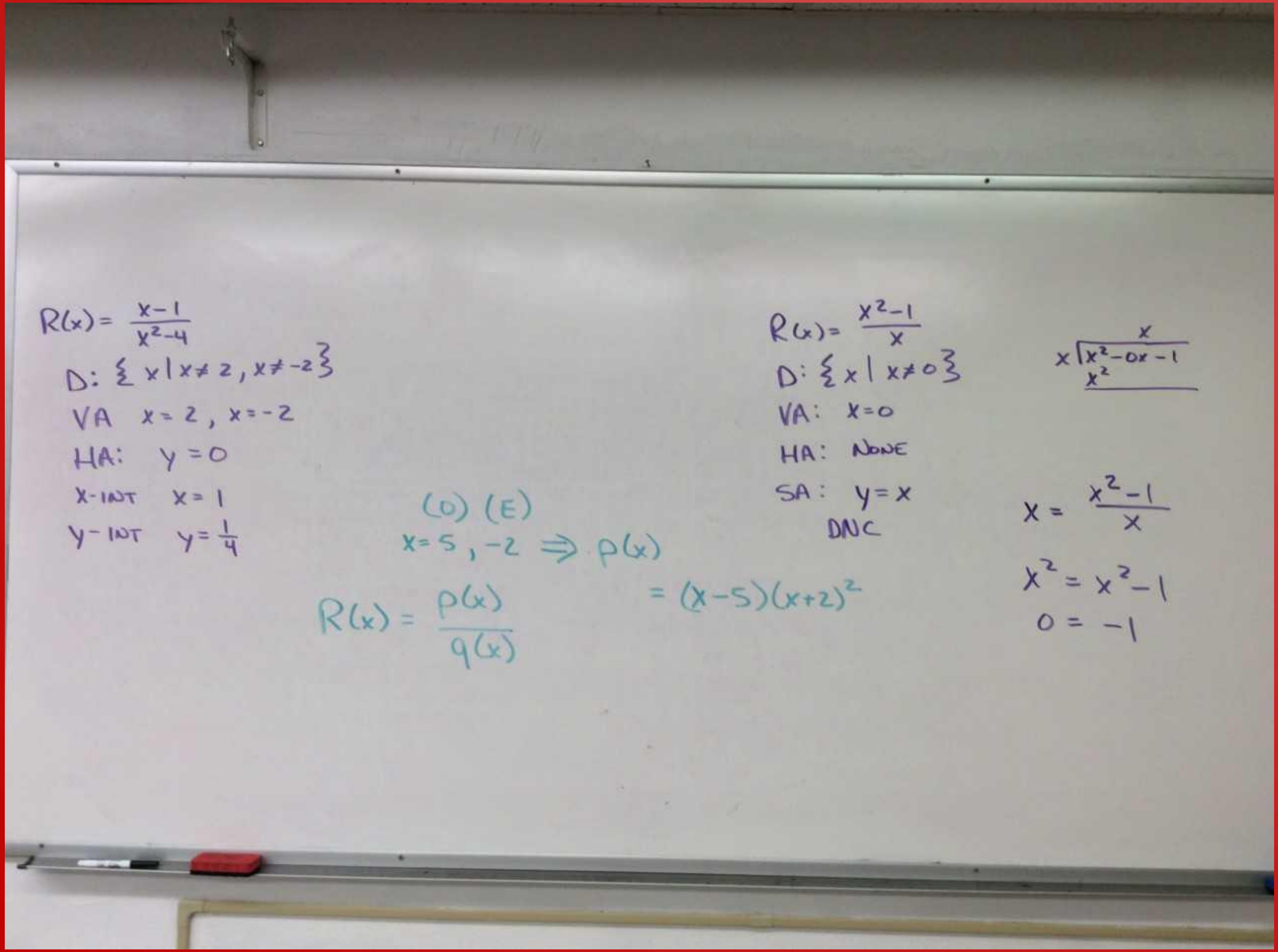
(b) Odd multiplicity
$\lim_{x \to 2^-} R_2(x) = \infty$
$\lim_{x \to 2^+} R_2(x) = -\infty$

(c) Even multiplicity
$\lim_{x \to 2^-} R_3(x) = \infty$
$\lim_{x \to 2^+} R_3(x) = \infty$

(d) Even multiplicity
$\lim_{x \to 2^-} R_4(x) = -\infty$
$\lim_{x \to 2^+} R_4(x) = -\infty$



R(x) = (3x^4 − x^2) / (x^3 − x^2 + 1)
• D: {x | x ≠ −.75}
VA: x = −.75
HA: None 4>3
Where does it cross if at all
• SA: y = 3x+3
Crosses @ x = −.68 ? 2.18
• X-INT −√3/3, 0, √3/3
(−.57735)
Y-INT 0
Your Graph
Calc Graph
3x + 3
x^3 − x^2 + 1 | 3x^4 − 0x^3 − x^2 − 0x − 0
3x^4 − 3x^3 + 3x
3x^3 − x^2 − 3x
3x^3 − 3x^2 +3
F2 4 Zeros(x^3 − x^2 + 1, x) Enter
F2 1 Solve((3x^4 − x^2)/(x^3 − x^2 + 1) = 3x+3, x)
Enter
F2 4 Zeros(3x^4 − x^2, x) Enter
x


R(x) = (x−1)/(x²−4)
D: {x | x ≠ 2, x ≠ −2}
VA x = 2, x = −2
HA: y = 0
X-INT x = 1
Y-INT y = 1/4
(O) (E)
x = 5, −2 ⇒ p(x)
R(x) = p(x)/q(x)
= (x−5)(x+2)²
R(x) = (x²−1)/x
D: {x | x ≠ 0}
VA: x = 0
HA: NONE
SA: y = x
DNC
x = (x²−1)/x
x² = x² − 1
0 = −1

CC PRECALCULUS CHAPTER 4 – POLYNOMIAL AND RATIONAL FUNCTIONS

- SECTION 4.3 - THE GRAPH OF A RATIONAL FUNCTION

Objectives:

- Analyze the graph of a rational function
- Solve applied problems involving rational functions

Analyzing the Graph of a Rational Function R

Step 1: Factor the numerator and denominator of R. Find the domain of the rational function.

Step 2: Write R in lowest terms.

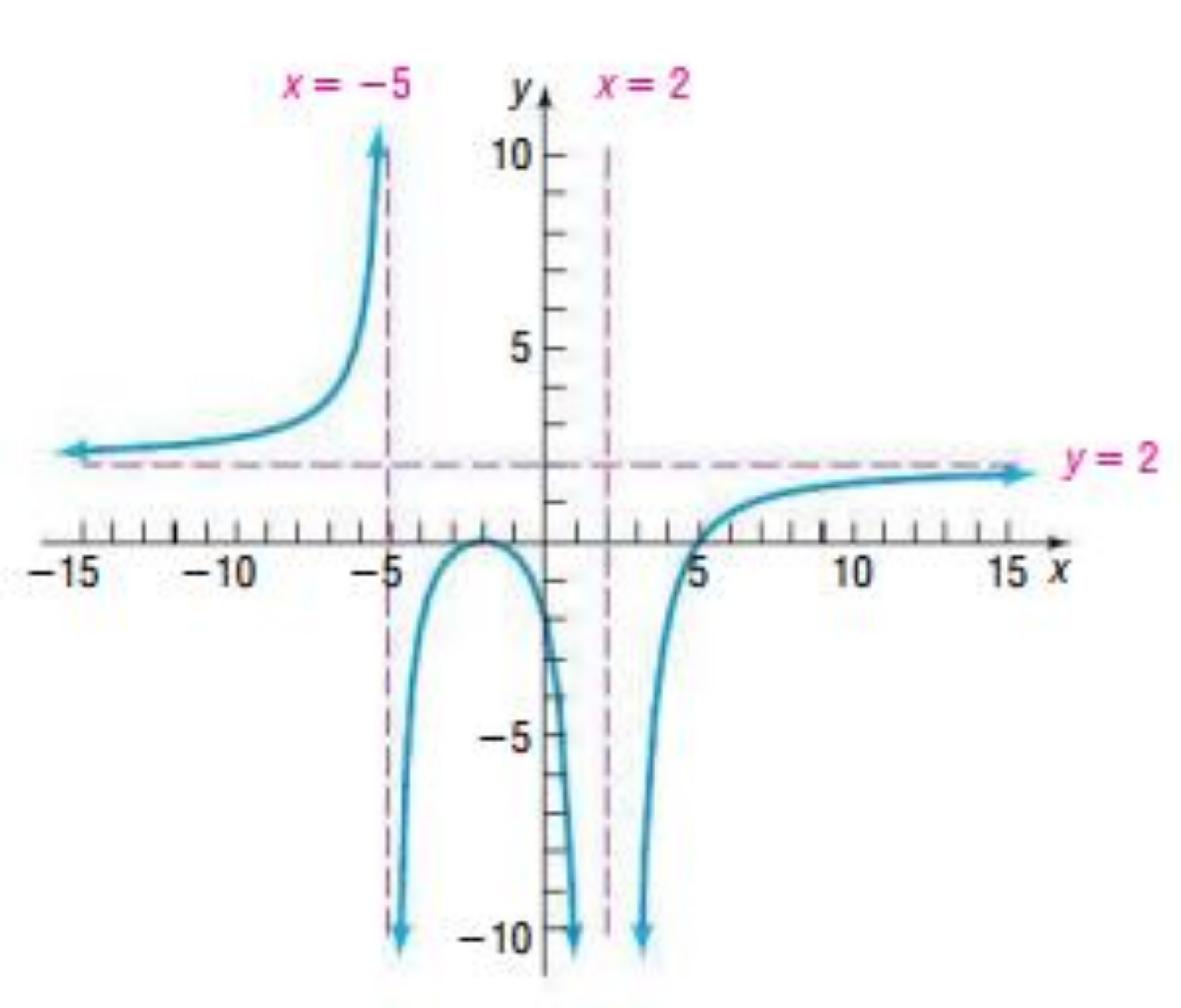
Step 3: Find and plot the intercepts of the graph. Use multiplicity to determine the behavior of the graph of R at each x-intercept.

Step 4: Find the vertical asymptotes. Graph each vertical asymptote using a dashed line. Determine the behavior of the graph of R on either side of each vertical asymptote.

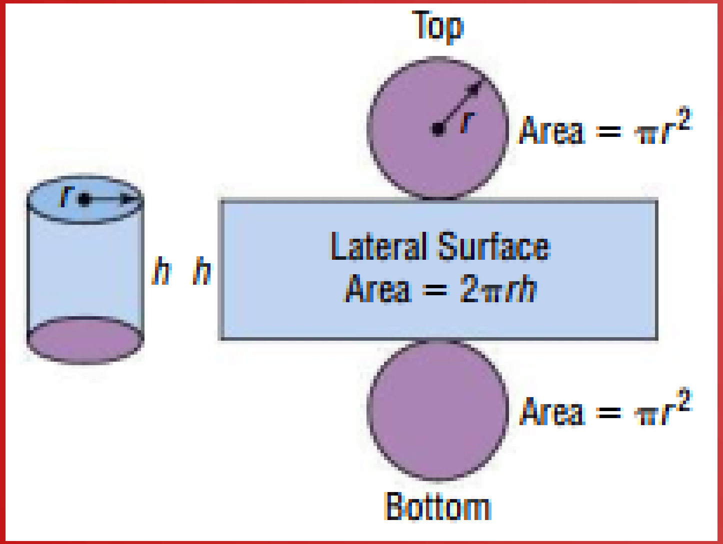
Step 5: Find the horizontal or oblique asymptote, if one exists. Find points, if any, at which the graph of R intersects this asymptote. Graph the asymptote using a dashed line. Plot any points at which the graph of R intersects the asymptote.

Step 6: Use the zeros of the numerator and denominator of R to divide the x-axis into intervals. Determine where the graph of R is above or below the x-axis by choosing a number in each interval and evaluating R there. Plot the points found.

Step 7: Use the results obtained in Steps 1 through 6 to graph R.



x = −5
y
x = 2
10
5
−5
−10
y = 2
−15
−10
−5
5
10
15
x



Finding the Least Cost of a Can

Reynolds Metal Company manufactures aluminum cans in the shape of a cylinder with a capacity of 500 cubic centimeters $\left(\frac{1}{2}\text{ liter}\right)$. The top and bottom of the can are made of a special aluminum alloy that costs 0.05¢ per square centimeter. The sides of the can are made of material that costs 0.02¢ per square centimeter.

(a) Express the cost of material for the can as a function of the radius r of the can.
(b) Use a graphing utility to graph the function $C = C(r)$.
(c) What value of r will result in the least cost?
(d) What is this least cost?