BOARD NOTES

16 OCTOBER 2018

CC ALGEBRA
CHAPTER 3 –
POLYNOMIAL AND RATIONAL FUNCTIONS

- SECTION 3.3 - DIVIDING POLYNOLIALS; REMAINDER AND FACTOR THEOREM

Objectives:

- Use long division to divide polynomials
- Use synthetic division to divide polynomials
- Evaluate a polynomial using the remainder theorem
- Use the factor theorem to solve a polynomial equation

The Division Algorithm

If $f(x)$ and $d(x)$ are polynomials, with $d(x) \neq 0$, and the degree of $d(x)$ is less than or equal to the degree of $f(x)$, then there exist unique polynomials $q(x)$ and $r(x)$ such that

$$\underbrace{f(x)}_{\text{Dividend}} = \underbrace{d(x)}_{\text{Divisor}} \cdot \underbrace{q(x)}_{\text{Quotient}} + \underbrace{r(x)}_{\text{Remainder}}.$$

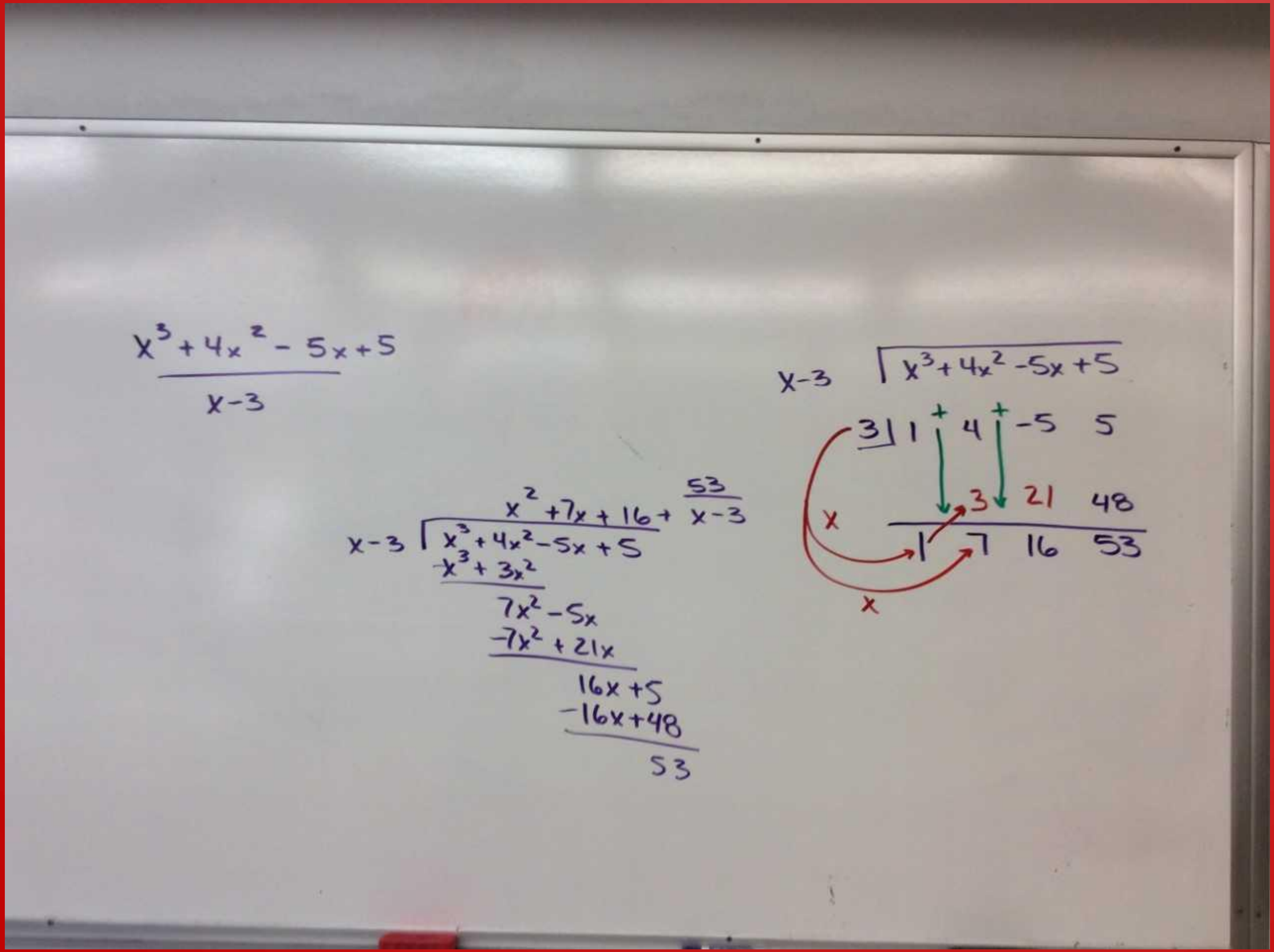
The remainder, $r(x)$, equals 0 or it is of degree less than the degree of $d(x)$. If $r(x) = 0$, we say that $d(x)$ **divides evenly** into $f(x)$ and that $d(x)$ and $q(x)$ are **factors** of $f(x)$.

Long Division of Polynomials

1. Arrange the terms of both the dividend and the divisor in descending powers of any variable.
2. **Divide** the first term in the dividend by the first term in the divisor. The result is the first term of the quotient.
3. **Multiply** every term in the divisor by the first term in the quotient. Write the resulting product beneath the dividend with like terms lined up.
4. **Subtract** the product from the dividend.
5. **Bring down** the next term in the original dividend and write it next to the remainder to form a new dividend.
6. Use this new expression as the dividend and repeat this process until the remainder can no longer be divided. This will occur when the degree of the remainder (the highest exponent on a variable in the remainder) is less than the degree of the divisor.

FRANK PHILLIPS COLLEGE Plainsmen

BORGER INDEPENDENT SCHOOL DISTRICT 1926



Synthetic Division

To divide a polynomial by $x - c$:

Example

1. Arrange the polynomial in descending powers, with a 0 coefficient for any missing term.

$$x - 3\overline{)x^3 + 4x^2 - 5x + 5}$$

2. Write c for the divisor, $x - c$. To the right, write the coefficients of the dividend.

$$\underline{3|}\quad 1 \quad 4 \quad -5 \quad 5$$

3. Write the leading coefficient of the dividend on the bottom row.

$$\underline{3|}\quad 1 \quad 4 \quad -5 \quad 5$$

Bring down 1.

$$1$$

4. Multiply c (in this case, 3) times the value just written on the bottom row. Write the product in the next column in the second row.

$$\underline{3|}\quad 1 \quad 4 \quad -5 \quad 5$$
$$\quad\quad\quad 3$$
$$1$$

Multiply by 3: $3 \cdot 1 = 3$.

5. Add the values in this new column, writing the sum in the bottom row.

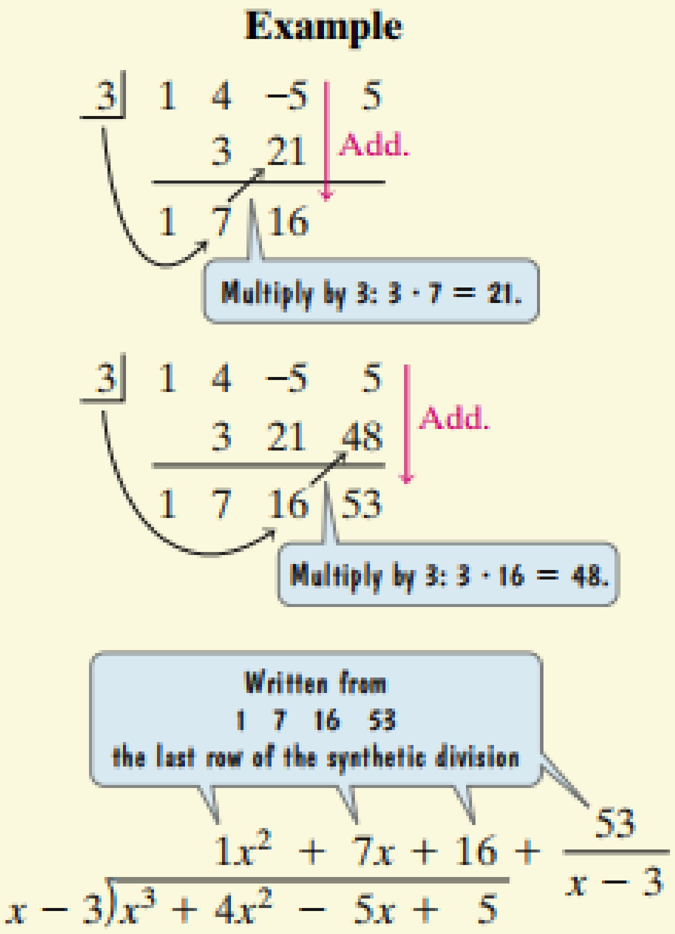
$$\underline{3|}\quad 1 \quad 4 \quad -5 \quad 5$$
$$\quad\quad\quad 3$$
$$1 \quad 7$$

Add.

6. Repeat this series of multiplications and additions until all columns are filled in.

7. Use the numbers in the last row to write the quotient, plus the remainder above the divisor. **The degree of the first term of the quotient is one less than the degree of the first term of the dividend.** The final value in this row is the remainder.

Example



The Remainder Theorem

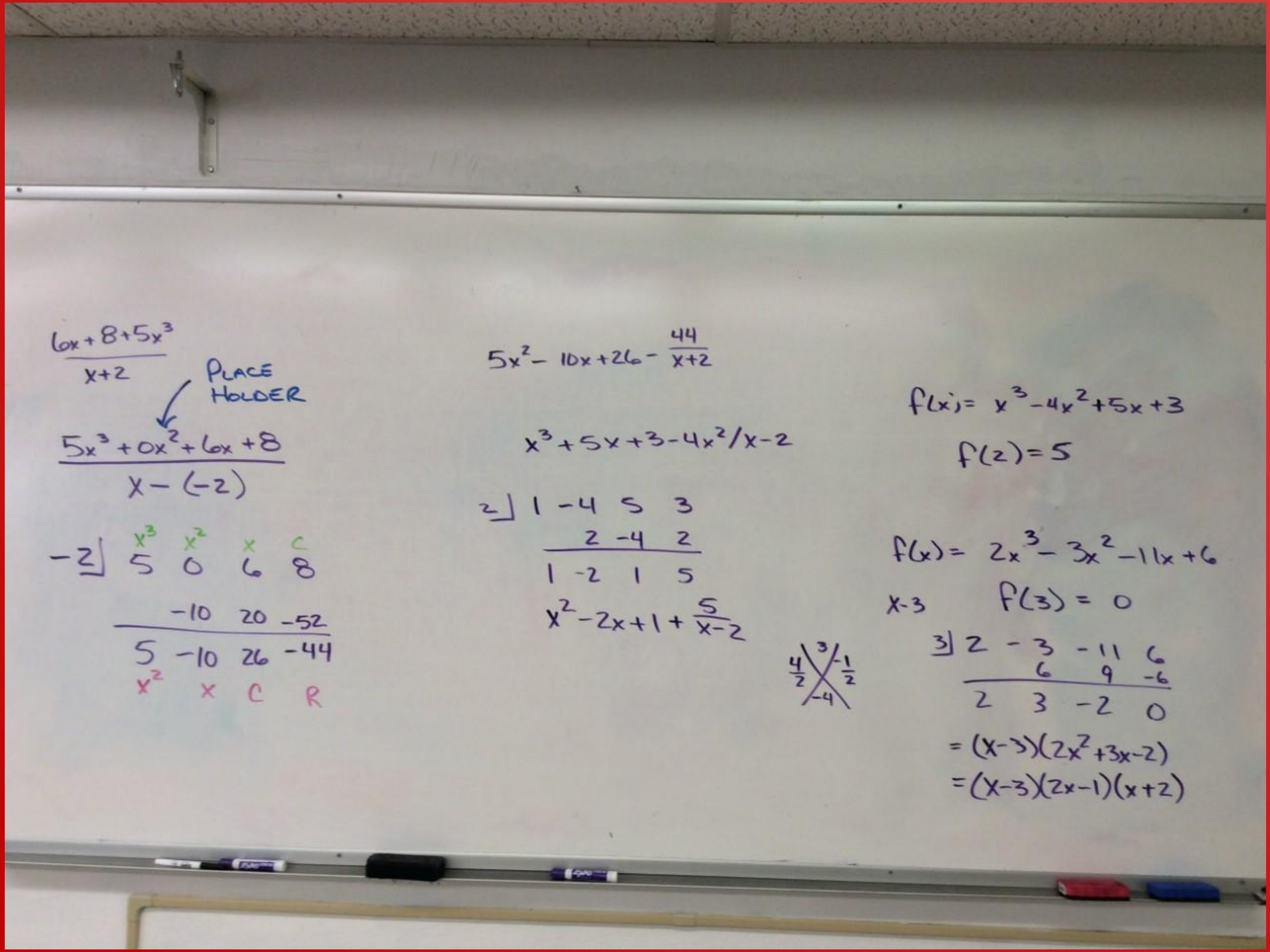
If the polynomial $f(x)$ is divided by $x - c$, then the remainder is $f(c)$.

The Factor Theorem

Let $f(x)$ be a polynomial.

a. If $f(c) = 0$, then $x - c$ is a factor of $f(x)$.

b. If $x - c$ is a factor of $f(x)$, then $f(c) = 0$.


6x+8+5x3 / x+2
PLACE HOLDER
5x3+0x2+6x+8 / X−(−2)
−2| 5 0 6 8
x3 x2 x c
−10 20 −52
5 −10 26 −44
x2 x C R
5x2 − 10x+26 − 44/x+2
x3+5x+3−4x2/x−2
2| 1 −4 5 3
2 −4 2
1 −2 1 5
x2−2x+1+ 5/x−2
f(x)= x3−4x2+5x+3
f(2)=5
f(x)= 2x3−3x2−11x+6
X-3
f(3) = 0
3| 2 −3 −11 6
6 9 −6
2 3 −2 0
=(x−3)(2x2+3x−2)
=(x−3)(2x−1)(x+2)