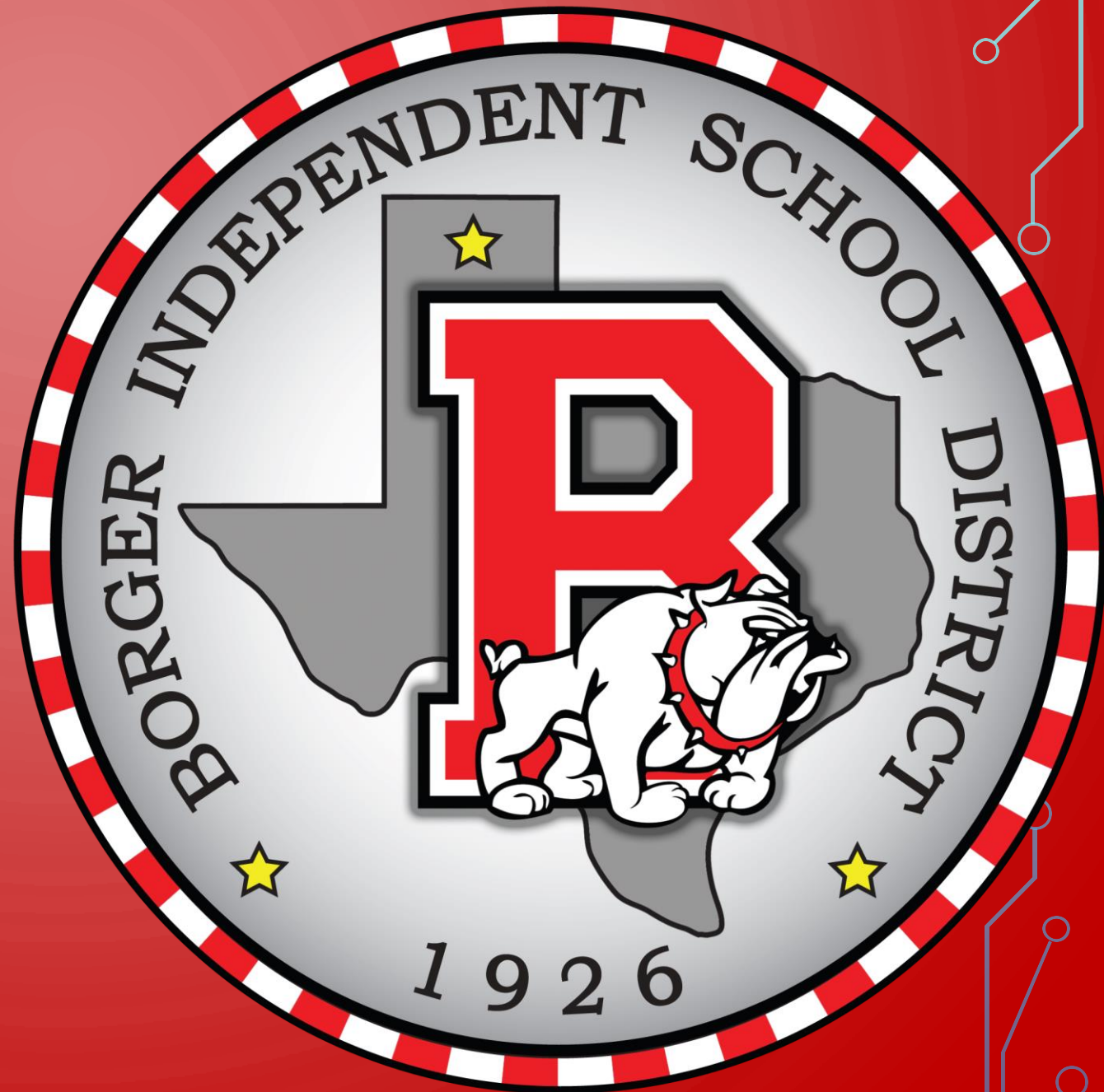
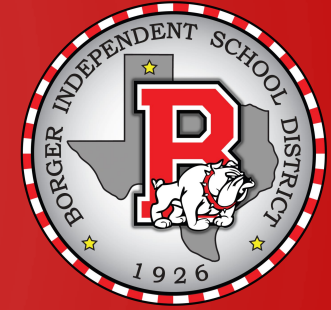


BOARD NOTES

17 OCTOBER 2018



CC ALGEBRA CHAPTER 3 – POLYNOMIAL AND RATIONAL FUNCTIONS



- SECTION 3.4 - ZEROS
OF POLYNOMIAL FUNCTIONS

Objectives:

- Use the remainder and factor theorem
- Use Descartes' rule of signs to determine the number of positive and negative real zeros of a polynomial function
- Find the real zeros of a polynomial function
- Solve polynomial equations
- Use the Intermediate Value Theorem

Division Algorithm

If $f(x)$ and $g(x)$ denote polynomial functions and if $g(x)$ is a polynomial whose degree is greater than zero, then there are unique polynomial functions $q(x)$ and $r(x)$ such that

$$\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)} \quad \text{or} \quad f(x) = q(x)g(x) + r(x) \quad (1)$$

dividend quotient divisor remainder

where $r(x)$ is either the zero polynomial or a polynomial of degree less than that of $g(x)$.



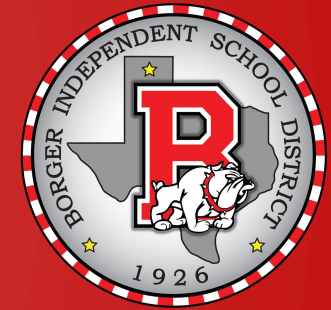
Remainder Theorem

Let f be a polynomial function. If $f(x)$ is divided by $x - c$, then the remainder is $f(c)$.

Factor Theorem

Let f be a polynomial function. Then $x - c$ is a factor of $f(x)$ if and only if $f(c) = 0$.

1. If $f(c) = 0$, then $x - c$ is a factor of $f(x)$.
2. If $x - c$ is a factor of $f(x)$, then $f(c) = 0$.



Number of Real Zeros

A polynomial function cannot have more real zeros than its degree.

Descartes' Rule of Signs

Let f denote a polynomial function written in standard form.

The number of positive real zeros of f either equals the number of variations in the sign of the nonzero coefficients of $f(x)$ or else equals that number less an even integer.

The number of negative real zeros of f either equals the number of variations in the sign of the nonzero coefficients of $f(-x)$ or else equals that number less an even integer.



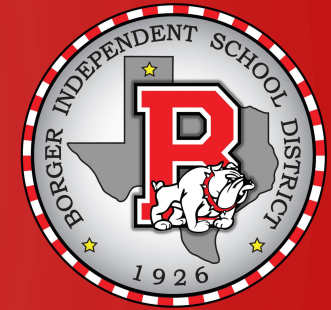
Rational Zeros Theorem

Let f be a polynomial function of degree 1 or higher of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 \quad a_n \neq 0 \quad a_0 \neq 0$$

where each coefficient is an integer. If $\frac{p}{q}$, in lowest terms, is a rational zero of f , then p must be a factor of a_0 , and q must be a factor of a_n .

$$\text{Possible Rational Zeros (PRZ)} = \frac{p \text{ which is all factors of } a_0}{q \text{ which is all factors of } a_n}$$



Steps for Finding the Real Zeros of a Polynomial Function

STEP 1: Use the degree of the polynomial to determine the maximum number of real zeros.

STEP 2: Use Descartes' Rule of Signs to determine the possible number of positive zeros and negative zeros.

STEP 3: (a) If the polynomial has integer coefficients, use the Rational Zeros Theorem to identify those rational numbers that potentially could be zeros.

(b) Use substitution, synthetic division, or long division to test each potential rational zero. Each time that a zero (and thus a factor) is found, repeat Step 3 on the depressed equation.

In attempting to find the zeros, remember to use (if possible) the factoring techniques that you already know (special products, factoring by grouping, and so on).



Intermediate Value Theorem

Let f denote a polynomial function. If $a < b$ and if $f(a)$ and $f(b)$ are of opposite sign, there is at least one real zero of f between a and b .

DA, RT, $\frac{1}{3}$ FT

$$f(x) = \overset{9}{0}x^3 - 4x^2 - \overset{P}{5}$$

ZEROS = 3

$$f(1) = -8$$

$$x-3$$

1 + REAL

$$f(5) = 125 - 100 - 5 = 20$$

$$f(3) = 3^3 - 4(3)^2 - 5 = -14$$

$$f(-x) = -x^3 - 4x^2 - 5$$

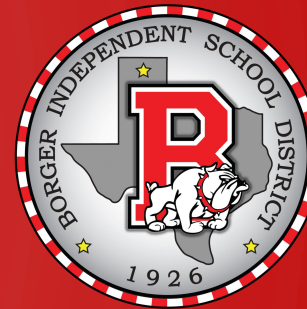
0 - REAL

$$\text{PRZ: } P \pm 1 \pm 5$$

$$\text{PRZ: } \frac{\pm 1}{\pm 1}, \frac{\pm 5}{\pm 1}$$

$$9 \pm 1$$

$$\boxed{\pm 1, \pm 5}$$



$$g(x) = x^5 - 7x^4 + 19x^3 - 37x^2 + 60x - 36$$

ZEROS = $n = 5$

5, 3, 1 + REAL

$$g(-x) = -x^5 - 7x^4 - 19x^3 - 37x^2 - 60x - 36$$

0 - REAL

$$p: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, 18, \pm 36$$

$$q: \pm 1$$

$$h(x) = 2x^2 - 3x^7 - 4x^4 - 3x^3 + x - 3$$

ZEROS $n = 7$

$$h(x) = -3x^7 - 4x^4 - 3x^3 + 2x^2 + x - 3$$

2 OR 0 + REAL

3 OR 1 - REAL

$$\text{PRZ: } p \begin{matrix} \pm 1 & \pm 3 \\ 9 & \pm 1 & \pm 3 \end{matrix}$$

$$\text{PRZ} = \begin{matrix} \pm 1 & \pm 3 & \pm 1 & \pm 3 \\ \pm 1 & \pm 1 & \pm 3 & \pm 3 \end{matrix}$$

$$\pm 1, \pm 3, \pm \frac{1}{3}$$

