BOARD NOTES

25 OCTOBER 2018

BORGER INDEPENDENT SCHOOL DISTRICT

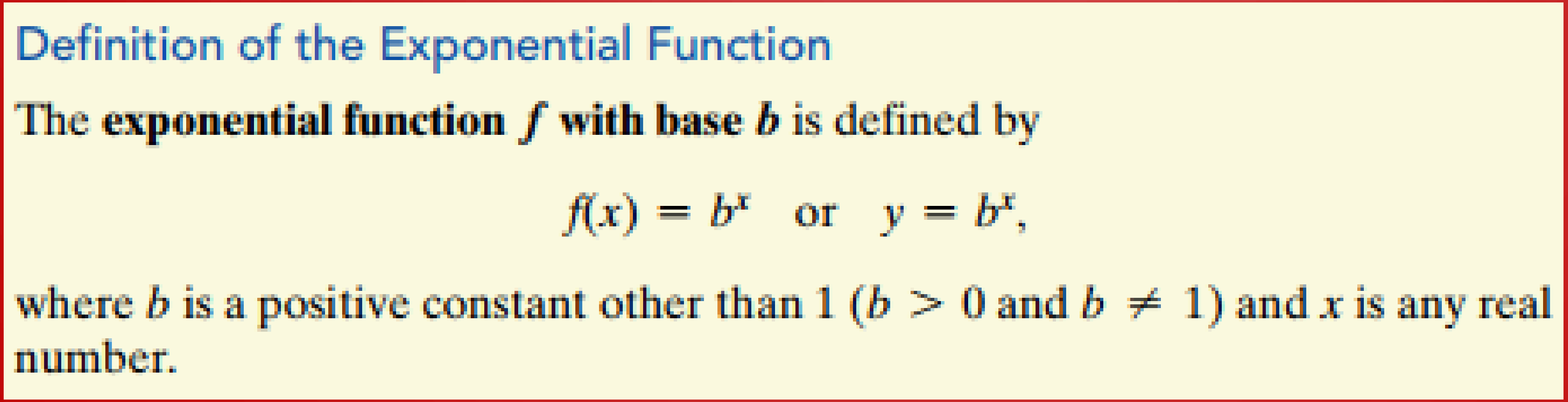
1926

CC ALGEBRA
CHAPTER 4 –
EXPONENTIAL AND LOGARITHMIC FUNCTIONS

- SECTION 4.1 - EXPONENTIAL FUNCTIONS

Objectives:

- Evaluate exponential functions
- Graph exponential functions
- Evaluate functions with base e
- Use compound interest formula

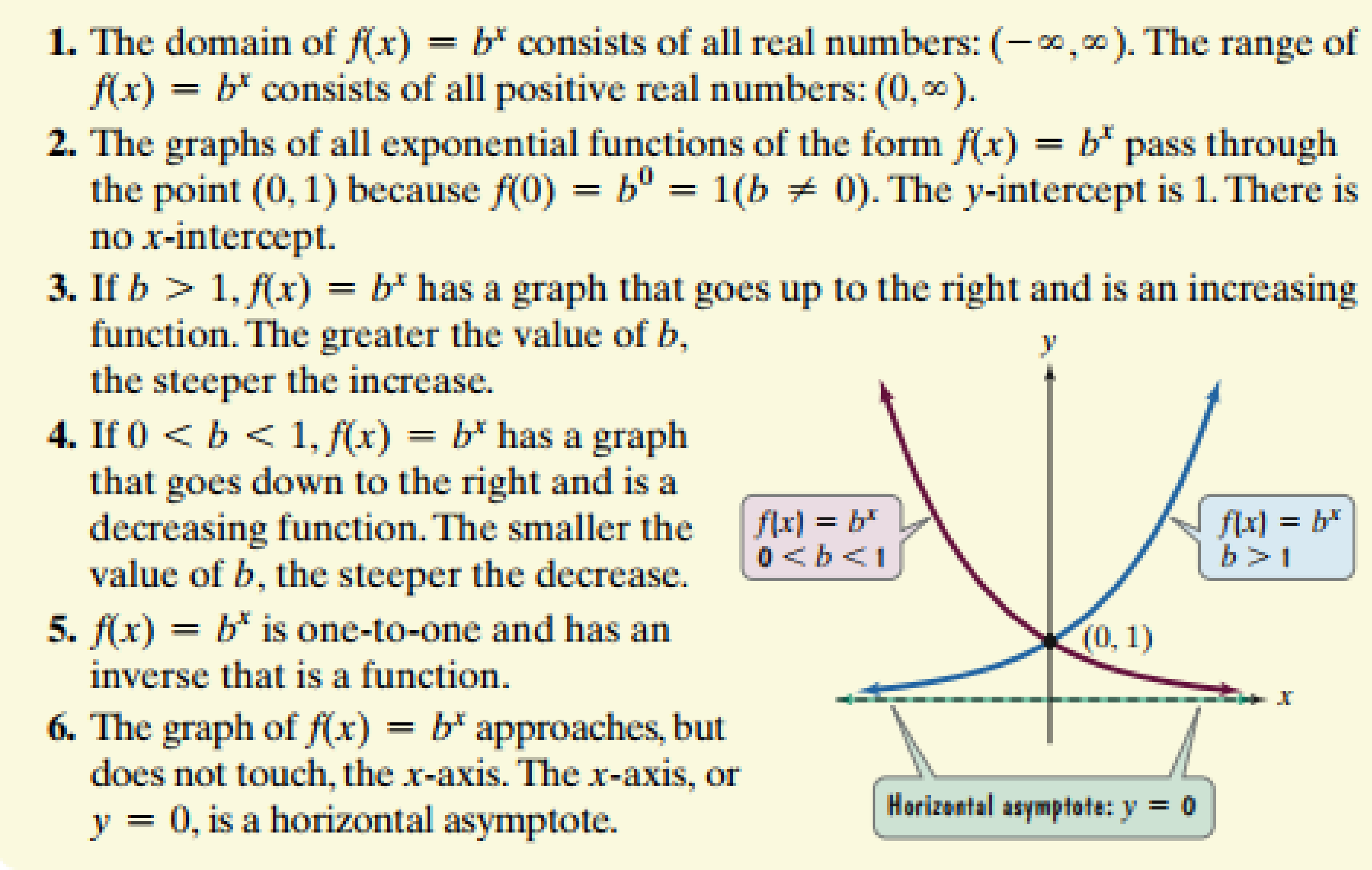


Definition of the Exponential Function

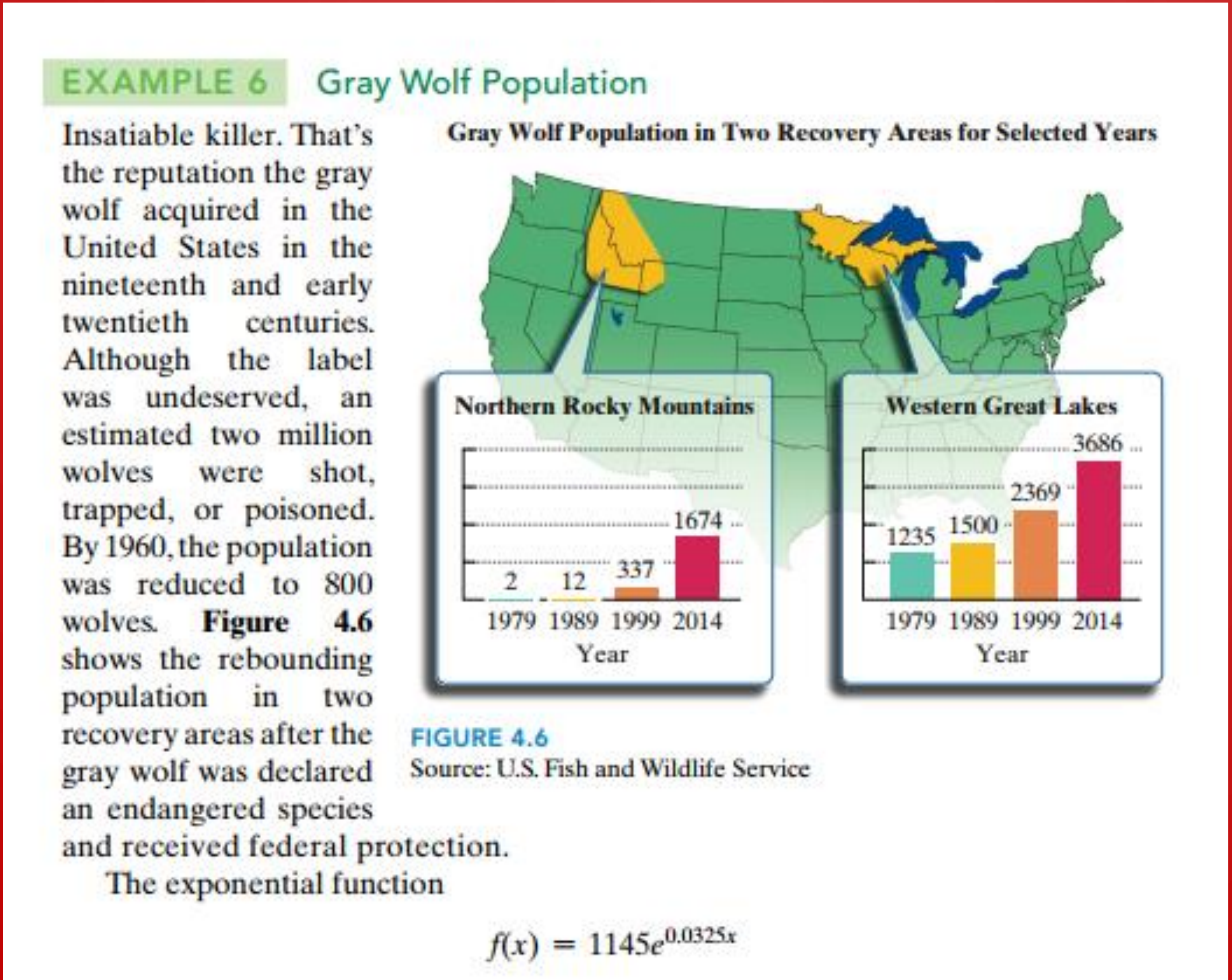
The **exponential function** f **with base** b is defined by

$$f(x) = b^x \quad \text{or} \quad y = b^x,$$

where b is a positive constant other than 1 ($b > 0$ and $b \neq 1$) and x is any real number.



1. The domain of $f(x) = b^x$ consists of all real numbers: $(-\infty, \infty)$. The range of $f(x) = b^x$ consists of all positive real numbers: $(0, \infty)$.
2. The graphs of all exponential functions of the form $f(x) = b^x$ pass through the point $(0, 1)$ because $f(0) = b^0 = 1 (b \neq 0)$. The y-intercept is 1. There is no x-intercept.
3. If $b > 1$, $f(x) = b^x$ has a graph that goes up to the right and is an increasing function. The greater the value of b, the steeper the increase.
4. If $0 < b < 1$, $f(x) = b^x$ has a graph that goes down to the right and is a decreasing function. The smaller the value of b, the steeper the decrease.
5. $f(x) = b^x$ is one-to-one and has an inverse that is a function.
6. The graph of $f(x) = b^x$ approaches, but does not touch, the x-axis. The x-axis, or $y = 0$, is a horizontal asymptote.



EXAMPLE 6 Gray Wolf Population

Insatiable killer. That's the reputation the gray wolf acquired in the United States in the nineteenth and early twentieth centuries. Although the label was undeserved, an estimated two million wolves were shot, trapped, or poisoned. By 1960, the population was reduced to 800 wolves. **Figure 4.6** shows the rebounding population in two recovery areas after the gray wolf was declared an endangered species and received federal protection.

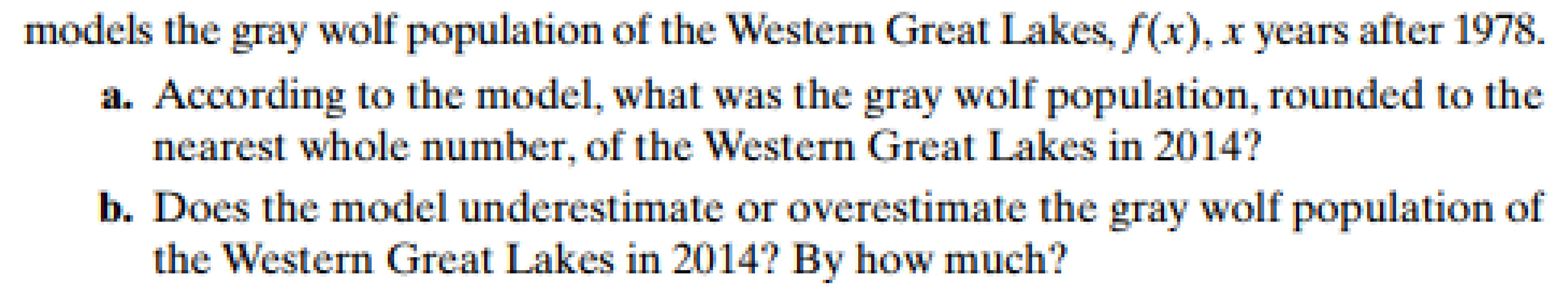
Gray Wolf Population in Two Recovery Areas for Selected Years

FIGURE 4.6

Source: U.S. Fish and Wildlife Service

The exponential function

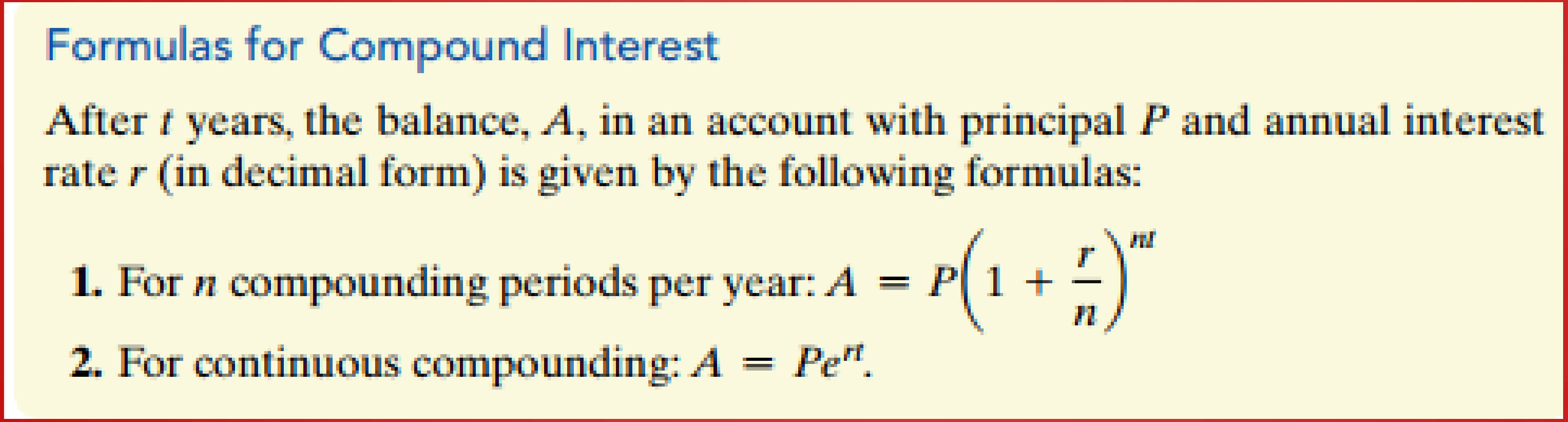
$$f(x) = 1145e^{0.0325x}$$



models the gray wolf population of the Western Great Lakes, $f(x)$, x years after 1978.

a. According to the model, what was the gray wolf population, rounded to the nearest whole number, of the Western Great Lakes in 2014?

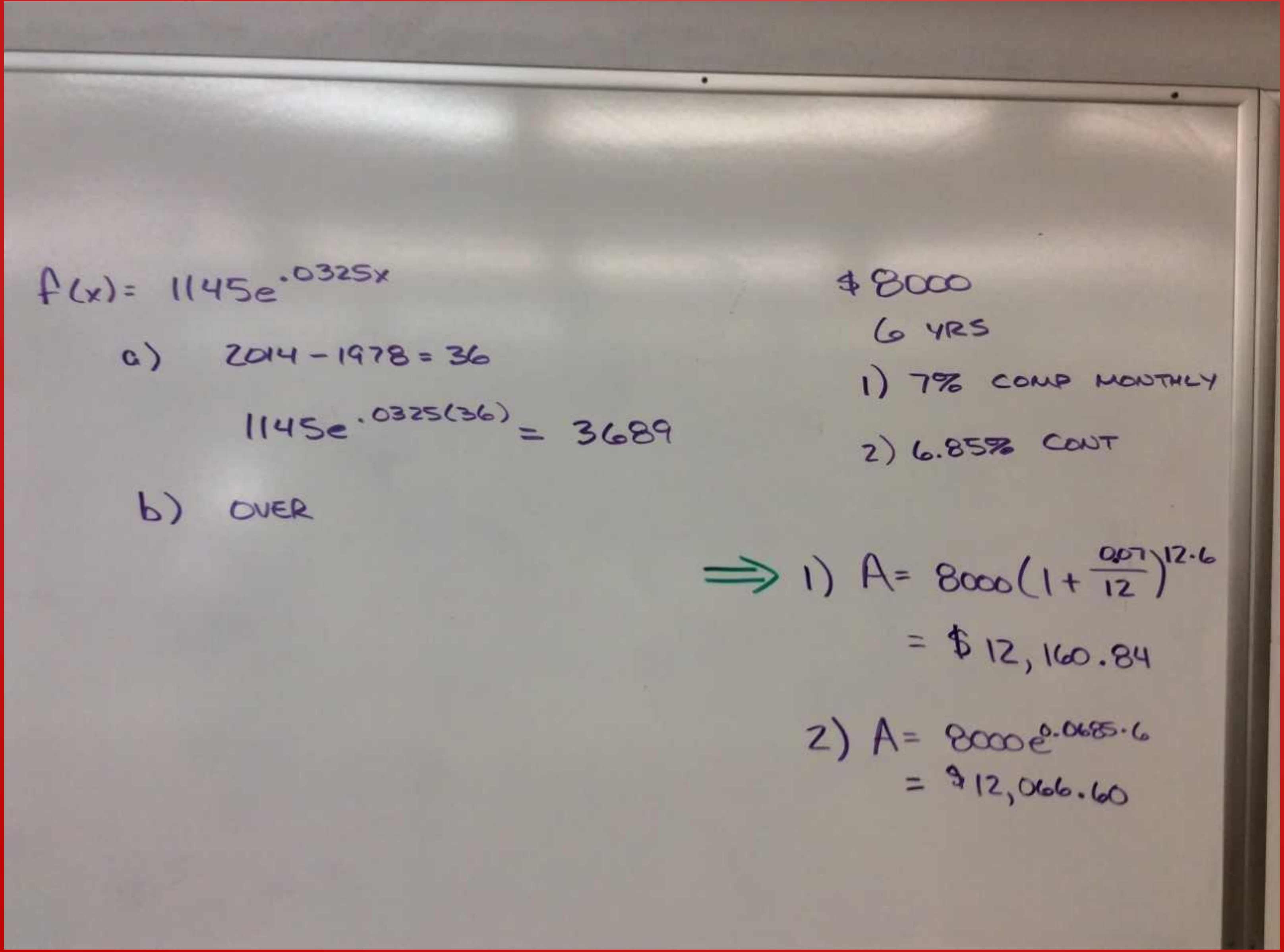
b. Does the model underestimate or overestimate the gray wolf population of the Western Great Lakes in 2014? By how much?



Formulas for Compound Interest

After t years, the balance, A, in an account with principal P and annual interest rate r (in decimal form) is given by the following formulas:

1. For n compounding periods per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$
2. For continuous compounding: $A = Pe^{rt}$.



$f(x) = 1145e^{.0325x}$

a) $2014 - 1978 = 36$

$1145e^{.0325(36)} = 3689$

b) OVER

$\$8000$

6 YRS

1) 7% COMP MONTHLY

2) 6.85% CONT

$\Longrightarrow$ 1) $A = 8000\left(1 + \frac{0.07}{12}\right)^{12 \cdot 6}$

$= \$12,160.84$

2) $A = 8000e^{0.0685 \cdot 6}$

$= \$12,066.60$

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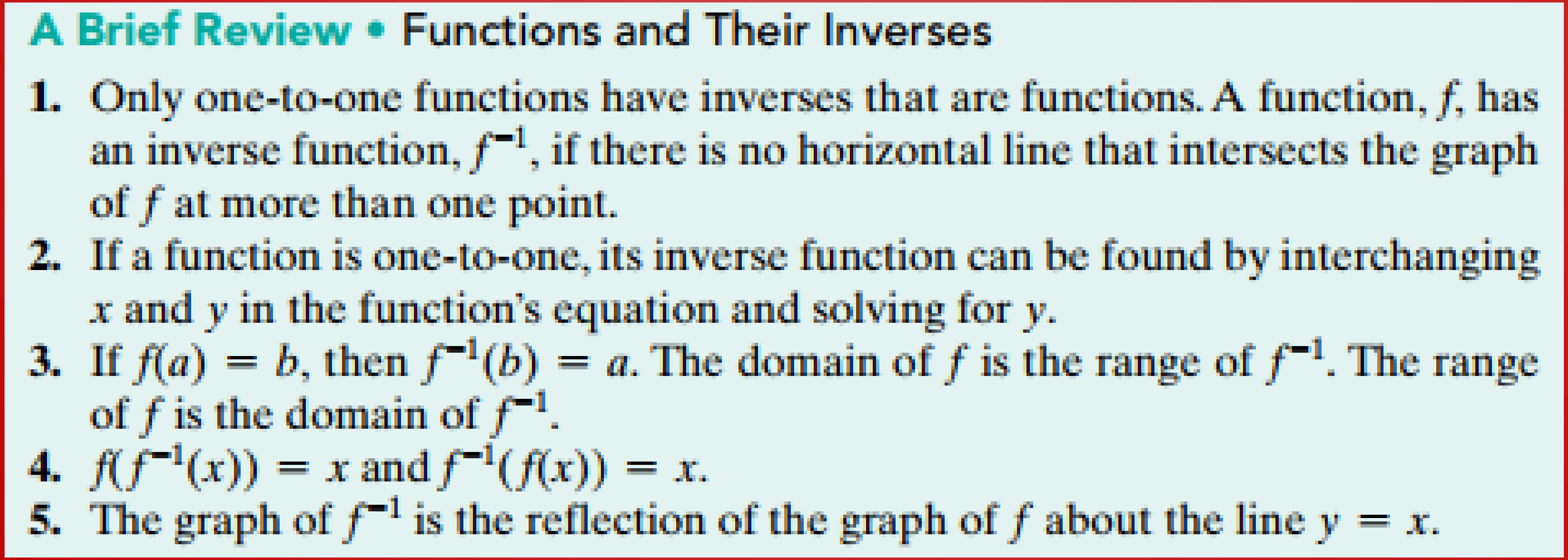
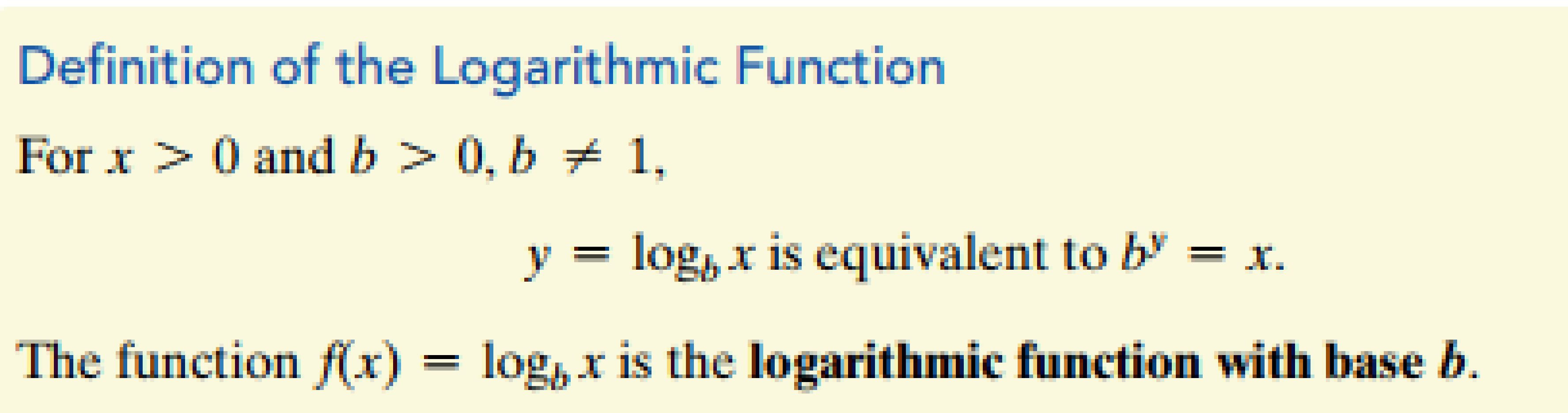


A Brief Review • Functions and Their Inverses

1. Only one-to-one functions have inverses that are functions. A function, f, has an inverse function, f^{-1}, if there is no horizontal line that intersects the graph of f at more than one point.
2. If a function is one-to-one, its inverse function can be found by interchanging x and y in the function's equation and solving for y.
3. If $f(a) = b$, then $f^{-1}(b) = a$. The domain of f is the range of f^{-1}. The range of f is the domain of f^{-1}.
4. $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.
5. The graph of f^{-1} is the reflection of the graph of f about the line $y = x$.

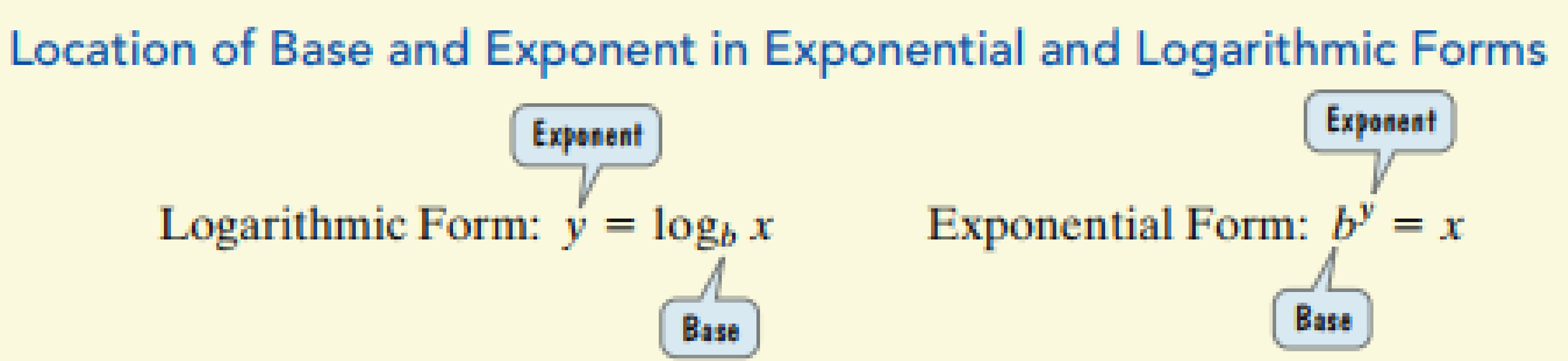


Definition of the Logarithmic Function

For $x > 0$ and $b > 0, b \neq 1,$

$$y = \log_b x \text{ is equivalent to } b^y = x.$$

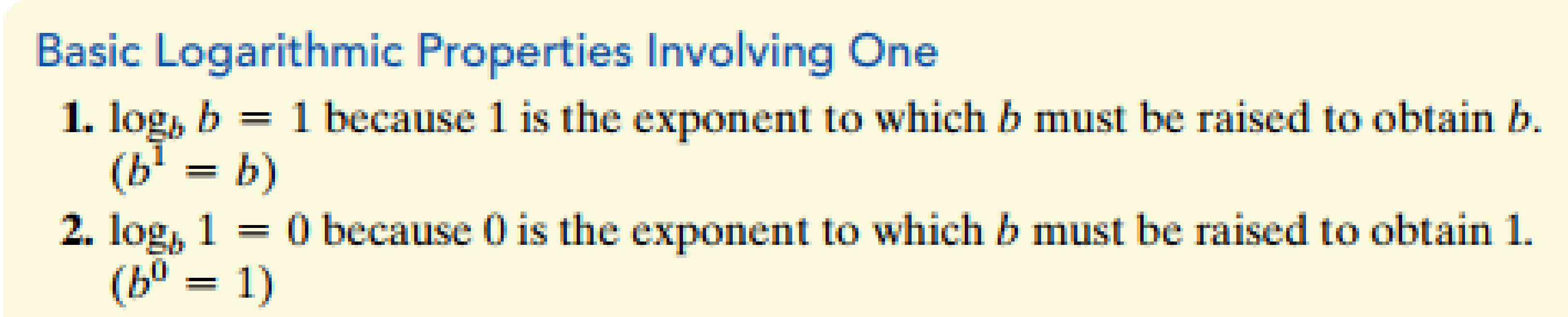
The function $f(x) = \log_b x$ is the **logarithmic function with base b.**



Location of Base and Exponent in Exponential and Logarithmic Forms

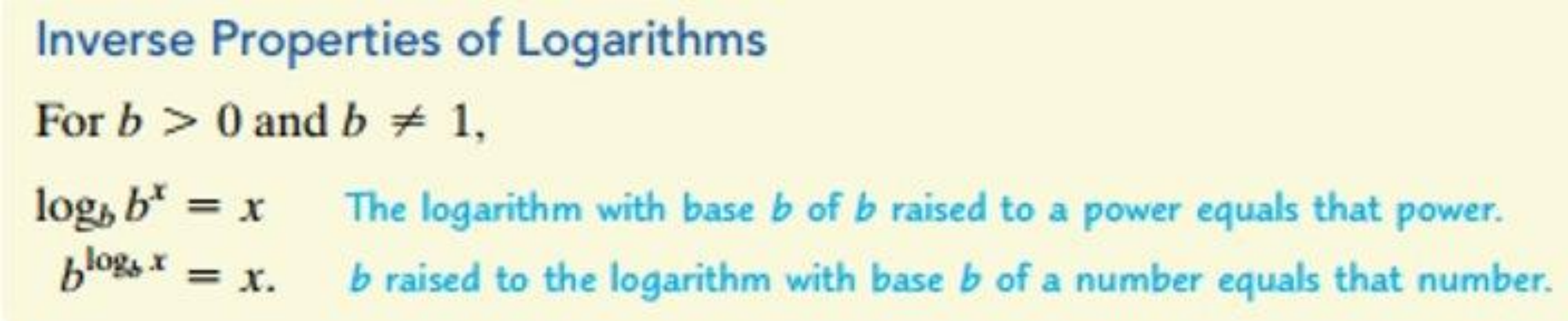
Logarithmic Form: $\overset{\text{Exponent}}{y} = \log_{\underset{\text{Base}}{b}} x$

Exponential Form: $\underset{\text{Base}}{b}^{\overset{\text{Exponent}}{y}} = x$



Basic Logarithmic Properties Involving One

1. $\log_b b = 1$ because 1 is the exponent to which b must be raised to obtain b. $(b^1 = b)$
2. $\log_b 1 = 0$ because 0 is the exponent to which b must be raised to obtain 1. $(b^0 = 1)$

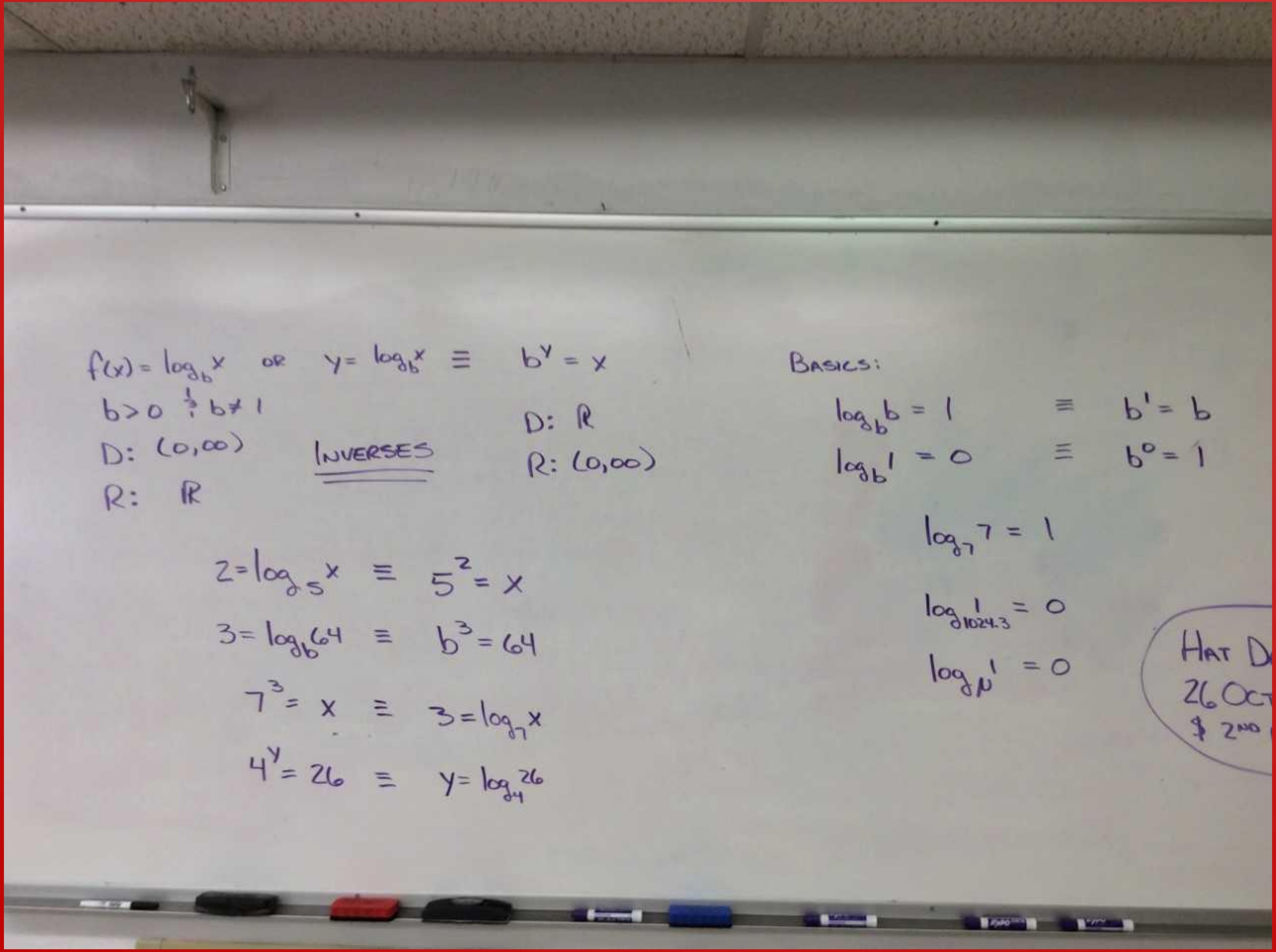


Inverse Properties of Logarithms

For $b > 0$ and $b \neq 1$,

$\log_b b^x = x$ The logarithm with base b of b raised to a power equals that power.

$b^{\log_b x} = x.$ b raised to the logarithm with base b of a number equals that number.



$f(x)=\log_b x$ OR $y=\log_b x \equiv b^y=x$

$b>0$ & $b\neq 1$

D: $(0,\infty)$

R: $\mathbb{R}$

INVERSES

D: $\mathbb{R}$

R: $(0,\infty)$

$2=\log_5 x \equiv 5^2=x$

$3=\log_b 64 \equiv b^3=64$

$7^3=x \equiv 3=\log_7 x$

$4^y=26 \equiv y=\log_4 26$

BASICS:

$\log_b b=1 \equiv b^1=b$

$\log_b 1=0 \equiv b^0=1$

$\log_7 7=1$

$\log_{1024.3} 1=0$

$\log_\mu 1=0$

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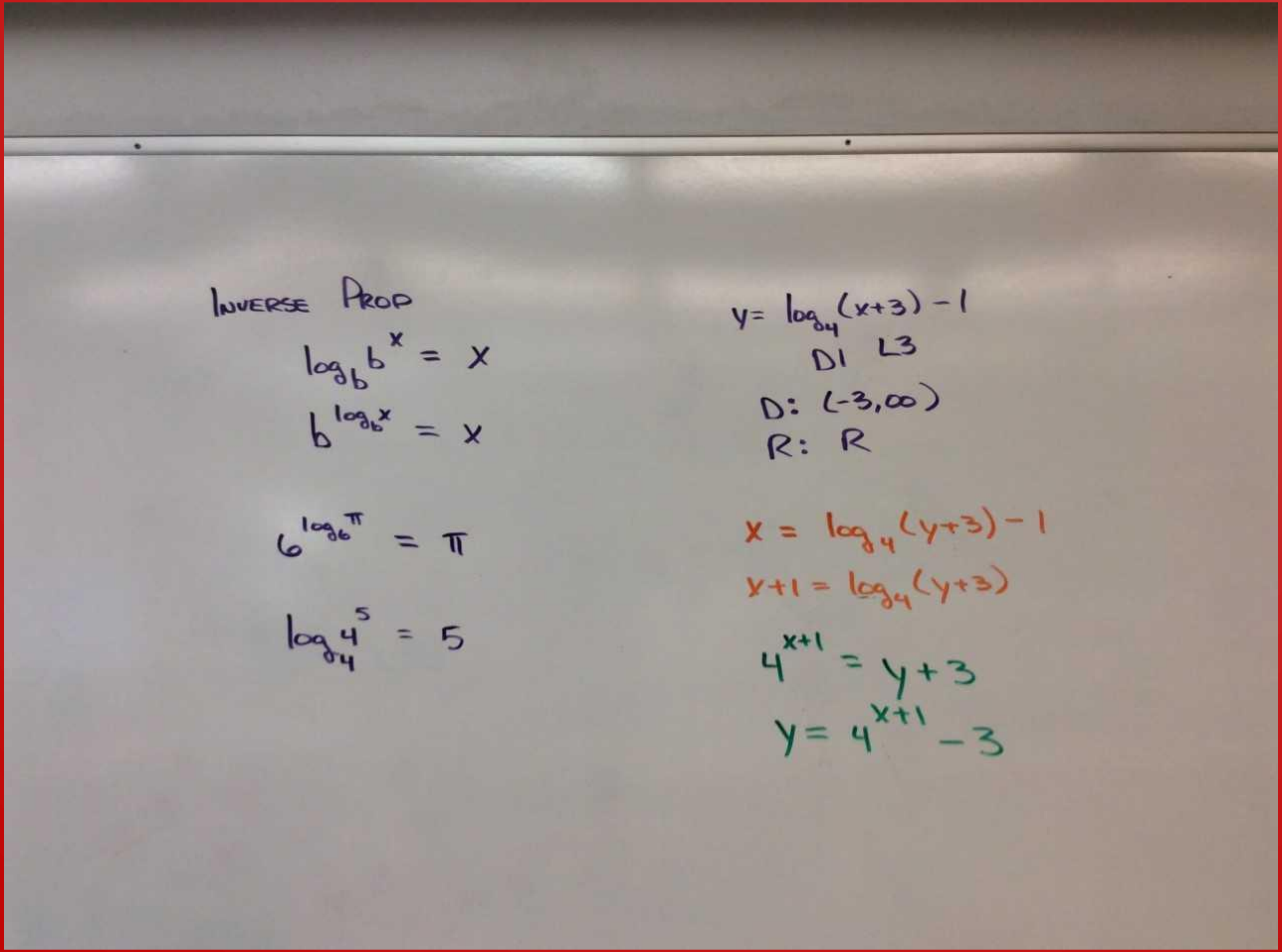


INVERSE PROP

$$\log_b b^x = x$$

$$b^{\log_b x} = x$$

$$6^{\log_6 \pi} = \pi$$

$$\log_4 4^5 = 5$$

$$y = \log_4(x+3) - 1$$

DI L3

D: $(-3, \infty)$

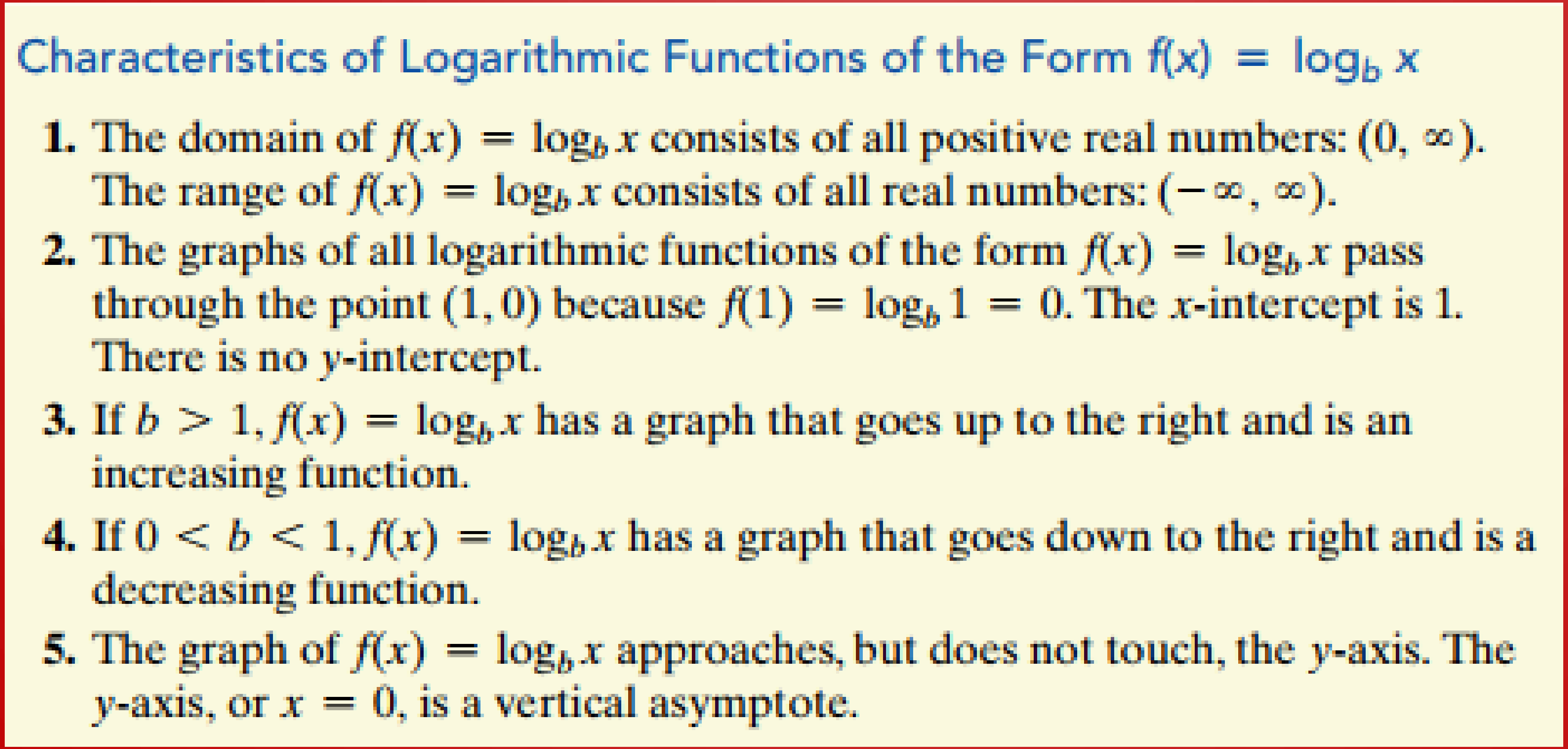
R: R

$$x = \log_4(y+3) - 1$$

$$x+1 = \log_4(y+3)$$

$$4^{x+1} = y+3$$

$$y = 4^{x+1} - 3$$



Characteristics of Logarithmic Functions of the Form $f(x) = \log_b x$

1. The domain of $f(x) = \log_b x$ consists of all positive real numbers: $(0, \infty)$. The range of $f(x) = \log_b x$ consists of all real numbers: $(-\infty, \infty)$.
2. The graphs of all logarithmic functions of the form $f(x) = \log_b x$ pass through the point $(1, 0)$ because $f(1) = \log_b 1 = 0$. The x-intercept is 1. There is no y-intercept.
3. If $b > 1$, $f(x) = \log_b x$ has a graph that goes up to the right and is an increasing function.
4. If $0 < b < 1$, $f(x) = \log_b x$ has a graph that goes down to the right and is a decreasing function.
5. The graph of $f(x) = \log_b x$ approaches, but does not touch, the y-axis. The y-axis, or $x = 0$, is a vertical asymptote.

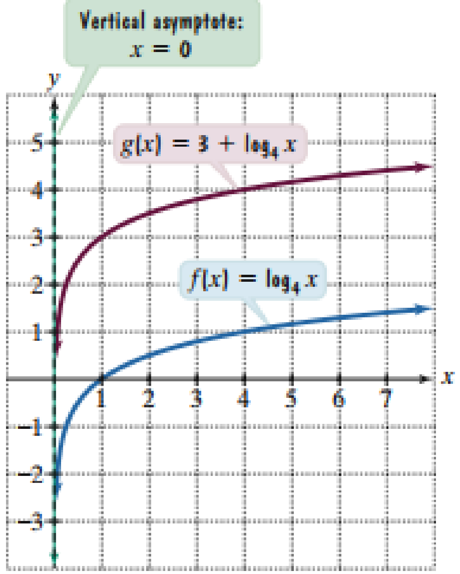


FIGURE 4.10 Shifting vertically up three units

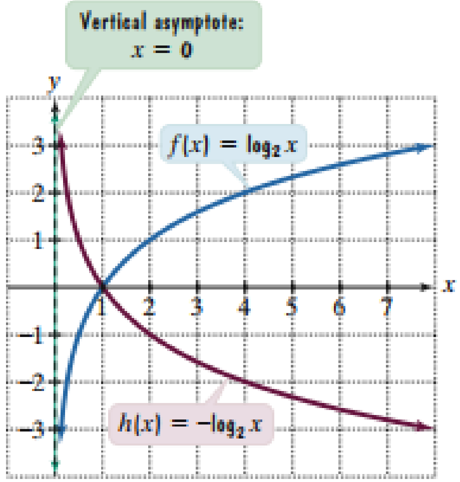


FIGURE 4.11 Reflection about the x-axis

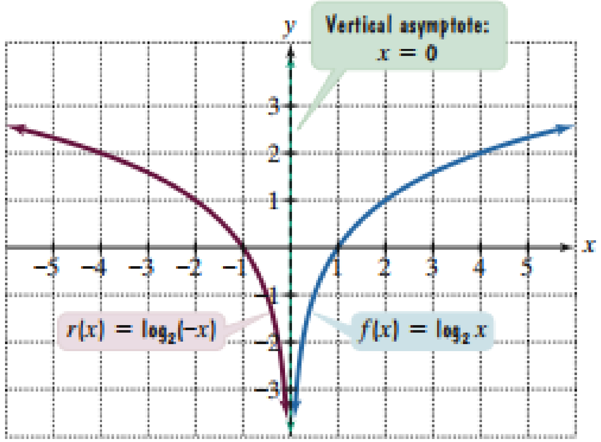
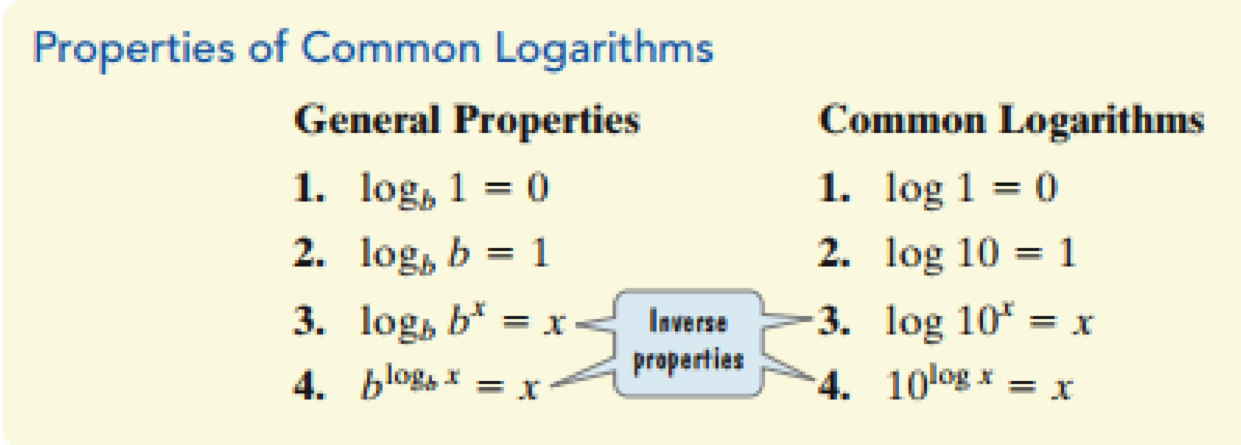


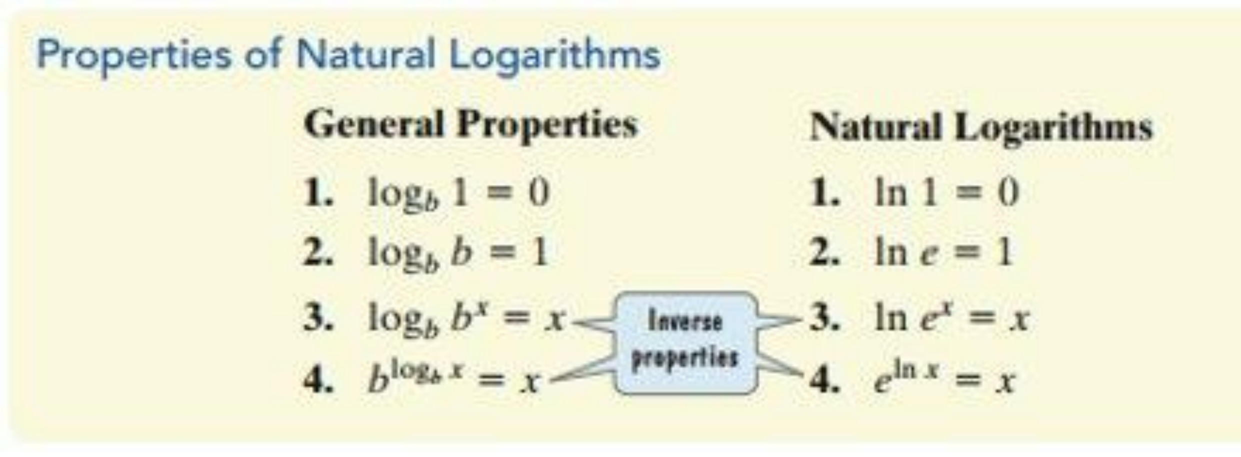
FIGURE 4.12 Reflection about the y-axis



Properties of Common Logarithms

General Properties	Common Logarithms
1. $\log_b 1 = 0$	**1.** $\log 1 = 0$
2. $\log_b b = 1$	**2.** $\log 10 = 1$
3. $\log_b b^x = x$	**3.** $\log 10^x = x$
4. $b^{\log_b x} = x$	**4.** $10^{\log x} = x$

Inverse properties



Properties of Natural Logarithms

General Properties	Natural Logarithms
1. $\log_b 1 = 0$	**1.** $\ln 1 = 0$
2. $\log_b b = 1$	**2.** $\ln e = 1$
3. $\log_b b^x = x$	**3.** $\ln e^x = x$
4. $b^{\log_b x} = x$	**4.** $e^{\ln x} = x$

Inverse properties