# BOARD NOTES

25 OCTOBER 2018

# CC PRECALCULUS
# CHAPTER 5 –
# EXPONENTIAL AND LOGARITHMIC FUNCTIONS

- SECTION 5.1 - COMPOSITE FUNCTIONS

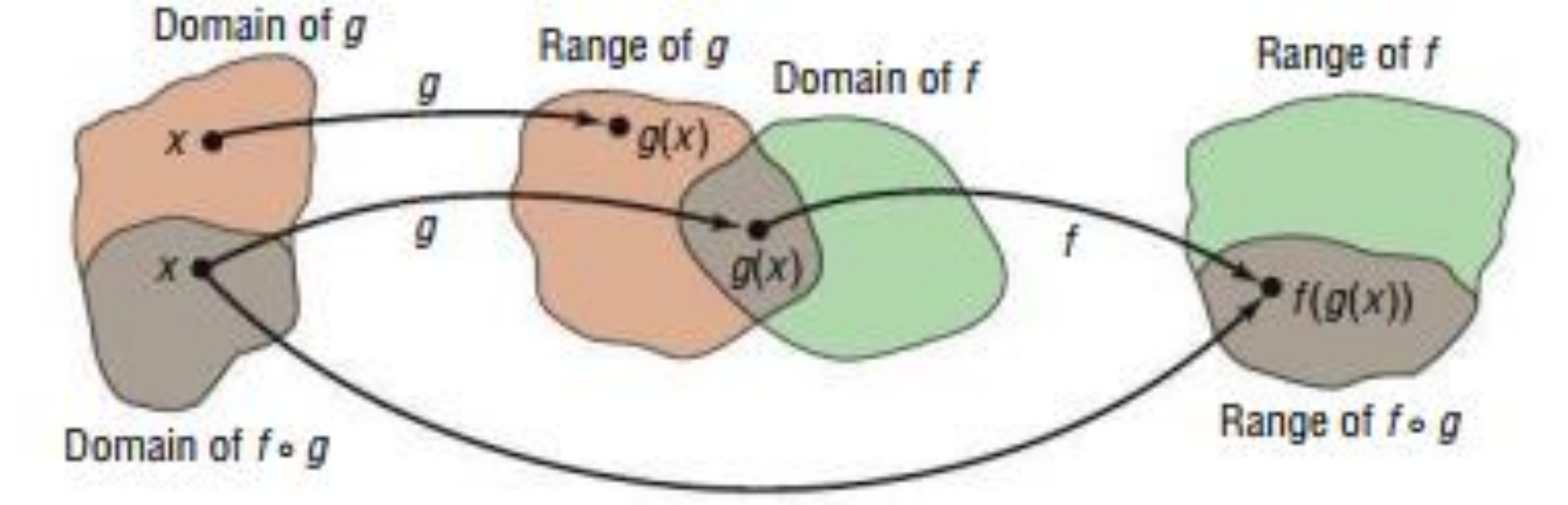
Objectives:

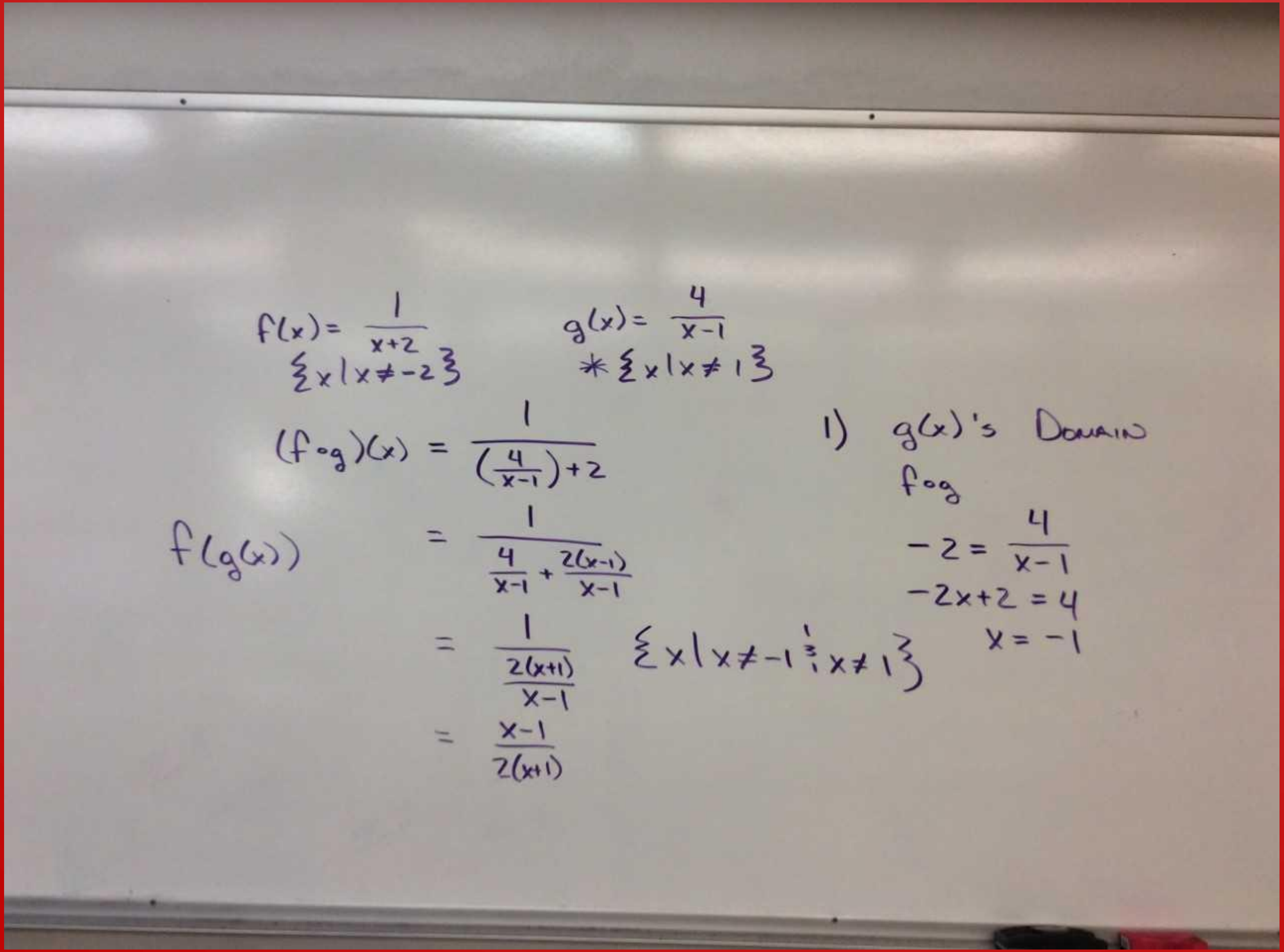
- Form a composite function
- Find the domain of a composite function

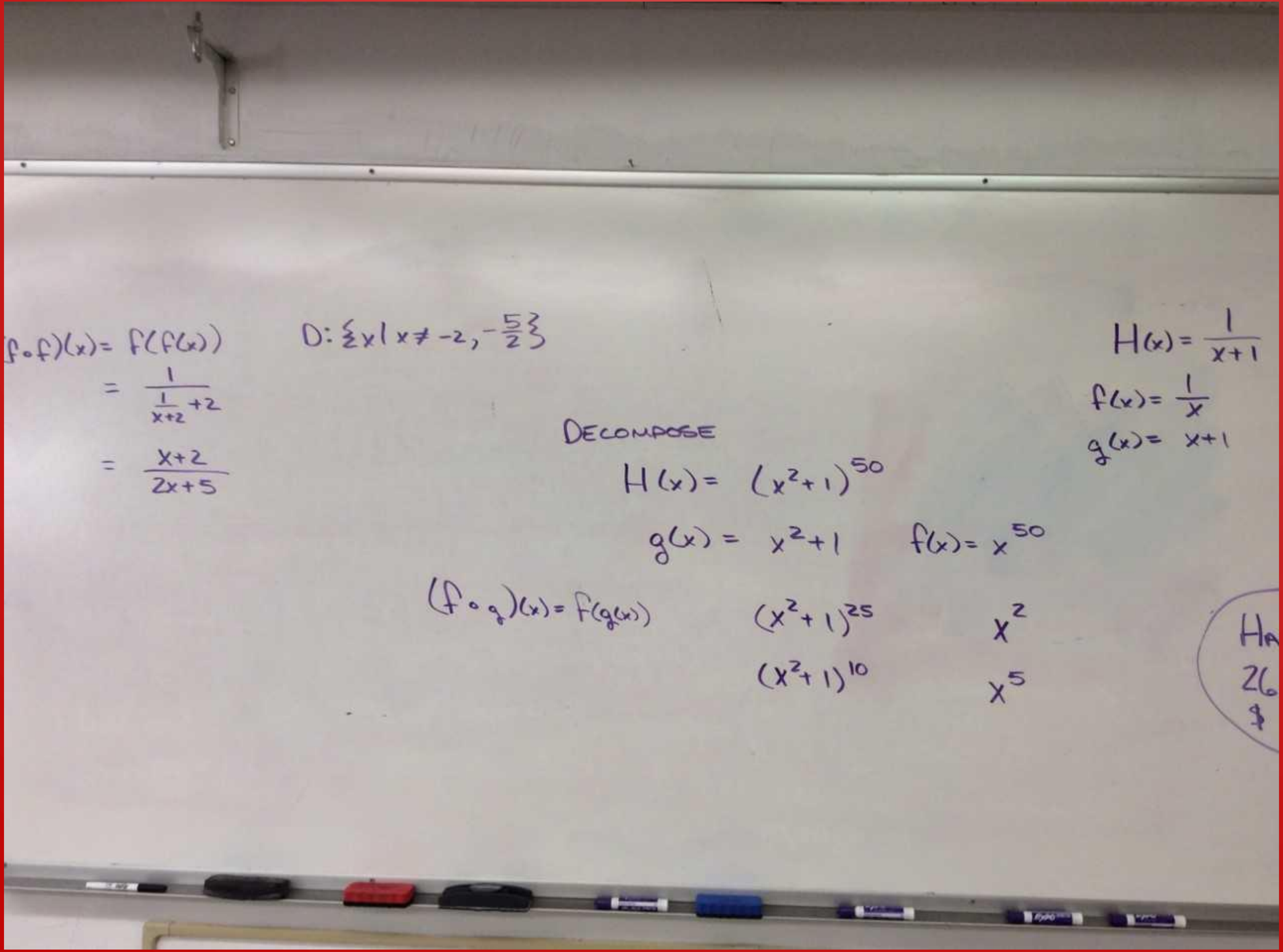
Given two functions $f$ and $g$, the **composite function**, denoted by $f \circ g$ (read as "$f$ composed with $g$"), is defined by

$$(f \circ g)(x) = f(g(x))$$

The domain of $f \circ g$ is the set of all numbers $x$ in the domain of $g$ such that $g(x)$ is in the domain of $f$.




f(x) = 1/(x+2)
{x | x ≠ -2}
g(x) = 4/(x-1)
* {x | x ≠ 1}
(f∘g)(x) = 1/((4/(x-1)) + 2)
f(g(x))
= 1/(4/(x-1) + 2(x-1)/(x-1))
= 1/(2(x+1)/(x-1))
{x | x ≠ -1 ; x ≠ 1}
= (x-1)/(2(x+1))
1) g(x)'s Domain
f∘g
-2 = 4/(x-1)
-2x+2 = 4
x = -1


(f∘f)(x) = f(f(x))
= 1 / (1/(x+2) + 2)
= (x+2) / (2x+5)
D: {x | x ≠ -2, -5/2}
DECOMPOSE
H(x) = (x²+1)^50
g(x) = x²+1
f(x) = x^50
(f∘g)(x) = f(g(x))
(x²+1)^25
x²
(x²+1)^10
x^5
H(x) = 1/(x+1)
f(x) = 1/x
g(x) = x+1

# CC PRECALCULUS
# CHAPTER 5 –
# EXPONENTIAL AND LOGARITHMIC FUNCTIONS

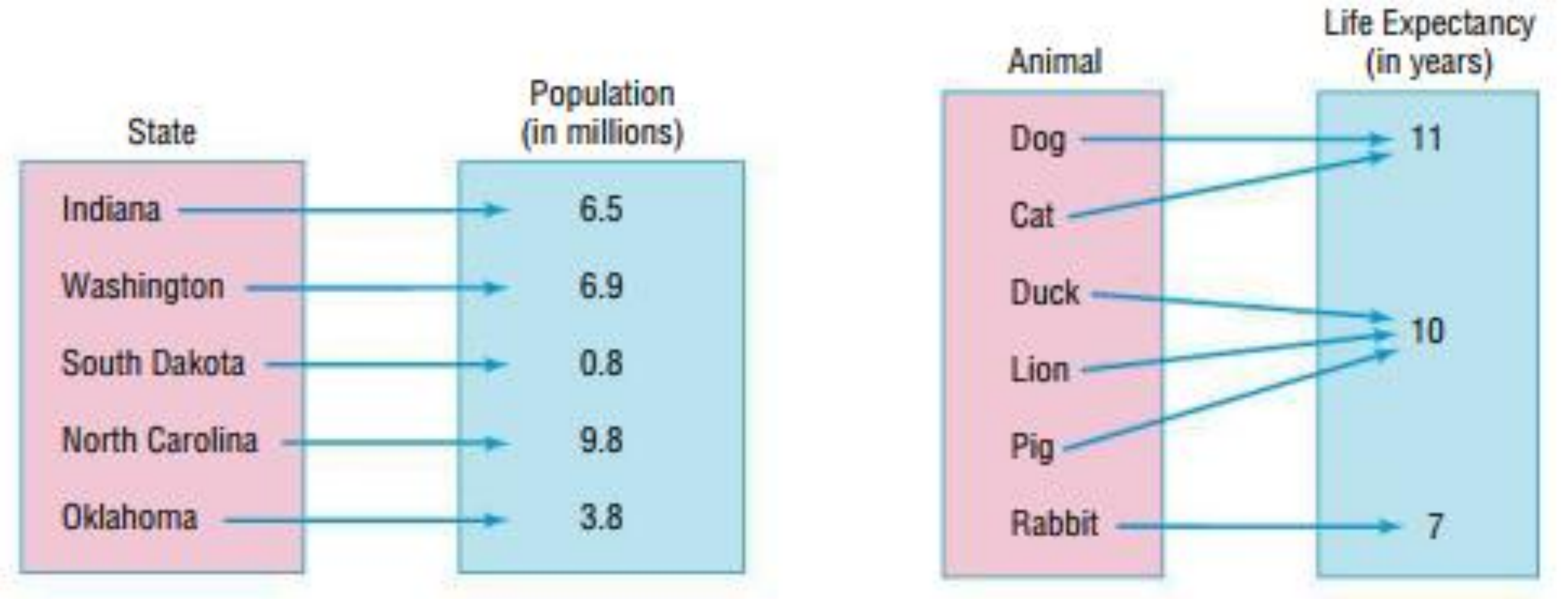
- SECTION 5.2 - ONE-TO-ONE FUNCTIONS; INVERSE FUNCTIONS

Objectives:

- Determine if a function is injective
- Determine the inverse of a function defined by a map or a set of ordered pairs
- Obtain the graph of the inverse function from the graph of the function
- Find the inverse of a function defined by an equation

Let $X$ and $Y$ be two nonempty sets.* A **function** from $X$ into $Y$ is a relation that associates with each element of $X$ exactly one element of $Y$.

A function is **one-to-one** if any two different inputs in the domain correspond to two different outputs in the range. That is, if $x_1$ and $x_2$ are two different inputs of a function $f$, then $f$ is one-to-one if $f(x_1) \neq f(x_2)$.

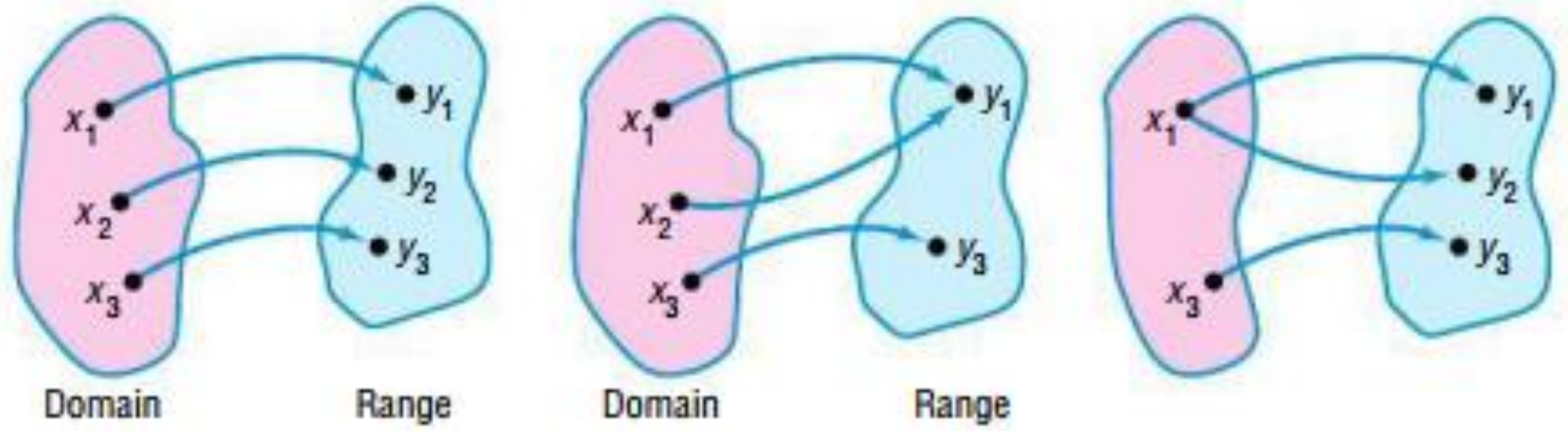


## Vertical-Line Test

A set of points in the *xy*-plane is the graph of a function if and only if every vertical line intersects the graph in at most one point.

## Horizontal-line Test

If every horizontal line intersects the graph of a function $f$ in at most one point, then $f$ is one-to-one.

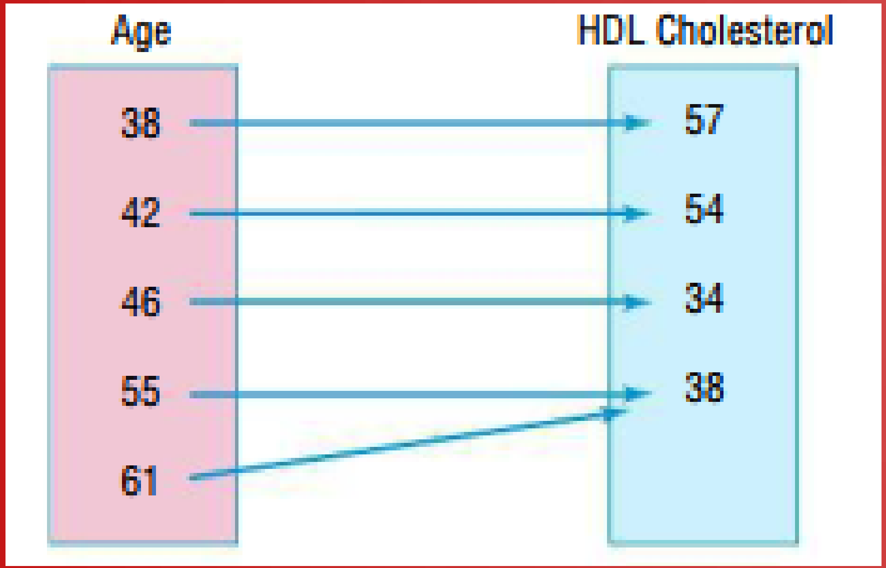



**(a)** One-to-one function: Each $x$ in the domain has one and only one image in the range.

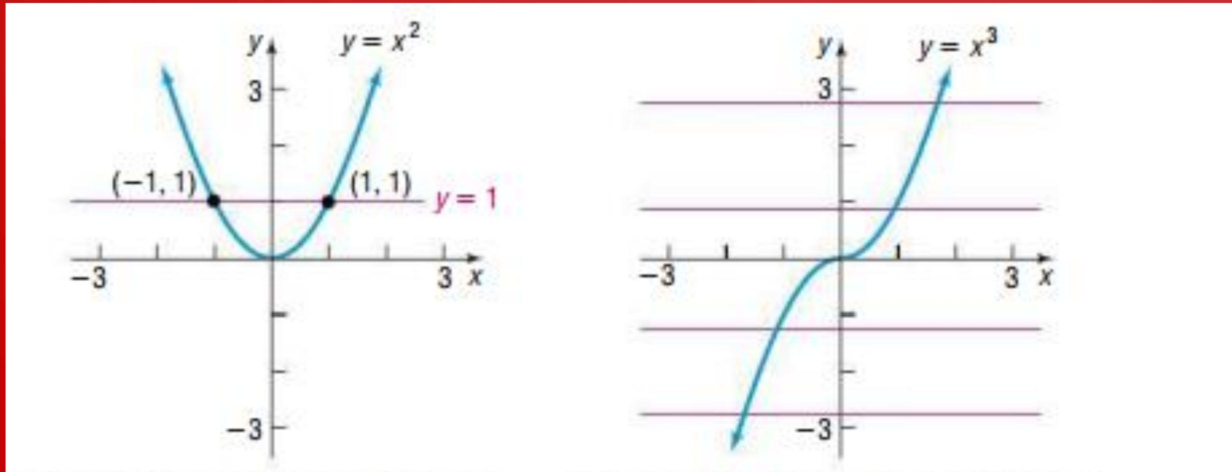
**(b)** Not a one-to-one function: $y_1$ is the image of both $x_1$ and $x_2$.

**(c)** Not a function: $x_1$ has two images, $y_1$ and $y_2$.

A function that is increasing on an interval $I$ is a one-to-one function on $I$.
A function that is decreasing on an interval $I$ is a one-to-one function on $I$.




{ (-2, 6), (-1, 3), (0, 2), (1,5), (2, 8) }




(a) A horizontal line intersects the graph twice; *f* is not one-to-one.

(b) Every horizontal line intersects the graph exactly once; *g* is one-to-one.

## Definition of Inverse

Suppose that $f$ is a one-to-one function. Then, corresponding to each $x$ in the domain of $f$, there is exactly one $y$ in the range (because $f$ is a function); and corresponding to each $y$ in the range of $f$, there is exactly one $x$ in the domain (because $f$ is one-to-one). The correspondence from the range of $f$ back to the domain of $f$ is called the **inverse function of $f$**. The symbol $f^{-1}$ is used to denote the inverse function of $f$.

## Relationship of Domain and Range

Domain of $f$ = Range of $f^{-1}$      Range of $f$ = Domain of $f^{-1}$

## Verifying an Inverse

$f^{-1}(f(x)) = x$ where $x$ is in the domain of $f$

$f(f^{-1}(x)) = x$ where $x$ is in the domain of $f^{-1}$

## Symmetry of an Inverse

The graph of a one-to-one function $f$ and the graph of its inverse function $f^{-1}$ are symmetric with respect to the line $y = x$.

## Procedure for Finding the Inverse of a One-to-One Function

**STEP 1:** In $y = f(x)$, interchange the variables $x$ and $y$ to obtain

$$x = f(y)$$

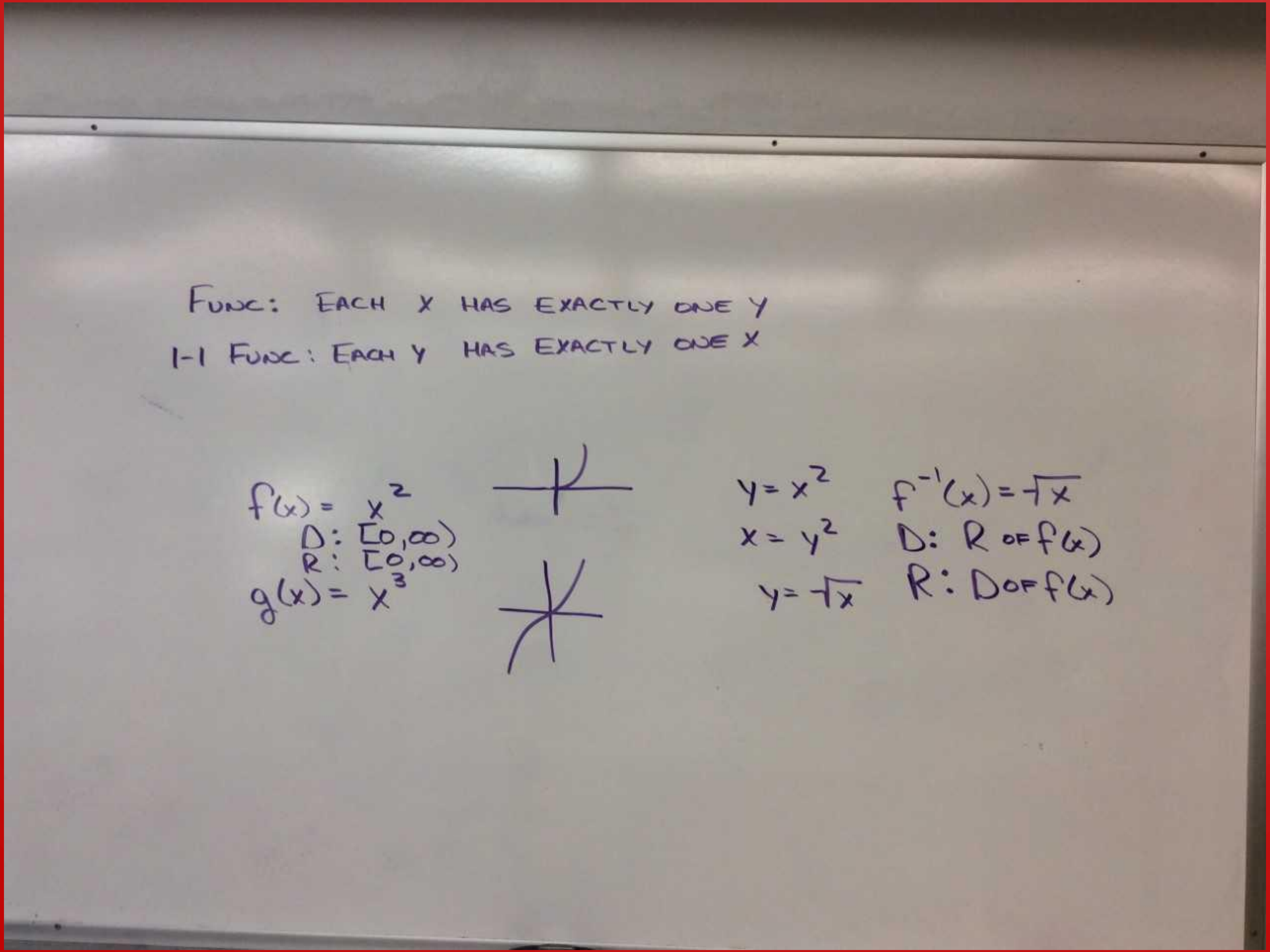
This equation defines the inverse function $f^{-1}$ implicitly.

**STEP 2:** If possible, solve the implicit equation for $y$ in terms of $x$ to obtain the explicit form of $f^{-1}$:

$$y = f^{-1}(x)$$

**STEP 3:** Check the result by showing that

$$f^{-1}(f(x)) = x \quad \text{and} \quad f(f^{-1}(x)) = x$$

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$$f(x) = \frac{1}{x-1}$$

$$D: \{x \mid x \neq 1\}$$

$$f^{-1}(x) = \frac{x+1}{x}$$

$$D: \{x \mid x \neq 0\}$$

$$y = \frac{1}{x-1}$$

$$x = \frac{1}{y-1}$$

$$xy - x = 1$$

$$xy = x + 1$$

$$y = \frac{x+1}{x}$$