BOARD NOTES

29 OCTOBER 2018

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CHAPTER 4 –
EXPONENTIAL AND LOGARITHMIC FUNCTIONS

- SECTION 4.2 - LOGARITHMIC FUNCTIONS

Objectives:

- Change from logarithmic to exponential form
- Change from exponential to logarthimic form
- Evaluate logarithms
- Use basic logarithmic propertites
- Use inverse logarithmic propertites
- Use common and natural logs

Definition of the Logarithmic Function

For $x > 0$ and $b > 0$, $b \neq 1$,

$$y = \log_b x \text{ is equivalent to } b^y = x.$$

The function $f(x) = \log_b x$ is the **logarithmic function with base b.**

Location of Base and Exponent in Exponential and Logarithmic Forms

Exponent

Logarithmic Form: $y = \log_b x$

Base

Exponent

Exponential Form: $b^y = x$

Base

Basic Logarithmic Properties Involving One

1. $\log_b b = 1$ because 1 is the exponent to which b must be raised to obtain b. $(b^1 = b)$
2. $\log_b 1 = 0$ because 0 is the exponent to which b must be raised to obtain 1. $(b^0 = 1)$

Inverse Properties of Logarithms

For $b > 0$ and $b \neq 1$,

$\log_b b^x = x$ — The logarithm with base b of b raised to a power equals that power.

$b^{\log_b x} = x.$ — b raised to the logarithm with base b of a number equals that number.

Characteristics of Logarithmic Functions of the Form $f(x) = \log_b x$

1. The domain of $f(x) = \log_b x$ consists of all positive real numbers: $(0, \infty)$. The range of $f(x) = \log_b x$ consists of all real numbers: $(-\infty, \infty)$.
2. The graphs of all logarithmic functions of the form $f(x) = \log_b x$ pass through the point $(1, 0)$ because $f(1) = \log_b 1 = 0$. The x-intercept is 1. There is no y-intercept.
3. If $b > 1$, $f(x) = \log_b x$ has a graph that goes up to the right and is an increasing function.
4. If $0 < b < 1$, $f(x) = \log_b x$ has a graph that goes down to the right and is a decreasing function.
5. The graph of $f(x) = \log_b x$ approaches, but does not touch, the y-axis. The y-axis, or $x = 0$, is a vertical asymptote.

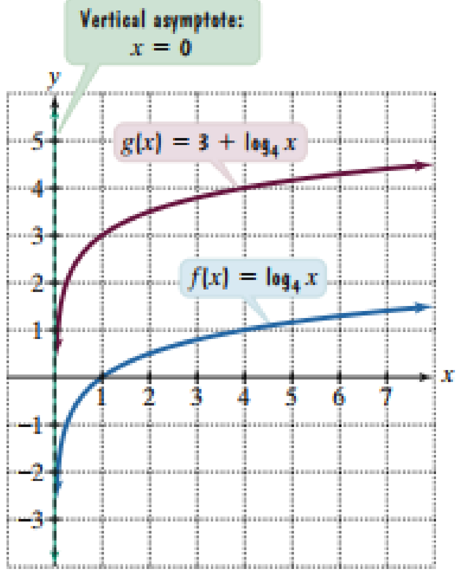


FIGURE 4.10 Shifting vertically up three units

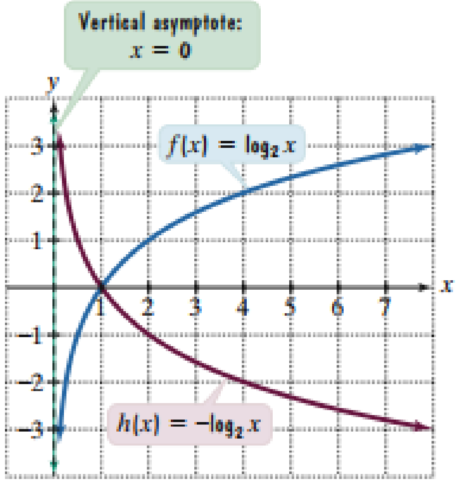


FIGURE 4.11 Reflection about the x-axis

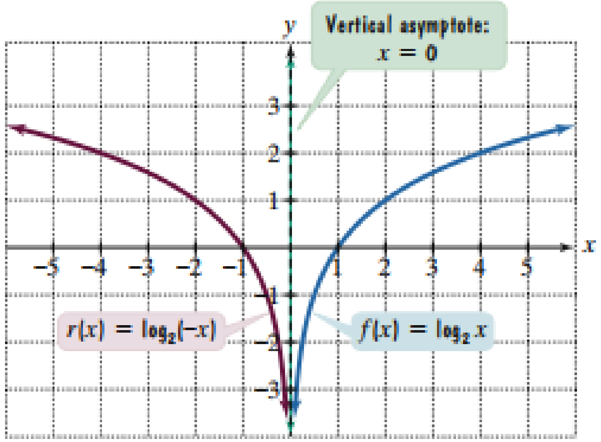


FIGURE 4.12 Reflection about the y-axis

Properties of Common Logarithms

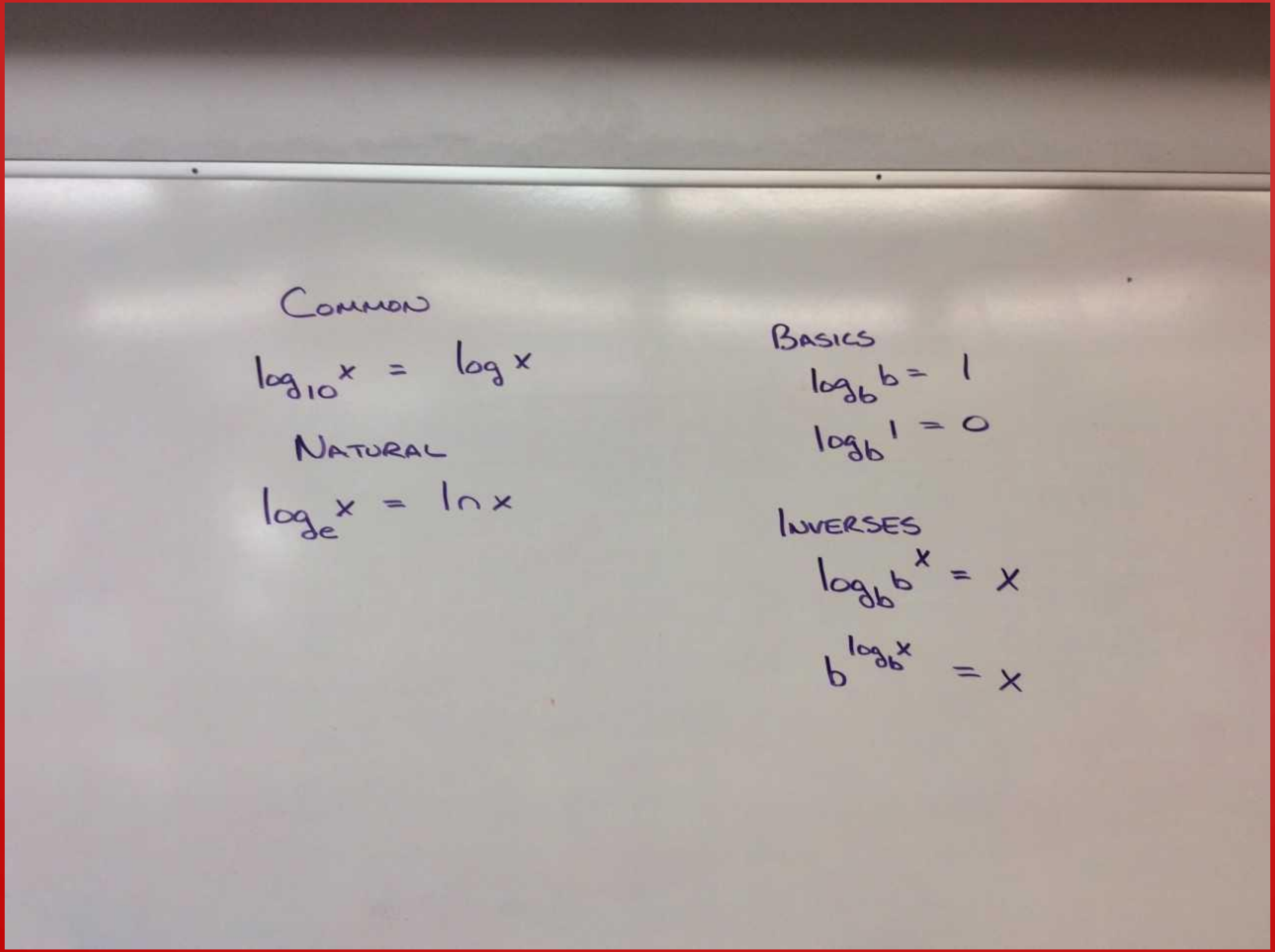
General Properties	Common Logarithms
1. $\log_b 1 = 0$	1. $\log 1 = 0$
2. $\log_b b = 1$	2. $\log 10 = 1$
3. $\log_b b^x = x$	3. $\log 10^x = x$
4. $b^{\log_b x} = x$	4. $10^{\log x} = x$

Inverse properties (3 and 4)

Properties of Natural Logarithms

General Properties	Natural Logarithms
1. $\log_b 1 = 0$	1. $\ln 1 = 0$
2. $\log_b b = 1$	2. $\ln e = 1$
3. $\log_b b^x = x$	3. $\ln e^x = x$
4. $b^{\log_b x} = x$	4. $e^{\ln x} = x$

Inverse properties (3 and 4)


Common
$\log_{10} x = \log x$
Natural
$\log_e x = \ln x$
Basics
$\log_b b = 1$
$\log_b 1 = 0$
Inverses
$\log_b b^x = x$
$b^{\log_b x} = x$

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CHAPTER 4 –
EXPONENTIAL AND LOGARITHMIC FUNCTIONS

- SECTION 4.3 - PROPERTIES OF LOGARITHMS

Objectives:

- Use the product rule
- Use the quotient rule
- Use the power rule
- Expand logarithmic expressions
- Condense logarithmic expressions
- Use the change of base property

The Product Rule

Let b, M, and N be positive real numbers with $b \neq 1$.

$$\log_b(MN) = \log_b M + \log_b N$$

The logarithm of a product is the sum of the logarithms.

The Quotient Rule

Let b, M, and N be positive real numbers with $b \neq 1$.

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

The logarithm of a quotient is the difference of the logarithms.

The Power Rule

Let b and M be positive real numbers with $b \neq 1$, and let p be any real number.

$$\log_b M^p = p \log_b M$$

The logarithm of a number with an exponent is the product of the exponent and the logarithm of that number.

Properties for Expanding Logarithmic Expressions

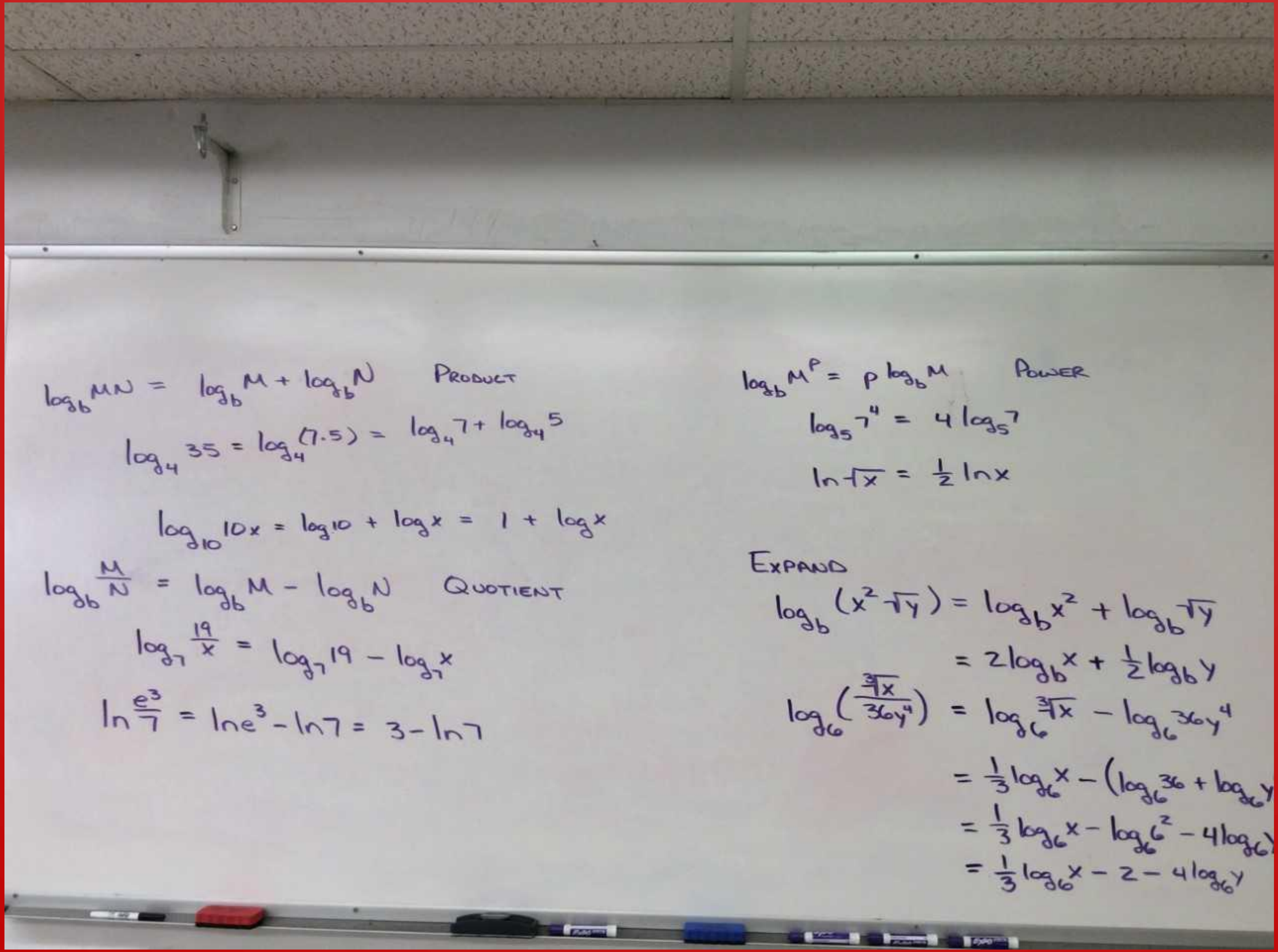
For $M > 0$ and $N > 0$:

1. $\log_b (MN) = \log_b M + \log_b N$ Product rule
2. $\log_b \left(\frac{M}{N}\right) = \log_b M - \log_b N$ Quotient rule
3. $\log_b M^p = p \log_b M$ Power rule

Properties for Condensing Logarithmic Expressions

For $M > 0$ and $N > 0$:

1. $\log_b M + \log_b N = \log_b(MN)$ Product rule
2. $\log_b M - \log_b N = \log_b\left(\frac{M}{N}\right)$ Quotient rule
3. $p \log_b M = \log_b M^p$ Power rule


$\log_b MN = \log_b M + \log_b N$ PRODUCT
$\log_4 35 = \log_4 (7 \cdot 5) = \log_4 7 + \log_4 5$
$\log_{10} 10x = \log 10 + \log x = 1 + \log x$
$\log_b \frac{M}{N} = \log_b M - \log_b N$ QUOTIENT
$\log_7 \frac{19}{x} = \log_7 19 - \log_7 x$
$\ln \frac{e^3}{7} = \ln e^3 - \ln 7 = 3 - \ln 7$
$\log_b M^p = p \log_b M$ POWER
$\log_5 7^4 = 4 \log_5 7$
$\ln \sqrt{x} = \frac{1}{2} \ln x$
EXPAND
$\log_b (x^2 \sqrt{y}) = \log_b x^2 + \log_b \sqrt{y}$
$= 2 \log_b x + \frac{1}{2} \log_b y$
$\log_6 \left(\frac{\sqrt[3]{x}}{36y^4}\right) = \log_6 \sqrt[3]{x} - \log_6 36y^4$
$= \frac{1}{3} \log_6 x - (\log_6 36 + \log_6 y$
$= \frac{1}{3} \log_6 x - \log_6 6^2 - 4 \log_6 y$
$= \frac{1}{3} \log_6 x - 2 - 4 \log_6 y$

Condense

$$\log_4 2 + \log_4 32 = \log_4 64 = \log_4 4^3 = 3$$

$$\log(4x-3) - \log x = \log \frac{4x-3}{x}$$

$$\frac{1}{2}\log x + 4\log(x-1) + 1 = \log x^{\frac{1}{2}} + \log(x-1)^4 + \log 10$$

$$= \log \sqrt{x}(x-1)^4 + \log 10$$

$$= \log 10\sqrt{x}(x-1)^4$$

Change of Base

$$\log_b M = \frac{\log M}{\log b}$$

OR

$$\log_b M = \frac{\ln M}{\ln b}$$

The Change-of-Base Property

For any logarithmic bases a and b, and any positive number M,

$$\log_b M = \frac{\log_a M}{\log_a b}.$$

The logarithm of M with base b is equal to the logarithm of M with any new base divided by the logarithm of b with that new base.

The Change-of-Base Property: Introducing Common and Natural Logarithms

Introducing Common Logarithms

$$\log_b M = \frac{\log M}{\log b}$$

Introducing Natural Logarithms

$$\log_b M = \frac{\ln M}{\ln b}$$