BOARD NOTES

29 OCTOBER 2018

CC PRECALCULUS
CHAPTER 5 –
EXPONENTIAL AND LOGARITHMIC FUNCTIONS

- SECTION 5.2 - ONE-TO-ONE FUNCTIONS; INVERSE FUNCTIONS

Objectives:

- Determine if a function is injective
- Determine the inverse of a function defined by a map or a set of ordered pairs
- Obtain the graph of the inverse function from the graph of the function
- Find the inverse of a function defined by an equation

Definition of Inverse

Suppose that f is a one-to-one function. Then, corresponding to each x in the domain of f, there is exactly one y in the range (because f is a function); and corresponding to each y in the range of f, there is exactly one x in the domain (because f is one-to-one). The correspondence from the range of f back to the domain of f is called the **inverse function of** f. The symbol f^{-1} is used to denote the inverse function of f.

Relationship of Domain and Range

Domain of f = Range of f^{-1} Range of f = Domain of f^{-1}

Verifying an Inverse

$f^{-1}(f(x)) = x$ where x is in the domain of f

$f(f^{-1}(x)) = x$ where x is in the domain of f^{-1}

Symmetry of an Inverse

The graph of a one-to-one function f and the graph of its inverse function f^{-1} are symmetric with respect to the line $y = x$.

Procedure for Finding the Inverse of a One-to-One Function

Step 1: In $y = f(x)$, interchange the variables x and y to obtain

$$x = f(y)$$

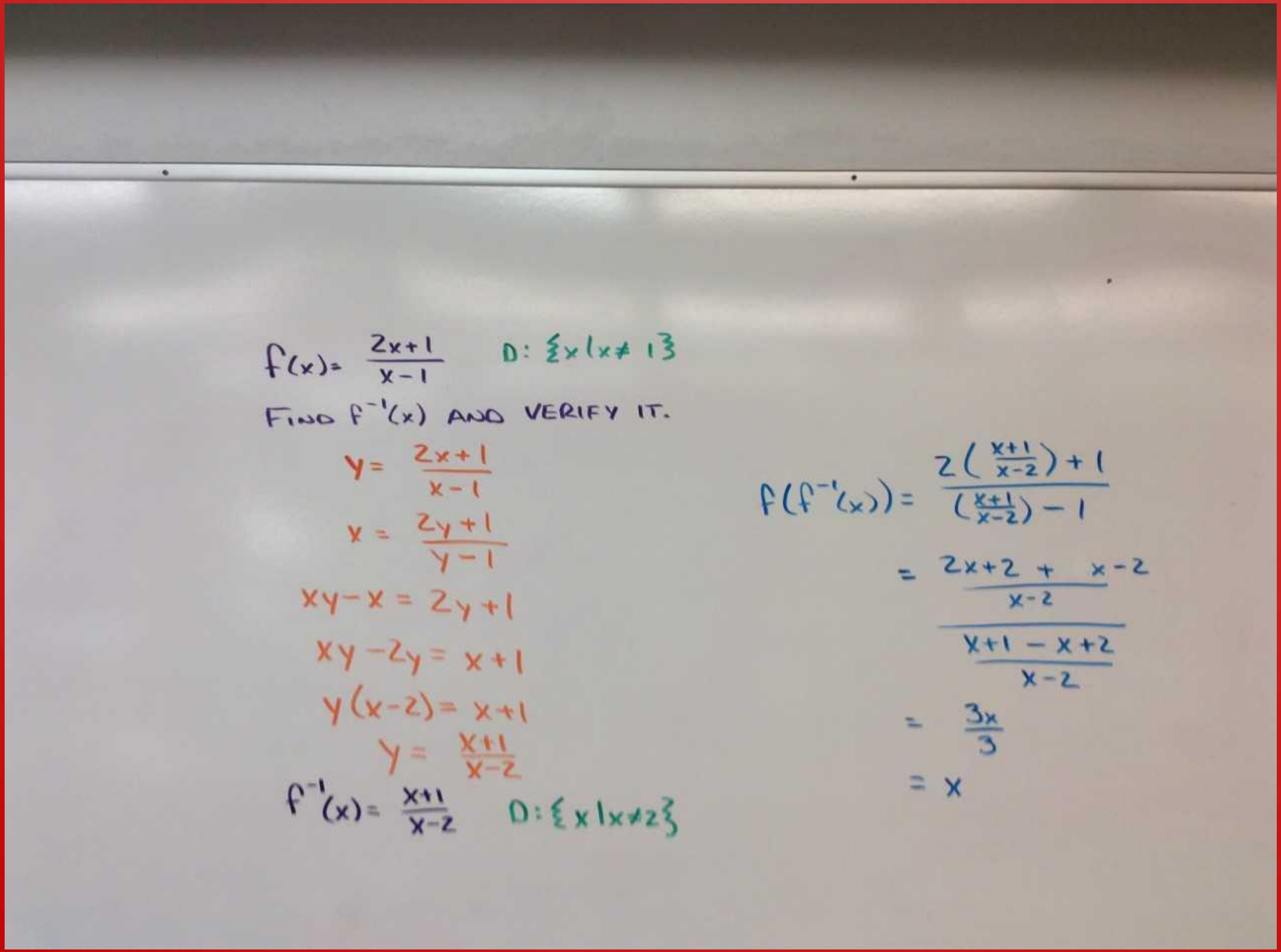
This equation defines the inverse function f^{-1} implicitly.

Step 2: If possible, solve the implicit equation for y in terms of x to obtain the explicit form of f^{-1}:

$$y = f^{-1}(x)$$

Step 3: Check the result by showing that

$$f^{-1}(f(x)) = x \quad \text{and} \quad f(f^{-1}(x)) = x$$


f(x) = (2x+1)/(x-1)
D: {x | x ≠ 1}
FIND f⁻¹(x) AND VERIFY IT.
y = (2x+1)/(x-1)
x = (2y+1)/(y-1)
xy - x = 2y + 1
xy - 2y = x + 1
y(x-2) = x+1
y = (x+1)/(x-2)
f⁻¹(x) = (x+1)/(x-2)
D: {x | x ≠ 2}
f(f⁻¹(x)) = (2((x+1)/(x-2)) + 1) / (((x+1)/(x-2)) - 1)
= ((2x+2 + x-2)/(x-2)) / ((x+1 - x+2)/(x-2))
= 3x/3
= x

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CHAPTER 5 –
EXPONENTIAL AND LOGARITHMIC FUNCTIONS

- SECTION 5.3 - EXPONENTIAL FUNCTIONS

Objectives:

- Evaluate exponential functions
- Graph exponential functions
- Define the number e
- Solve exponential equations

Brief Review: Law of Exponents

Laws of Exponents

If s, t, a, and b are real numbers with $a > 0$ and $b > 0$, then

$$a^s \cdot a^t = a^{s+t} \qquad (a^s)^t = a^{st} \qquad (ab)^s = a^s \cdot b^s$$

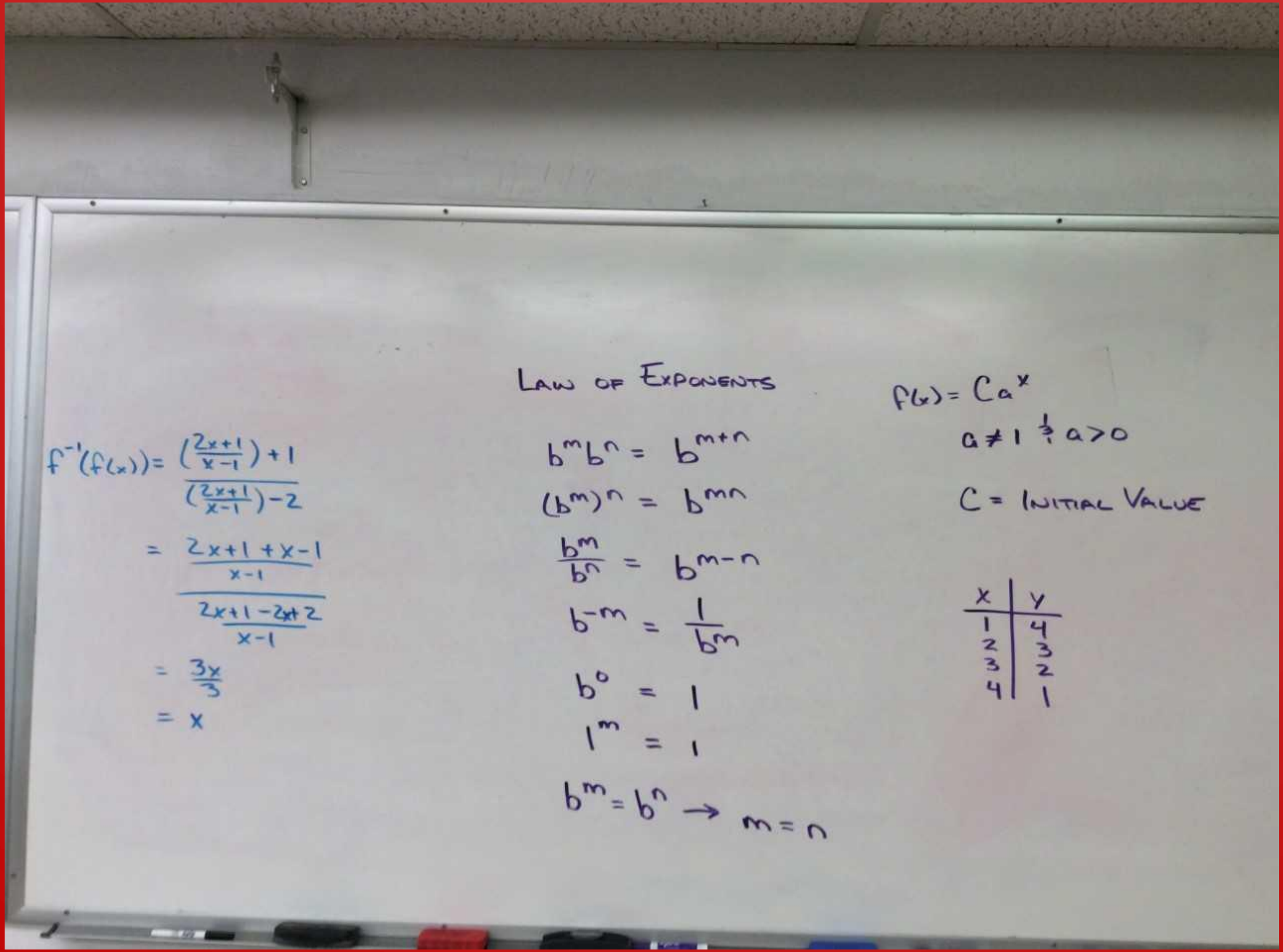
$$1^s = 1 \qquad a^{-s} = \frac{1}{a^s} = \left(\frac{1}{a}\right)^s \qquad a^0 = 1 \qquad \textbf{(1)}$$

Definition of Exponential Function

An **exponential function** is a function of the form

$$f(x) = Ca^x$$

where a is a positive real number $(a > 0)$, $a \neq 1$, and $C \neq 0$ is a real number. The domain of f is the set of all real numbers. The base a is the **growth factor**, and, because $f(0) = Ca^0 = C$, C is called the **initial value.**



f⁻¹(f(x)) = ((2x+1)/(x-1) + 1) / ((2x+1)/(x-1) - 2)
= ((2x+1+x-1)/(x-1)) / ((2x+1-2x+2)/(x-1))
= 3x/3
= x
LAW OF EXPONENTS
b^m b^n = b^(m+n)
(b^m)^n = b^(mn)
b^m / b^n = b^(m-n)
b^-m = 1/b^m
b^0 = 1
1^m = 1
b^m = b^n → m = n
f(x) = Ca^x
a ≠ 1 ¢ a > 0
C = INITIAL VALUE
x | y
1 | 4
2 | 3
3 | 2
4 | 1

Linear Model

Average Rate of Change of a Linear Function

Linear functions have a constant average rate of change. That is, the average rate of change of a linear function $f(x) = mx + b$ is

$$\frac{\Delta y}{\Delta x} = m$$

Quadratic Model - Have to visualize the data

Exponential Model

For an exponential function $f(x) = Ca^x$, $a > 0$, $a \neq 1$, and $C \neq 0$, if x is any real number, then

$$\frac{f(x+1)}{f(x)} = a \quad \text{or} \quad f(x+1) = af(x)$$

(a)

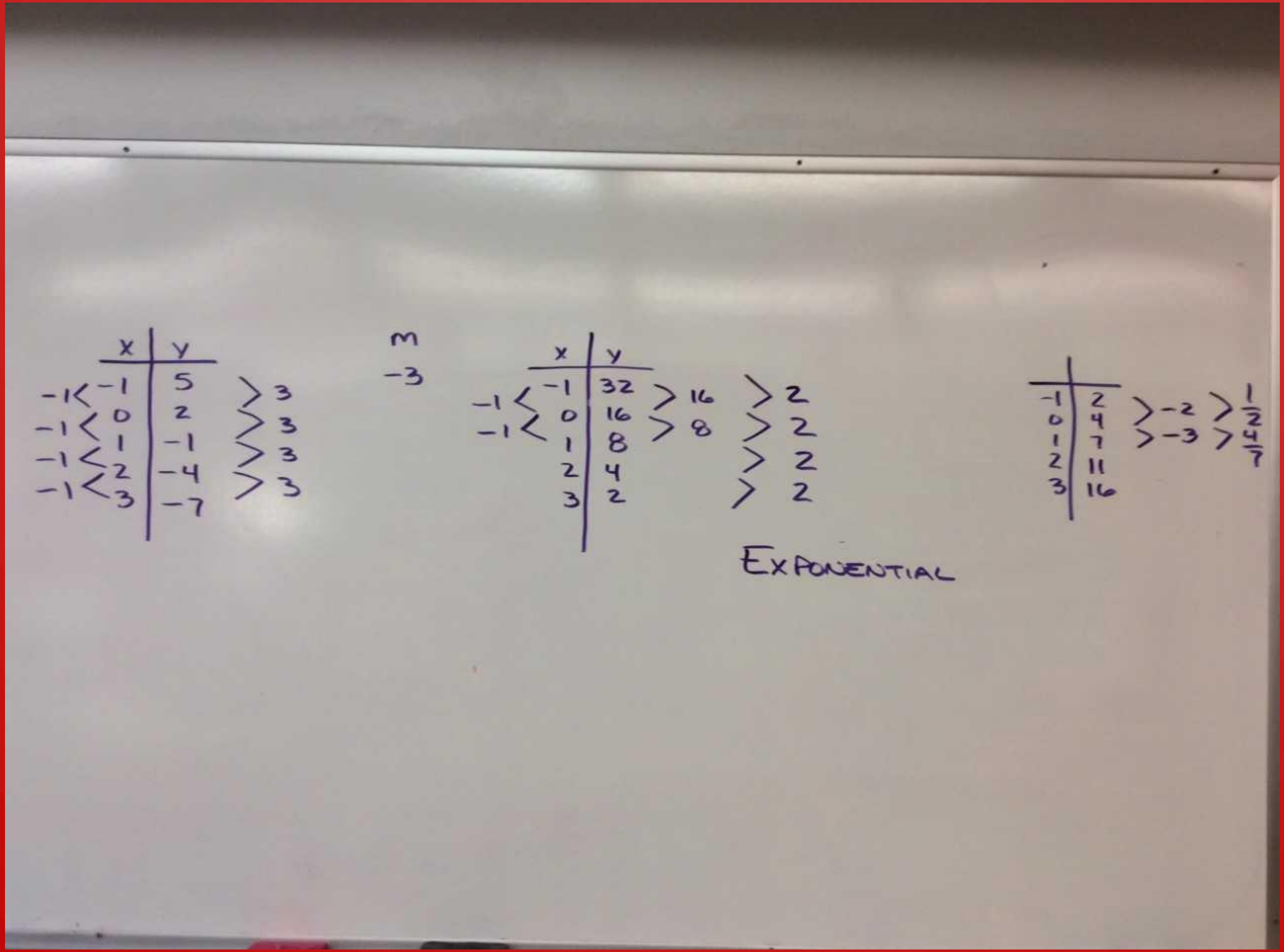
x	y
−1	5
0	2
1	−1
2	−4
3	−7

(b)

x	y
−1	32
0	16
1	8
2	4
3	2

(c)

x	y
−1	2
0	4
1	7
2	11
3	16



x | y
-1 | 5
0 | 2
1 | -1
2 | -4
3 | -7
-1 -1 -1 -1
3 3 3 3
m
-3
x | y
-1 | 32
0 | 16
1 | 8
2 | 4
3 | 2
-1 -1
16 8
2 2 2 2
EXPONENTIAL
-1 | 2
0 | 4
1 | 7
2 | 11
3 | 16
-2 -3
1/2 4/7

Exponential Growth Model

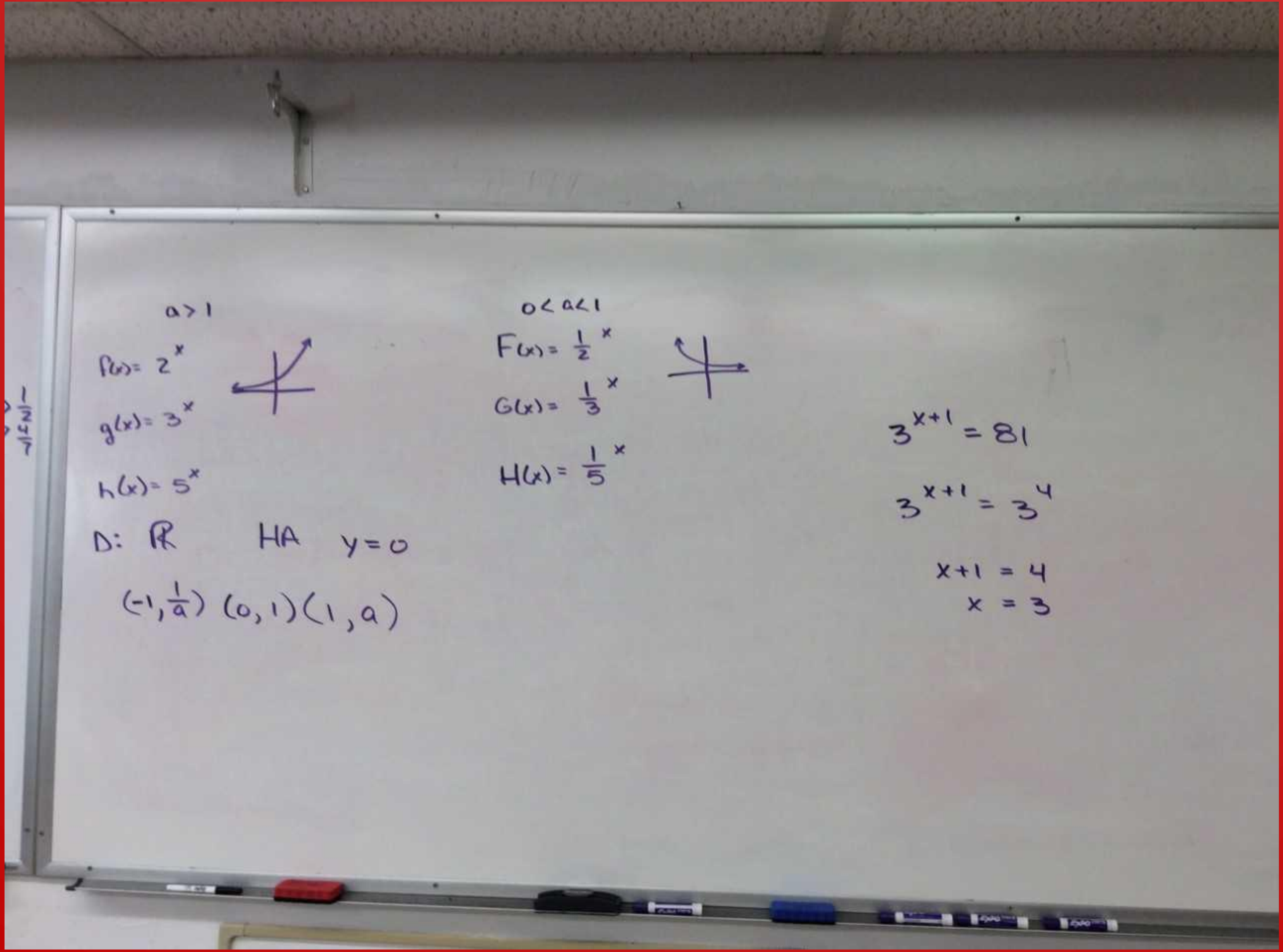
Properties of the Exponential Function $f(x) = a^x, a > 1$

1. The domain is the set of all real numbers, or $(-\infty, \infty)$ using interval notation; the range is the set of positive real numbers, or $(0, \infty)$ using interval notation.
2. There are no x-intercepts; the y-intercept is 1.
3. The x-axis $(y = 0)$ is a horizontal asymptote as $x \to -\infty$. $\left[\lim_{x \to -\infty} a^x = 0\right]$.
4. $f(x) = a^x, a > 1$, is an increasing function and is one-to-one.
5. The graph of f contains the points $\left(-1, \frac{1}{a}\right)$, $(0, 1)$, and $(1, a)$.
6. The graph of f is smooth and continuous, with no corners or gaps. See Figure 21.

Exponential Decay Model

Properties of the Exponential Function $f(x) = a^x, 0 < a < 1$

1. The domain is the set of all real numbers, or $(-\infty, \infty)$ using interval notation; the range is the set of positive real numbers, or $(0, \infty)$ using interval notation.
2. There are no x-intercepts; the y-intercept is 1.
3. The x-axis $(y = 0)$ is a horizontal asymptote as $x \to \infty$ $\left[\lim_{x\to\infty} a^x = 0\right]$.
4. $f(x) = a^x, 0 < a < 1$, is a decreasing function and is one-to-one.
5. The graph of f contains the points $\left(-1, \frac{1}{a}\right)$, $(0, 1)$, and $(1, a)$.
6. The graph of f is smooth and continuous, with no corners or gaps. See Figure 25.



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Euler's Number

The **number *e*** is defined as the number that the expression

$$\left(1 + \frac{1}{n}\right)^n \qquad \textbf{(2)}$$

approaches as $n \to \infty$. In calculus, this is expressed, using limit notation, as

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$