BOARD NOTES

30 OCTOBER 2018

CC ALGEBRA
CHAPTER 4 –
EXPONENTIAL AND LOGARITHMIC FUNCTIONS

- SECTION 4.3 - PROPERTIES OF LOGARITHMS

Objectives:

- Use the product rule
- Use the quotient rule
- Use the power rule
- Expand logarithmic expressions
- Condense logarithmic expressions
- Use the change of base property

The Product Rule

Let b, M, and N be positive real numbers with $b \neq 1$.

$$\log_b(MN) = \log_b M + \log_b N$$

The logarithm of a product is the sum of the logarithms.

The Quotient Rule

Let b, M, and N be positive real numbers with $b \neq 1$.

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

The logarithm of a quotient is the difference of the logarithms.

The Power Rule

Let b and M be positive real numbers with $b \neq 1$, and let p be any real number.

$$\log_b M^p = p \log_b M$$

The logarithm of a number with an exponent is the product of the exponent and the logarithm of that number.

Properties for Expanding Logarithmic Expressions

For $M > 0$ and $N > 0$:

1. $\log_b(MN) = \log_b M + \log_b N$ Product rule
2. $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$ Quotient rule
3. $\log_b M^p = p \log_b M$ Power rule

Properties for Condensing Logarithmic Expressions

For $M > 0$ and $N > 0$:

1. $\log_b M + \log_b N = \log_b(MN)$ Product rule
2. $\log_b M - \log_b N = \log_b\left(\frac{M}{N}\right)$ Quotient rule
3. $p \log_b M = \log_b M^p$ Power rule

The Change-of-Base Property

For any logarithmic bases a and b, and any positive number M,

$$\log_b M = \frac{\log_a M}{\log_a b}.$$

The logarithm of M with base b is equal to the logarithm of M with any new base divided by the logarithm of b with that new base.

The Change-of-Base Property: Introducing Common and Natural Logarithms

Introducing Common Logarithms

$$\log_b M = \frac{\log M}{\log b}$$

Introducing Natural Logarithms

$$\log_b M = \frac{\ln M}{\ln b}$$

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CHAPTER 4 –
EXPONENTIAL AND LOGARITHMIC FUNCTIONS

- SECTION 4.4 - EXPONENTIAL AND LOGARITHMIC EQUATIONS

Objectives:

- Use like bases to solve exponential equations
- Use logarithms to solve exponential equations
- Use the definition of logarithm to solve logarithmic equations
- Use one-to-one property of logarithms to solve logarithmic equations
- Solve applied problems involving exponential and logarithmic equations

Solving Exponential Equations by Expressing Each Side as a Power of the Same Base

$$\text{If } b^M = b^N, \text{ then } M = N.$$

Express each side as a power of the same base.

Set the exponents equal to each other.

1. Rewrite the equation in the form $b^M = b^N$.
2. Set $M = N$.
3. Solve for the variable.

Using Logarithms to Solve Exponential Equations

1. Isolate the exponential expression.
2. Take the common logarithm on both sides of the equation for base 10. Take the natural logarithm on both sides of the equation for bases other than 10.
3. Simplify using one of the following properties:

$$\ln b^x = x \ln b \quad \text{or} \quad \ln e^x = x \quad \text{or} \quad \log 10^x = x.$$

4. Solve for the variable.

Using the Definition of a Logarithm to Solve Logarithmic Equations

1. Express the equation in the form $\log_b M = c$.
2. Use the definition of a logarithm to rewrite the equation in exponential form:

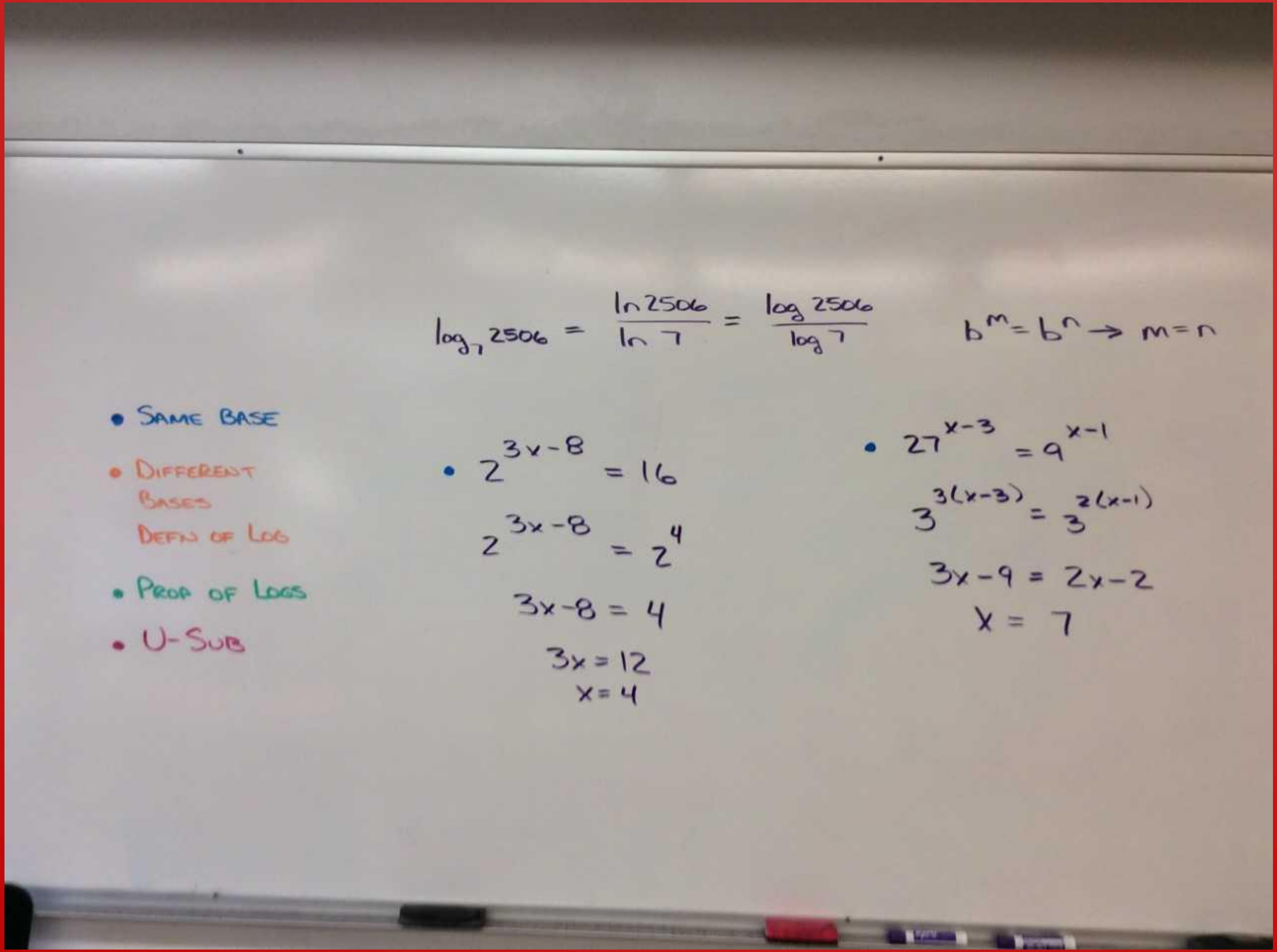
$$\log_b M = c \quad \text{means} \quad b^c = M.$$

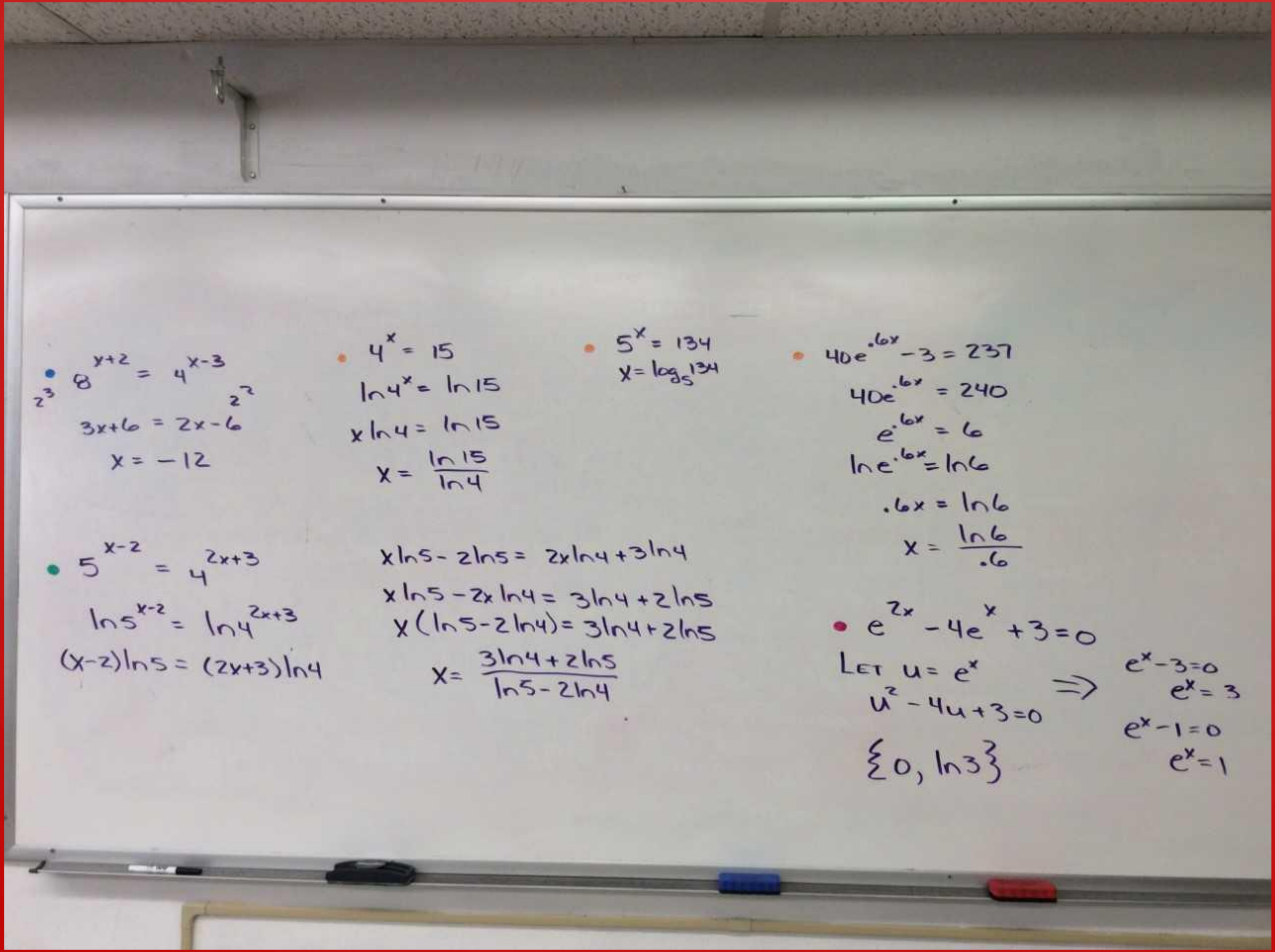
Logarithms are exponents.

3. Solve for the variable.
4. Check proposed solutions in the original equation. Include in the solution set only values for which $M > 0$.

Using the One-to-One Property of Logarithms to Solve Logarithmic Equations

1. Express the equation in the form $\log_b M = \log_b N$. This form involves a single logarithm whose coefficient is 1 on each side of the equation.
2. Use the one-to-one property to rewrite the equation without logarithms: If $\log_b M = \log_b N$, then $M = N$.
3. Solve for the variable.
4. Check proposed solutions in the original equation. Include in the solution set only values for which $M > 0$ and $N > 0$.


$\log_7 2506 = \frac{\ln 2506}{\ln 7} = \frac{\log 2506}{\log 7}$
$b^m = b^n \rightarrow m = n$
• Same Base
• Different Bases
Defn of Log
• Prop of Logs
• U-Sub
• $2^{3x-8} = 16$
$2^{3x-8} = 2^4$
$3x-8 = 4$
$3x = 12$
$x = 4$
• $27^{x-3} = 9^{x-1}$
$3^{3(x-3)} = 3^{2(x-1)}$
$3x-9 = 2x-2$
$x = 7$



$8^{x+2} = 4^{x-3}$
2^3 2^2
$3x+6 = 2x-6$
$x = -12$
$4^x = 15$
$\ln 4^x = \ln 15$
$x \ln 4 = \ln 15$
$x = \frac{\ln 15}{\ln 4}$
$5^x = 134$
$x = \log_5 134$
$40e^{.6x} - 3 = 237$
$40e^{.6x} = 240$
$e^{.6x} = 6$
$\ln e^{.6x} = \ln 6$
$.6x = \ln 6$
$x = \frac{\ln 6}{.6}$
$5^{x-2} = 4^{2x+3}$
$\ln 5^{x-2} = \ln 4^{2x+3}$
$(x-2)\ln 5 = (2x+3)\ln 4$
$x\ln 5 - 2\ln 5 = 2x\ln 4 + 3\ln 4$
$x\ln 5 - 2x\ln 4 = 3\ln 4 + 2\ln 5$
$x(\ln 5 - 2\ln 4) = 3\ln 4 + 2\ln 5$
$x = \frac{3\ln 4 + 2\ln 5}{\ln 5 - 2\ln 4}$
$e^{2x} - 4e^x + 3 = 0$
Let $u = e^x$
$u^2 - 4u + 3 = 0$ $\Rightarrow$
$e^x - 3 = 0$
$e^x = 3$
$e^x - 1 = 0$
$e^x = 1$
$\{0, \ln 3\}$