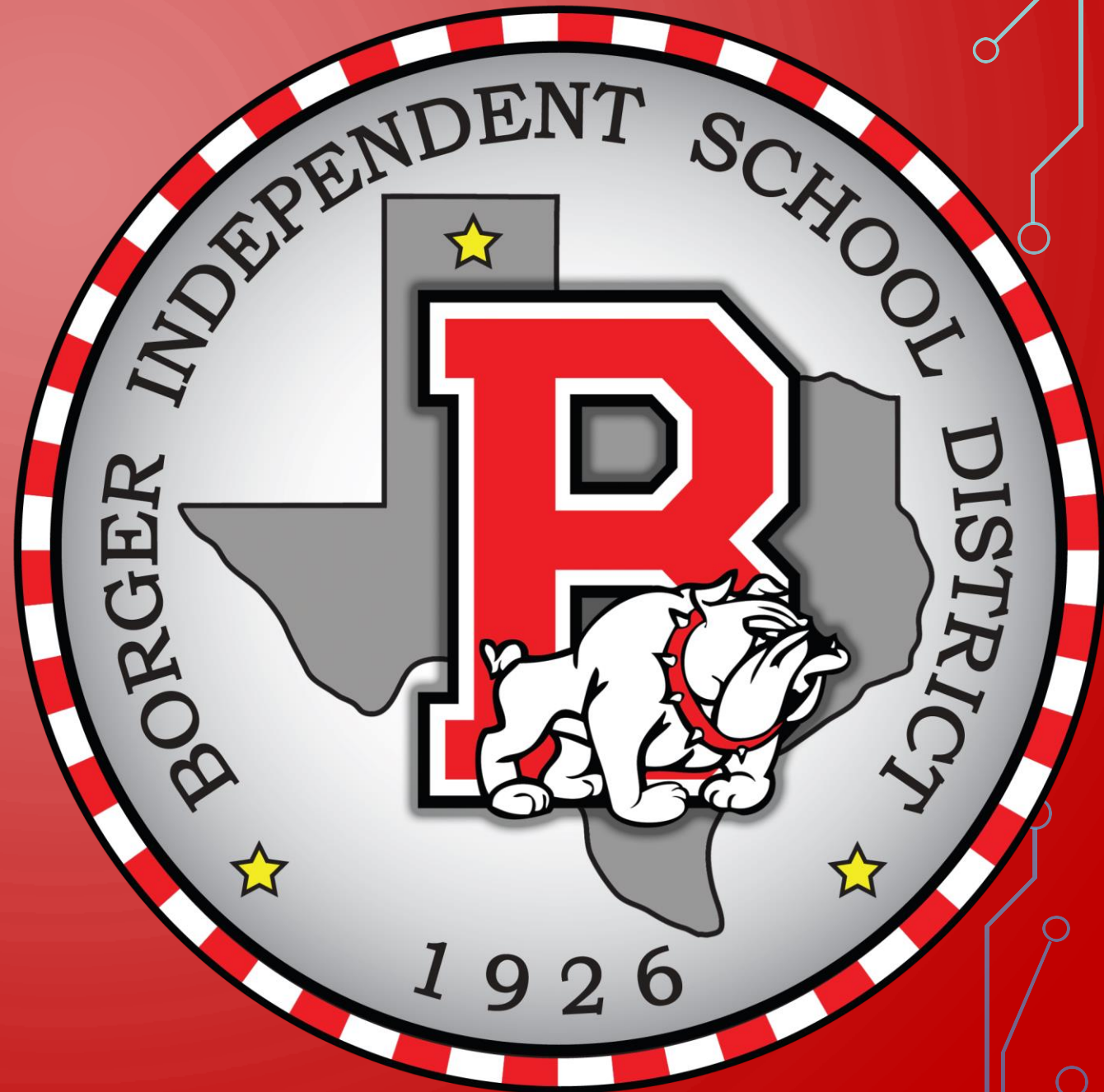


BOARD NOTES

30 OCTOBER 2018



CC PRECALCULUS

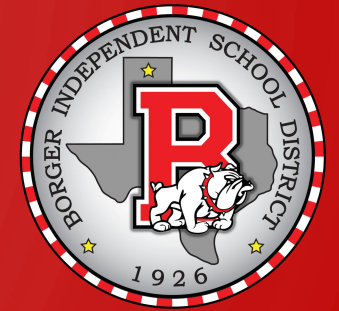
CHAPTER 5 –

EXPONENTIAL AND LOGARITHMIC FUNCTIONS

- SECTION 5.3 - EXPONENTIAL FUNCTIONS

Objectives:

- Evaluate exponential functions
- Graph exponential functions
- Define the number e
- Solve exponential equations



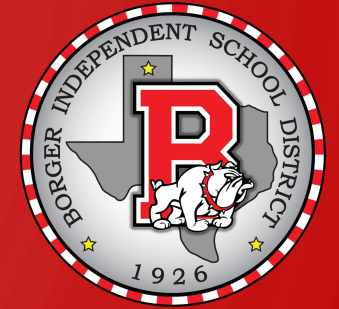
Euler's Number

The number e is defined as the number that the expression

$$\left(1 + \frac{1}{n}\right)^n \quad (2)$$

approaches as $n \rightarrow \infty$. In calculus, this is expressed, using limit notation, as

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$



SOLVE FOR X

$$4^{2x-1} = 8^{x+3}$$

$$2^{2(2x-1)} = 2^{3(x+3)}$$

$$4x-2 = 3x+9$$

$$x = 11$$

$$e^{-x^2} = (e^x)^2 \cdot \frac{1}{e^3}$$

$$e^{-x^2} = e^{2x} e^{-3}$$

$$-x^2 = 2x - 3$$

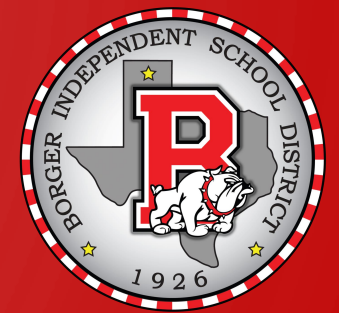
$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

$$x = -3, 1$$

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

$$\approx 2.7182$$



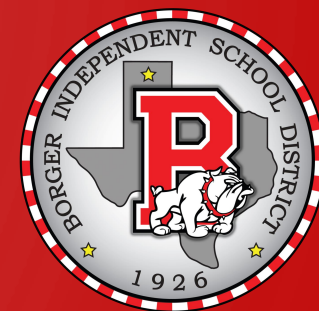
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CHAPTER 5 – EXPONENTIAL AND LOGARITHMIC FUNCTIONS

- SECTION 5.4 - LOGARITHMIC FUNCTIONS

Objectives:

- Change exponential statements to logarithmic statements and vice versa
- Evaluate logarithmic expressions
- Determine the domain of logarithmic functions
- Graph logarithmic functions
- Solve logarithmic equations



Definition of Logarithmic

The logarithmic function with base a , where $a > 0$ and $a \neq 1$, is denoted by $y = \log_a x$ (read as “ y is the logarithm with base a of x ”) and is defined by

$$y = \log_a x \quad \text{if and only if} \quad x = a^y$$

The domain of the logarithmic function $y = \log_a x$ is $x > 0$.

Definition of Natural Logarithmic

$$y = \ln x \quad \text{if and only if} \quad x = e^y$$

Definition of Common Logarithmic

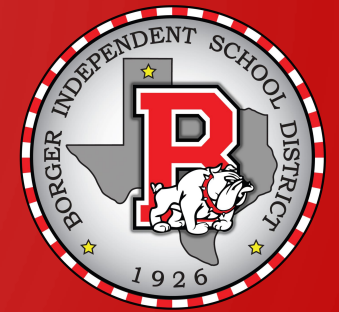
$$y = \log x \quad \text{if and only if} \quad x = 10^y$$

Similarities of Logarithmic and Exponential

Domain of the logarithmic function = Range of the exponential function = $(0, \infty)$

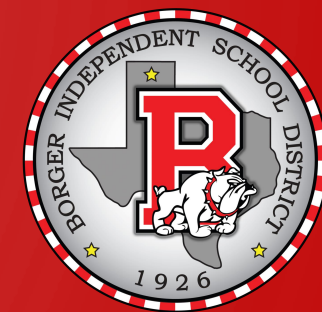
Range of the logarithmic function = Domain of the exponential function = $(-\infty, \infty)$





Properties of the Logarithmic Function $f(x) = \log_a x; a > 0, a \neq 1$

1. The domain is the set of positive real numbers, or $(0, \infty)$ using interval notation; the range is the set of all real numbers, or $(-\infty, \infty)$ using interval notation.
2. The x -intercept of the graph is 1. There is no y -intercept.
3. The y -axis ($x = 0$) is a vertical asymptote of the graph.
4. A logarithmic function is decreasing if $0 < a < 1$ and is increasing if $a > 1$.
5. The graph of f contains the points $(1, 0)$, $(a, 1)$, and $\left(\frac{1}{a}, -1\right)$.
6. The graph is smooth and continuous, with no corners or gaps.



EXPONENTIAL

$$f(x) = b^x$$
$$b > 0, b \neq 1$$

$$D: \mathbb{R}$$
$$R: (0, \infty)$$

$$y = b^x$$

$$\left(-1, \frac{1}{a}\right) (0, 1) (1, a)$$

$$HA \quad y = 0$$

LOGARITHMIC

$$f(x) = \log_b x$$

$$b \neq 1, b > 0$$

$$D: (0, \infty)$$

$$R: \mathbb{R}$$

$$y = \log_b x \equiv b^y = x$$

$$\left(\frac{1}{a}, -1\right) (1, 0) (a, 1)$$

$$VA \quad x = 0$$

MATH ALPH MATH

2, 10, e

$$y = \log_2 x$$

$$y = \log_{10} x = \log x$$

$$y = \log_e x = \ln x$$

$$1.2^3 = m \equiv \log_{1.2} m = 3$$

$$e^b = 9 \equiv \ln 9 = b$$

$$a^4 = 24 \equiv \log_a 24 = 4$$

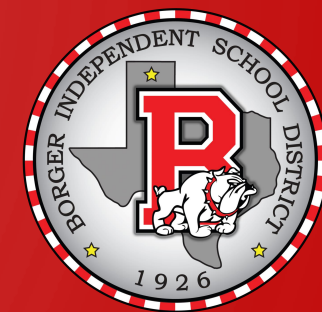
COMMON

NATURAL

$$\log_a 4 = 5 \equiv a^5 = 4$$

$$\ln b = -3 \equiv e^{-3} = b$$

$$\log 5 = c \equiv 10^c = 5$$



$$f(x) = -\ln(x-2)$$

$$y = -\ln(x-2)$$

$$-x = \ln(y-2)$$

$$e^{-x} = y-2$$

$$y = e^{-x} + 2$$

$$f^{-1}(x) = e^{-x} + 2$$

$$f(f^{-1}(x)) = -\ln(e^{-x} + 2 - 2)$$

$$= -\ln e^{-x}$$

$$= x$$

$$f^{-1}(f(x)) = e^{-(-\ln(x-2))} + 2$$

$$= e^{\ln(x-2)} + 2$$

$$= x - 2 + 2$$

$$= x$$

$$e^{2x} = 5$$

$$\ln e^{2x} = \ln 5$$

$$2x = \ln 5$$

$$x = \frac{\ln 5}{2}$$

$$\log_3(4x-7) = 2$$

$$3^2 = 4x-7$$

$$4x = 16$$

$$x = 4$$

