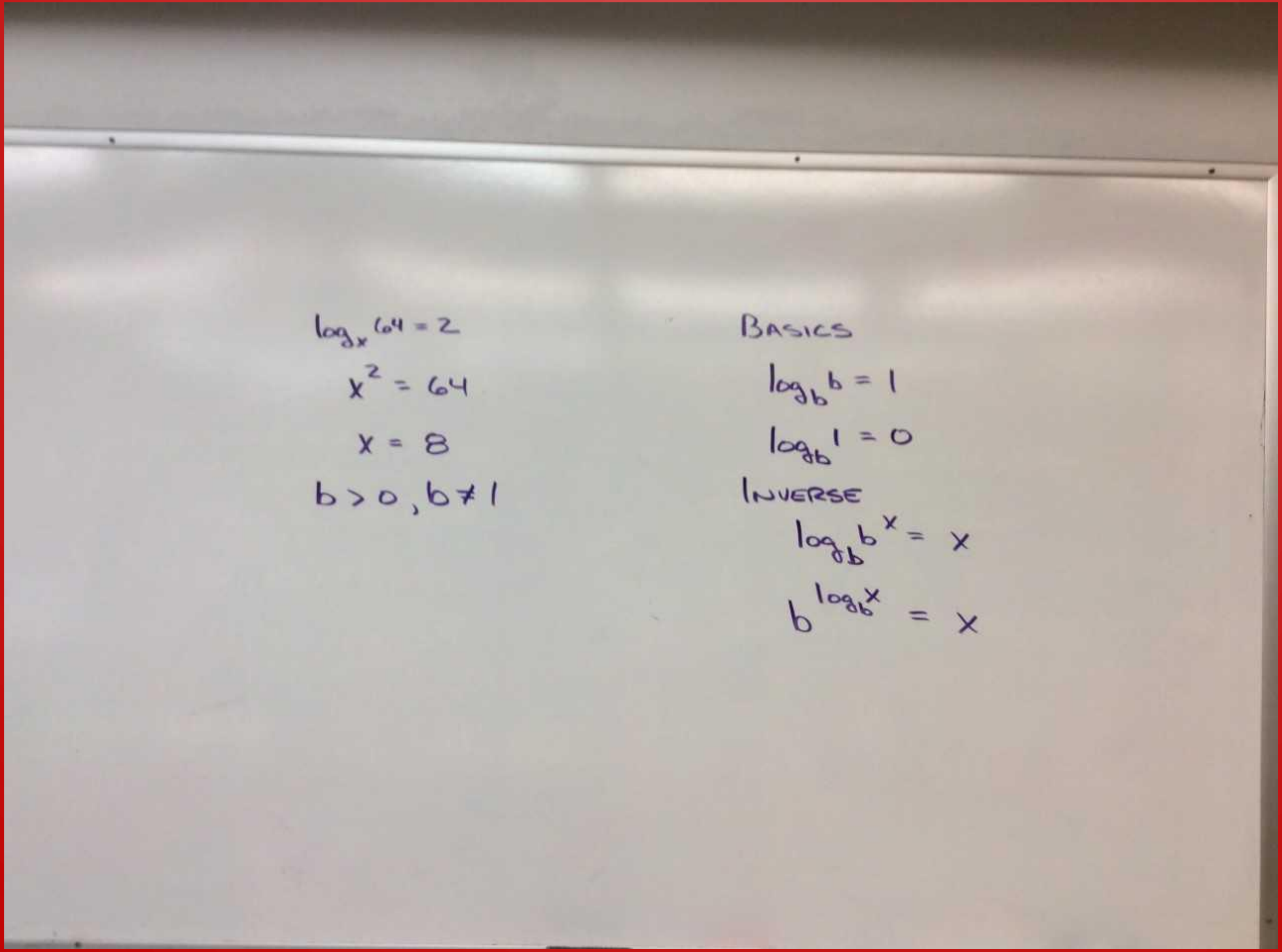
BOARD NOTES

30 OCTOBER 2018



FRANK PHILLIPS COLLEGE Plainsmen

BORGER INDEPENDENT SCHOOL DISTRICT 1926

CC PRECALCULUS
CHAPTER 5 –
EXPONENTIAL AND LOGARITHMIC FUNCTIONS

- SECTION 5.4 - PROPERTIES OF LOGARITHMS

Objectives:

- Work with the properties of logarithms
- Write a logarithmic expression as a sum or difference of logarithms
- Write a sum or difference of logarithms as single logarithmic expression
- Evaluate logarithms whose base is neither 10 nor e

Location of Base and Exponent in Exponential and Logarithmic Forms

Logarithmic Form: $y = \log_b x$ (Exponent: y; Base: b)

Exponential Form: $b^y = x$ (Exponent: y; Base: b)

Basic Logarithmic Properties Involving One

1. $\log_b b = 1$ because 1 is the exponent to which b must be raised to obtain b. $(b^1 = b)$
2. $\log_b 1 = 0$ because 0 is the exponent to which b must be raised to obtain 1. $(b^0 = 1)$

Inverse Properties of Logarithms

For $b > 0$ and $b \neq 1$,

$\log_b b^x = x$ The logarithm with base b of b raised to a power equals that power.

$b^{\log_b x} = x.$ b raised to the logarithm with base b of a number equals that number.

The Change-of-Base Property: Introducing Common and Natural Logarithms

Introducing Common Logarithms

$$\log_b M = \frac{\log M}{\log b}$$

Introducing Natural Logarithms

$$\log_b M = \frac{\ln M}{\ln b}$$

Properties of Logarithms

In the following properties, M, N, and a are positive real numbers, $a \neq 1$.

$$\text{If } M = N, \text{ then } \log_a M = \log_a N. \quad (7)$$

$$\text{If } \log_a M = \log_a N, \text{ then } M = N. \quad (8)$$

The Product Rule

Let b, M, and N be positive real numbers with $b \neq 1$.

$$\log_b(MN) = \log_b M + \log_b N$$

The logarithm of a product is the sum of the logarithms.

The Quotient Rule

Let b, M, and N be positive real numbers with $b \neq 1$.

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

The logarithm of a quotient is the difference of the logarithms.

The Power Rule

Let b and M be positive real numbers with $b \neq 1$, and let p be any real number.

$$\log_b M^p = p \log_b M$$

The logarithm of a number with an exponent is the product of the exponent and the logarithm of that number.

Properties for Expanding Logarithmic Expressions

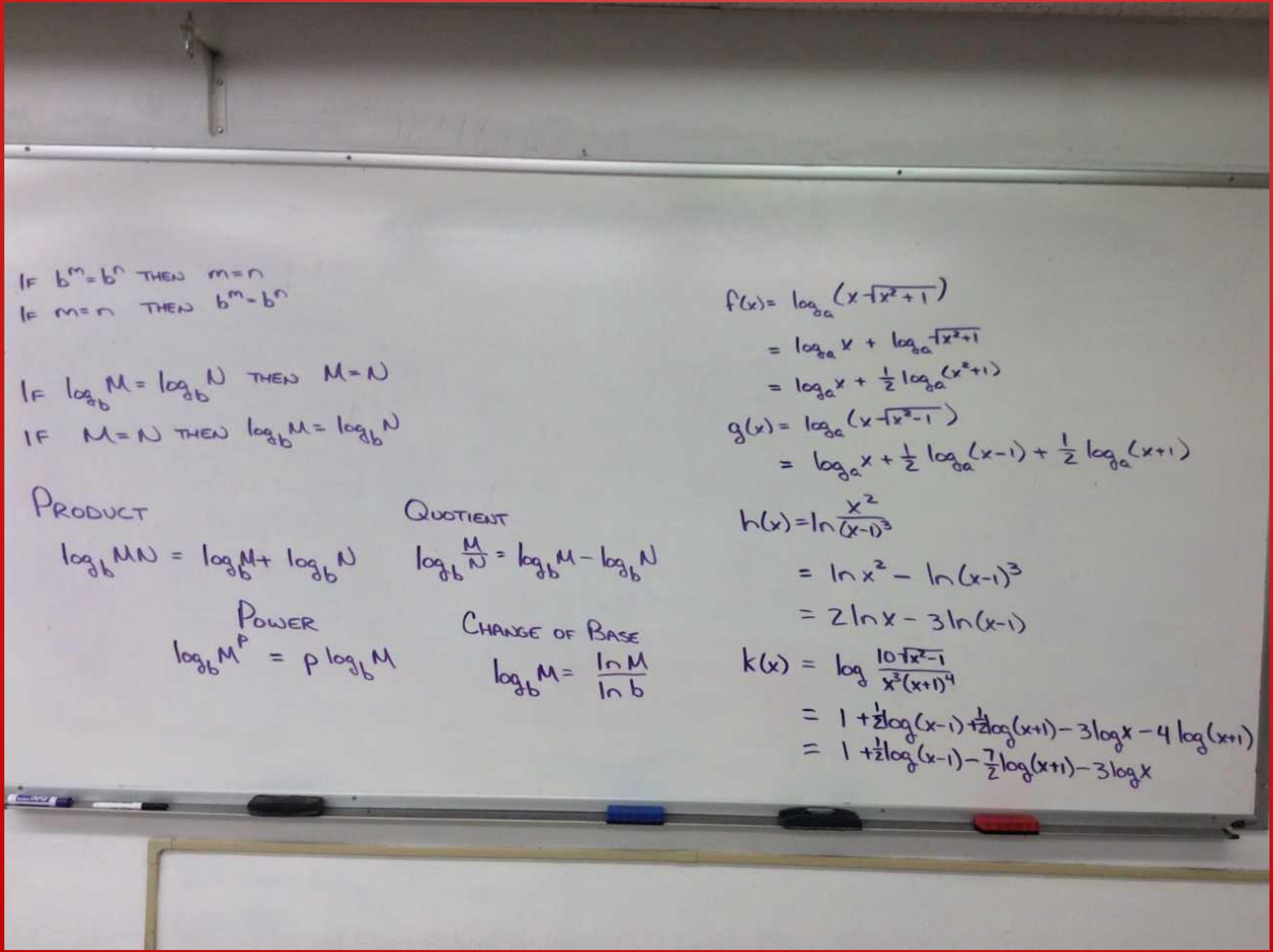
For $M > 0$ and $N > 0$:

1. $\log_b(MN) = \log_b M + \log_b N$ Product rule
2. $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$ Quotient rule
3. $\log_b M^p = p \log_b M$ Power rule

Properties for Condensing Logarithmic Expressions

For $M > 0$ and $N > 0$:

1. $\log_b M + \log_b N = \log_b(MN)$ Product rule
2. $\log_b M - \log_b N = \log_b\left(\frac{M}{N}\right)$ Quotient rule
3. $p \log_b M = \log_b M^p$ Power rule



IF b^m = b^n THEN m = n
IF m = n THEN b^m = b^n
IF log_b M = log_b N THEN M = N
IF M = N THEN log_b M = log_b N
PRODUCT
log_b MN = log_b M + log_b N
QUOTIENT
log_b M/N = log_b M − log_b N
POWER
log_b M^p = p log_b M
CHANGE OF BASE
log_b M = ln M / ln b
f(x) = log_a (x √(x²+1))
= log_a x + log_a √(x²+1)
= log_a x + ½ log_a (x²+1)
g(x) = log_a (x √(x²−1))
= log_a x + ½ log_a (x−1) + ½ log_a (x+1)
h(x) = ln x²/(x−1)³
= ln x² − ln (x−1)³
= 2 ln x − 3 ln (x−1)
k(x) = log 10√(x²−1) / x³(x+1)⁴
= 1 + ½ log (x−1) + ½ log (x+1) − 3 log x − 4 log (x+1)
= 1 + ½ log (x−1) − 7/2 log (x+1) − 3 log x

$$\log_a 7 + 4\log_a 3 = \log_a 567$$

$$\frac{2}{3}\ln 8 - \ln(5^2-1) = \ln\frac{1}{6}$$

$$\log x + \log 9 + \log(x^2+1) - \log 5 - \frac{1}{3}\log x = \log\frac{9x(x^2+1)}{5\sqrt[3]{x}}$$