BOARD NOTES

2 NOVEMBER 2018

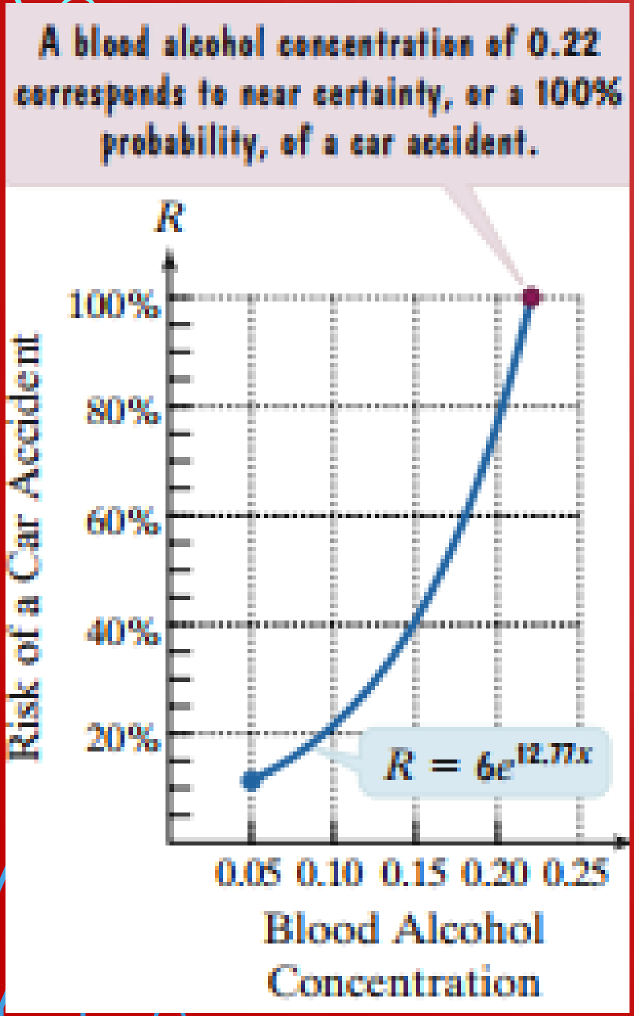
CC ALGEBRA
CHAPTER 4 –
EXPONENTIAL AND LOGARITHMIC FUNCTIONS

- SECTION 4.4 - EXPONENTIAL GROWTH AND DECAY; MODELING DATA

Objectives:

- Model exponential growth and decay
- Use logistic growth models
- Choose an appropriate model for data
- Express an exponential model in base e

Why is the Blood Alcohol Concentration set at 0.08 as the legal limit in Texas?



Medical Research indicates that the risk of having a car accident increases exponentially as the concentration of alcohol in the blood increases. The risk is modeled by

$$R = 6e^{12.77x},$$

where x is the blood alcohol concentration and R, given as a percent, is the risk of having a car accident. What blood alcohol concentration corresponds to a 17% risk of a car accident.

$$R = 6e^{12.77x}$$

$$R = 17\%$$

$$17 = 6e^{12.77x}$$

$$\frac{17}{6} = e^{12.77x}$$

$$\ln\frac{17}{6} = 12.77x$$

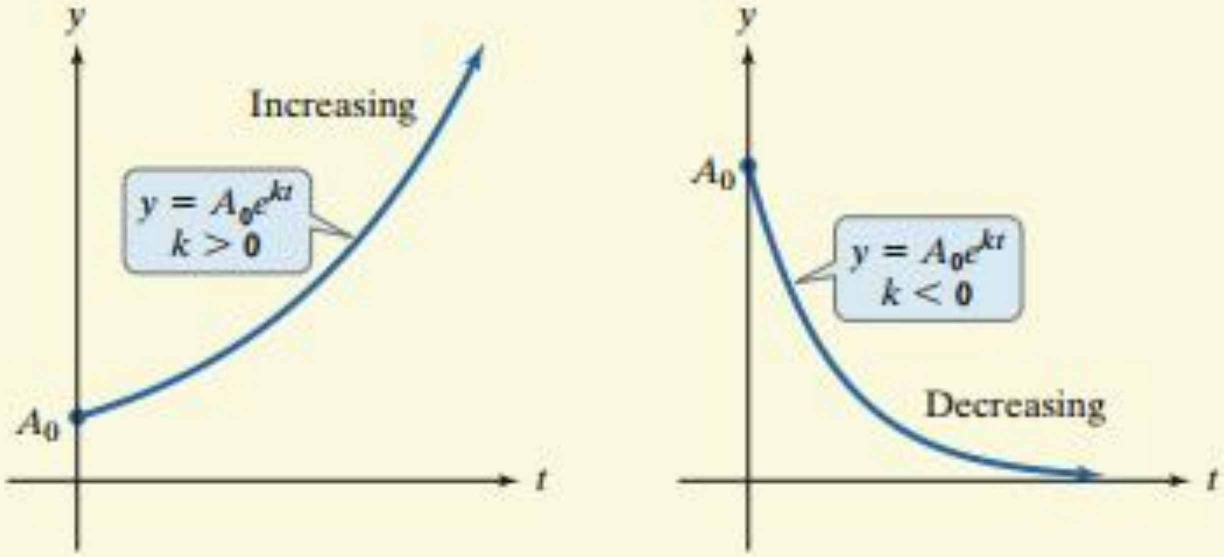
$$x = .08$$

Exponential Growth and Decay Models

The mathematical model for **exponential growth** or **decay** is given by

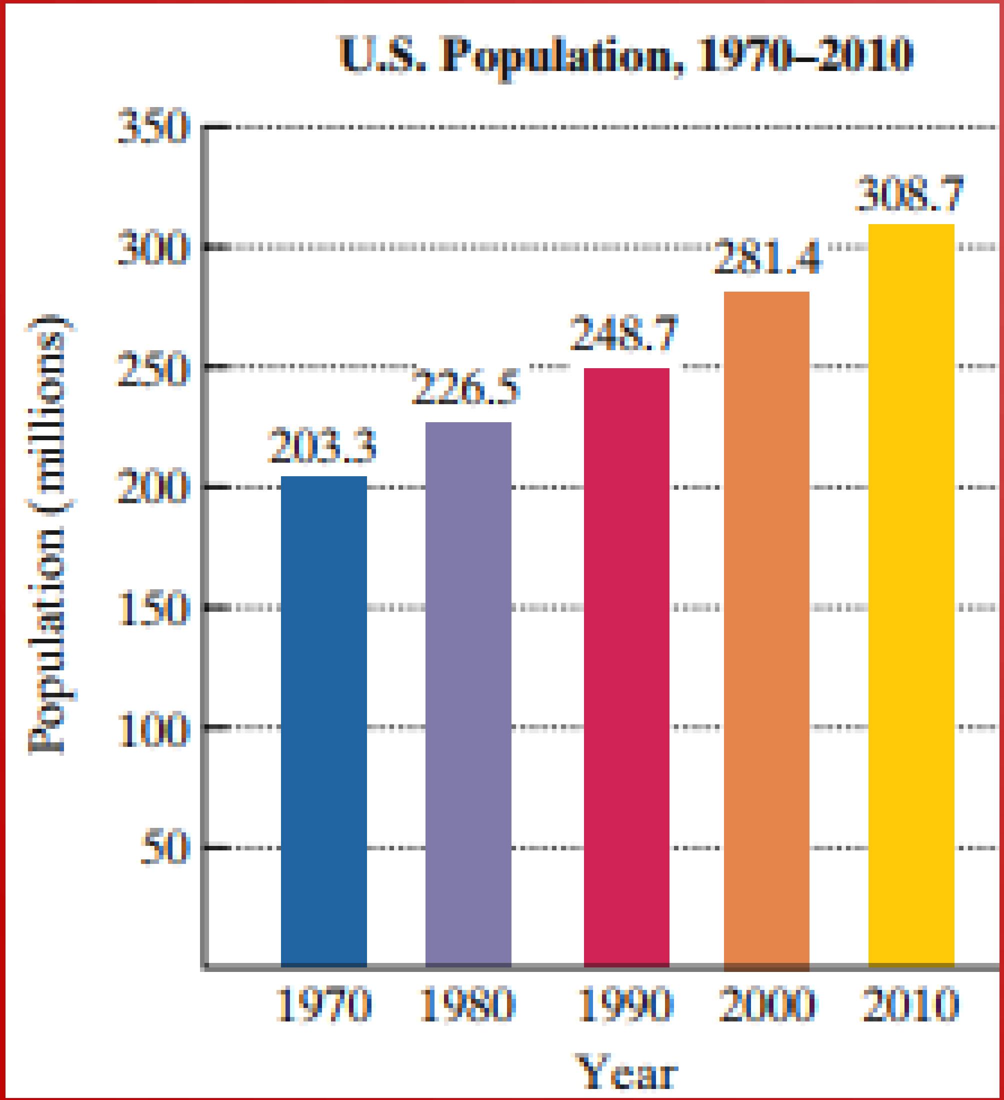
$$f(t) = A_0 e^{kt} \quad \text{or} \quad A = A_0 e^{kt}.$$

- **If $k > 0$, the function models the amount, or size, of a *growing* entity.** A_0 is the original amount, or size, of the growing entity at time $t = 0$, A is the amount at time t, and k is a constant representing the growth rate.
- **If $k < 0$, the function models the amount, or size, of a *decaying* entity.** A_0 is the original amount, or size, of the decaying entity at time $t = 0$, A is the amount at time t, and k is a constant representing the decay rate.



(a) Exponential growth **(b)** Exponential decay

Modeling the Growth of the U.S. Population



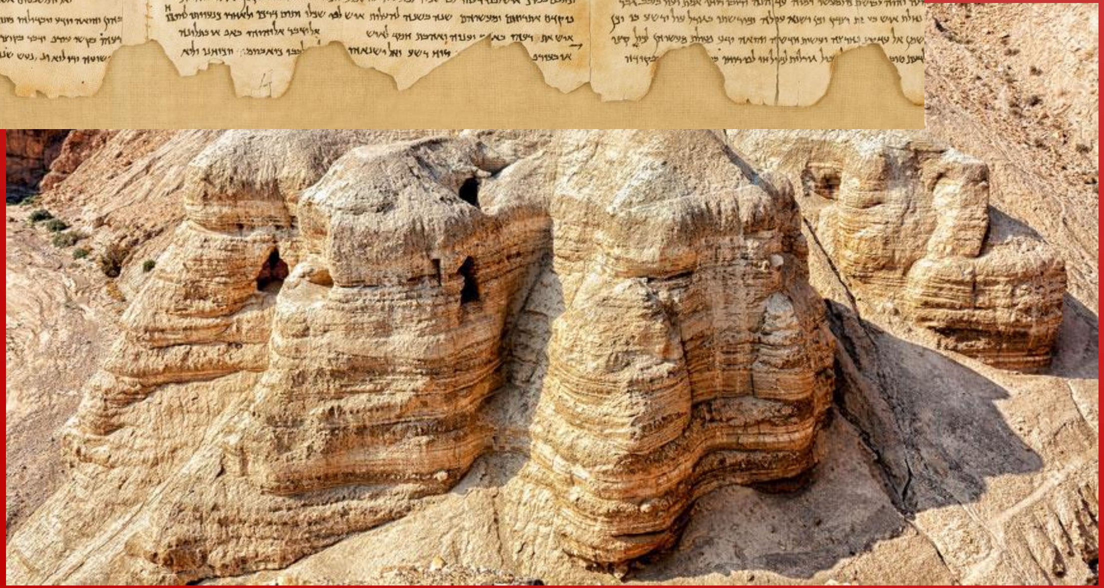
The graph shows the U.S. population, in millions, for five selected years from 1970 through 2010. In 1970, the U.S. population was 203.3 million. By 2010, it had grown to 308.7 million.

a. Find an exponential growth function that models the data.

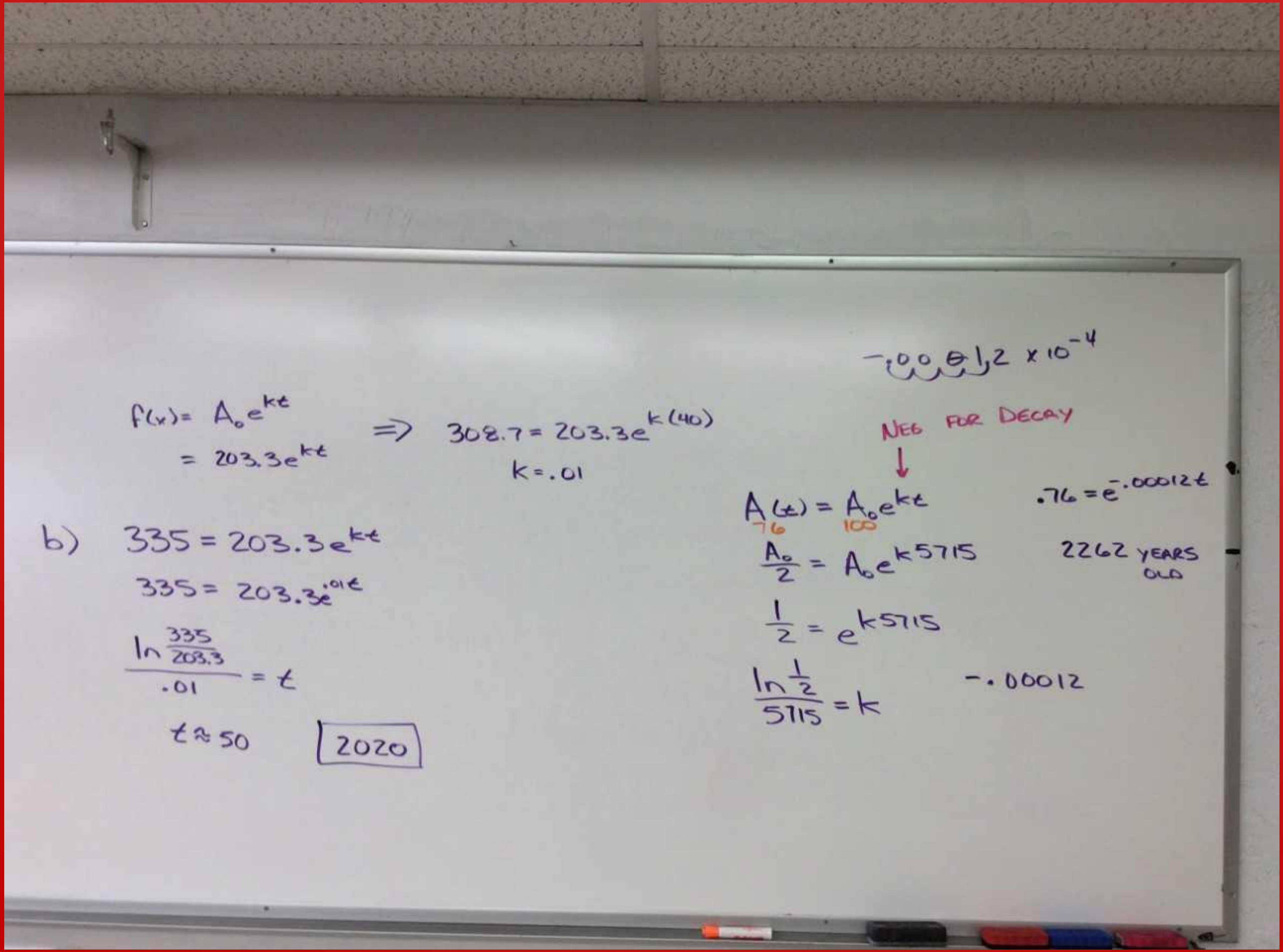
b. By which year will the U.S. population reach 335 million?

Carbon-14 decays exponentially with a *half-life* of approximately 5715 years. The *half-life* of a substance is the time required for half of a given sample to disintegrate. Thus, after 5715 years a given amount of carbon-14 will have decayed to half the original amount.

Carbon dating is useful for artifacts or fossils up to 80,000 years old. Older objects do not have enough carbon-14 to determine age accurately.



In 1947, earthenware jars containing what are known as the Dead Sea Scrolls were found by an Arab Bedouin herdsman. Analysis indicated that the scroll wrappings contained 76% of their original carbon-14. Estimate the age of the Dead Sea Scrolls.



f(x)= A₀e^kt
= 203.3e^kt
=> 308.7 = 203.3e^k(40)
k = .01
b) 335 = 203.3e^kt
335 = 203.3e^.01t
ln(335/203.3) / .01 = t
t ≈ 50
2020
-.00012 x 10^-4
NEG FOR DECAY
A(t) = A₀e^kt
76
100
A₀/2 = A₀e^k5715
1/2 = e^k5715
ln(1/2) / 5715 = k
-.00012
.76 = e^-.00012t
2262 YEARS OLD

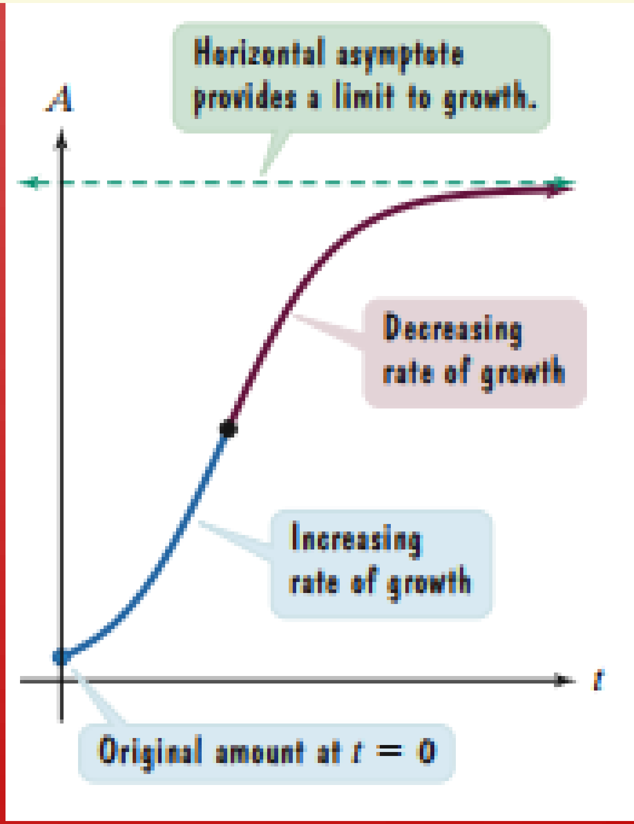
Logistic Growth Model

The mathematical model for limited logistic growth is given by

$$f(t) = \frac{c}{1 + ae^{-bt}} \quad \text{or} \quad A = \frac{c}{1 + ae^{-bt}},$$

where a, b, and c are constants, with $c > 0$ and $b > 0$.

As $t \to \infty$, the expression $ae^{-bt} \to 0$, and A gets closer and closer to c. This means $y = c$ is a horizontal asymptote for the graph of the function. Thus, the value of A can never exceed the value of c which is the limiting size.



Modeling the Spread of the Flu

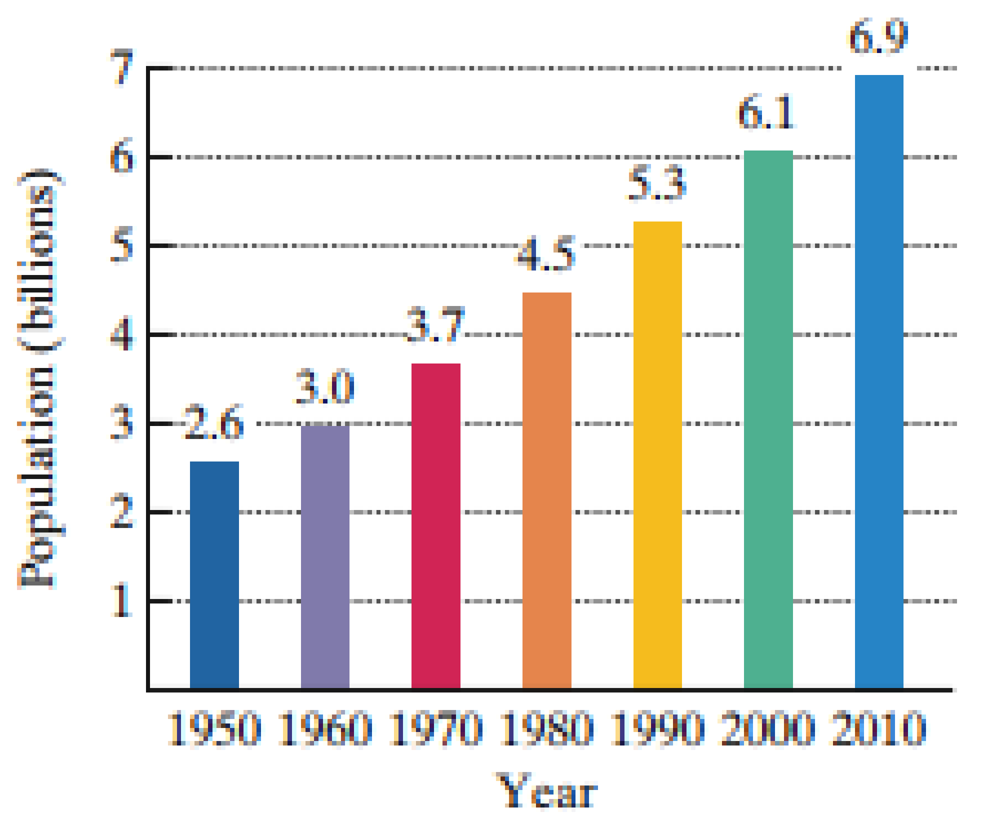
The function

$$P(t) = \frac{199{,}826}{1 + 20e^{-1.5t}}$$

describes the number of people, $P(t)$, who have become ill with influenza t weeks after its initial outbreak in Amarillo, TX.

a. How many people became ill with the flu when the epidemic began?
b. How many people were ill by the end of the fourth week?
c. What is the limiting size of $P(t)$ that becomes ill?

Modeling the World Population



The graph shows the world population, in billions, for seven selected years from 1950 through 2010.

a. Express two models in function notation to model the data. Round numbers to three decimal places.
b. How well do the functions model the population in 2000?
c. By one projection, the population is expected to reach 8 billion in 2026. Which function is a better model?