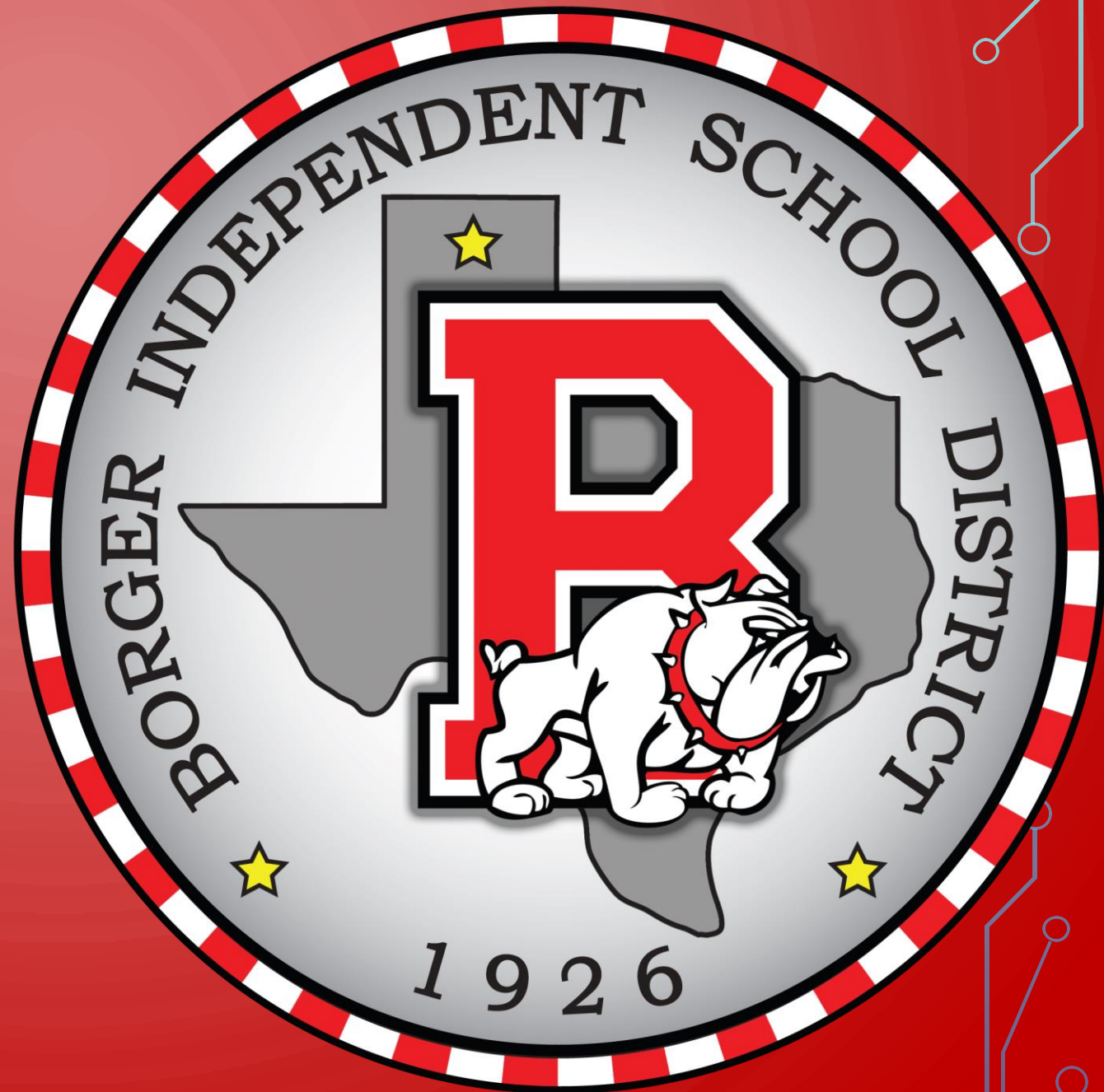


BOARD NOTES

5 NOVEMBER 2018



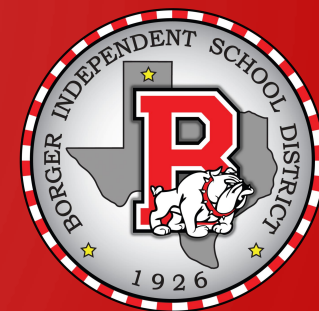
CC PRECALCULUS

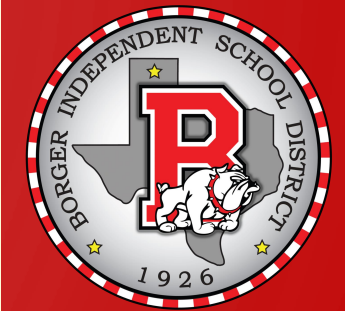
CHAPTER 5 – EXPONENTIAL AND LOGARITHMIC FUNCTIONS

- SECTION 5.6 - LOGARITHMIC AND EXPONENTIAL EQUATIONS

Objectives:

- Solve logarithmic equations
- Solve exponential equations
- Solve logarithmic and exponential equations using a calculator





Laws of Exponents

If $s, t, a,$ and b are real numbers with $a > 0$ and $b > 0$, then

$$\begin{array}{lll} a^s \cdot a^t = a^{s+t} & (a^s)^t = a^{st} & (ab)^s = a^s \cdot b^s \\ 1^s = 1 & a^{-s} = \frac{1}{a^s} = \left(\frac{1}{a}\right)^s & a^0 = 1 \end{array} \quad (1)$$

Properties of Logarithms

In the list that follows, $a, b, M, N,$ and r are real numbers. Also, $a > 0, a \neq 1, b > 0, b \neq 1, M > 0,$ and $N > 0.$

Definition

$$y = \log_a x \text{ means } x = a^y$$

Properties of Logarithms

$$\log_a 1 = 0$$

$$\log_a a = 1$$

$$\log_a M^r = r \log_a M$$

$$a^{\log_a M} = M$$

$$\log_a a^r = r$$

$$a^r = e^{r \ln a}$$

$$\log_a (MN) = \log_a M + \log_a N$$

$$\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$$

$$\text{If } M = N, \text{ then } \log_a M = \log_a N.$$

$$\text{If } \log_a M = \log_a N, \text{ then } M = N.$$

Change-of-Base Formula

$$\log_a M = \frac{\log_b M}{\log_b a}$$

$$\log_2(1-2x) = 3$$

$$2^3 = 1-2x$$

$$8 = 1-2x$$

$$7 = -2x$$

$$x = -\frac{7}{2}$$

$$2 \log_5 x = \log_5 9$$

$$\log_5 x^2 = \log_5 9$$

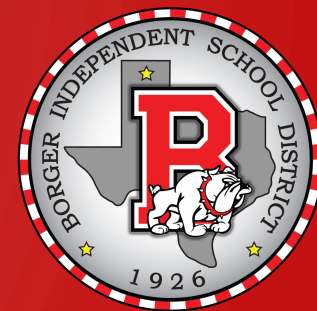
$$x^2 = 9$$

$$x = 3, -3$$

$$\log_4(x+2) = \log_4 8$$

$$x+2 = 8$$

$$x = 6$$

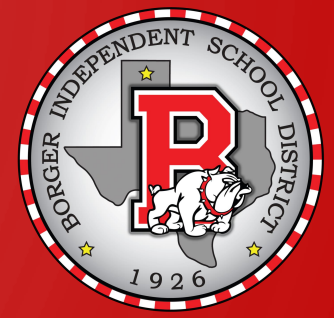


$$\begin{aligned} \overset{-1}{\log_5(x+6)} + \overset{+}{\log_5(x+2)} &= 1 \\ \log_5[(x+6)(x+2)] &= \log_5 5 \\ x^2 + 8x + 12 &= 5 \\ x^2 + 8x + 7 &= 0 \\ x &= \cancel{-1}, -1 \end{aligned}$$

$$\begin{aligned} (\log_3 x)^2 - 5(\log_3 x) &= 6 \\ \text{LET } u &= \log_3 x \\ u^2 - 5u &= 6 \\ u^2 - 5u - 6 &= 0 \\ (u-6)(u+1) &= 0 \end{aligned}$$

$$\begin{aligned} \Rightarrow \log_3 x &= 6 & \frac{1}{3} \log_3 x &= -1 \\ x &= 3^6 & x &= 3^{-1} \\ x &= 729 & \text{OR } & \frac{1}{3} \end{aligned}$$

$$\begin{aligned} 2^x &= 5 \\ x &= \frac{\ln 5}{\ln 2} \approx 2.322 \\ \log_2 5 &= x \\ 8 \cdot 3^x &= 5 \\ x &= \log_3 \frac{5}{8} = \frac{\log_3 5}{\log_3 8} = \frac{\ln 5}{\ln 8} \approx -.428 \end{aligned}$$



$$\begin{aligned}5^{x-2} &= 3^{3x+2} \\ \ln 5^{x-2} &= \ln 3^{3x+2} \\ (x-2)\ln 5 &= (3x+2)\ln 3 \\ x\ln 5 - 2\ln 5 &= 3x\ln 3 + 2\ln 3 \\ x\ln 5 - 3x\ln 3 &= 2(\ln 3 + \ln 5) \\ x(\ln 5 - 3\ln 3) &= 2(\ln 3 + \ln 5) \\ x &= \frac{2(\ln 3 + \ln 5)}{\ln 5 - 3\ln 3} \approx -3.212\end{aligned}$$

$$\begin{aligned}4^x - 2^x - 12 &= 0 \\ (2^2)^x - 2^x - 12 &= 0 \\ 2^{2x} - 2^x - 12 &= 0 \\ (2^x)^2 - 2^x - 12 &= 0 \\ u &= 2^x \\ u^2 - u - 12 &= 0\end{aligned}$$

