BOARD NOTES

6 NOVEMBER 2018

CC PRECALCULUS CHAPTER 5 – EXPONENTIAL AND LOGARITHMIC FUNCTIONS

- SECTION 5.7 - FINANCIAL MODELS

Objectives:

- Determine the future value of a lump sum of money
- Calculate the effective rates of return
- Determine the present value of a lump sum of money
- Determine the rate of interest or the time required to double a lump sum of money

Simple Interest Formula

If a principal of P dollars is borrowed for a period of t years at a per annum interest rate r, expressed as a decimal, the interest I charged is

$$I = Prt \tag{1}$$

Compound Interest Formula

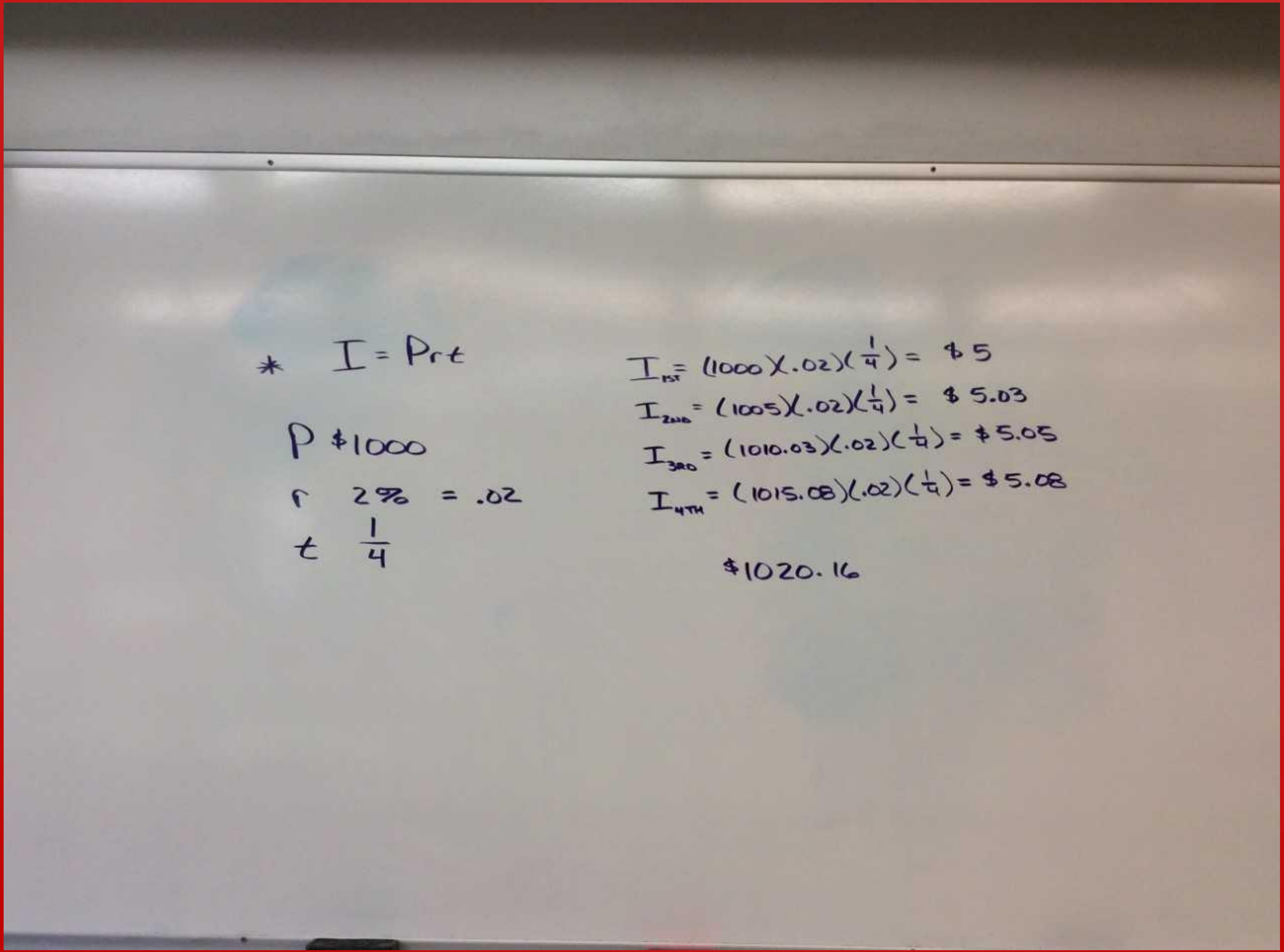
The amount A after t years due to a principal P invested at an annual interest rate r, expressed as a decimal, compounded n times per year is

$$A = P \cdot \left(1 + \frac{r}{n}\right)^{nt} \tag{2}$$

Continuous Compounding

The amount A after t years due to a principal P invested at an annual interest rate r compounded continuously is

$$A = Pe^{rt} \tag{4}$$


* I = Prt
P $1000
r 2% = .02
t 1/4
I1ST = (1000)(.02)(1/4) = $5
I2ND = (1005)(.02)(1/4) = $5.03
I3RD = (1010.03)(.02)(1/4) = $5.05
I4TH = (1015.08)(.02)(1/4) = $5.08
$1020.16

Effective Rate of Interest

The effective rate of interest r_e of an investment earning an annual interest rate r is given by

$$\text{Compounding } n \text{ times per year: } \quad r_e = \left(1 + \frac{r}{n}\right)^n - 1$$

$$\text{Continuous compounding: } \quad r_e = e^r - 1$$

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$= 1000\left(1+\frac{.02}{4}\right)^{4\cdot 1}$

$= \$1020.16$

$A = Pe^{rt}$

Effective Rate of Return

$r_e = \left(1+\frac{r}{n}\right)^{n} - 1$

$r_e = e^{r} - 1$

Annually	$n=1$	$t=2$	$P=\$1000$	\$1040.40
Semi-Annually	$n=2$	r 2%		\$1040.60
Quarterly	$n=4$			\$1040.70
Monthly	$n=12$			\$1040.78
Daily	$n=365$			\$1040.81
Continuous				\$1040.81

American Express rate of 2.15%
Comp monthly

$r_{AE} = \left(1+\frac{.0215}{12}\right)^{12} - 1 = .02171$ 2.171%

*Bank of America rate of 2.2%
Comp quarterly

$r_{BOA} = \left(1+\frac{.022}{4}\right)^{4} - 1 = .02218$ 2.218%*

Discover rate of 2.12% comp daily

$r_D = \left(1+\frac{.0212}{365}\right)^{365} - 1 = .02143$ 2.143%

Present Value Formulas

The present value P of A dollars to be received after t years, assuming a per annum interest rate r compounded n times per year, is

$$P = A \cdot \left(1 + \frac{r}{n}\right)^{-nt} \qquad (5)$$

If the interest is compounded continuously, then

$$P = Ae^{-rt} \qquad (6)$$

$P = A\left(1+\frac{r}{n}\right)^{-nt}$

$P = Ae^{-rt}$

$t = 10$

$r = 8\%$ $\qquad$ $r = 7\%$

$A = 1000$

Comp Monthly	Comp Cont
\$ 450.52	\$496.59

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = 2P$

$\Rightarrow 2P = P\left(1+\frac{r}{n}\right)^{nt}$

$n = 1 \quad t = 5$

$2 = \left(1+\frac{r}{1}\right)^{5}$

$\sqrt[5]{2} = 1 + r$

$.149 = r$

$r = 1.49\%$