BOARD NOTES

7 NOVEMBER 2018

CC PRECALCULUS
CHAPTER 5 –
EXPONENTIAL AND LOGARITHMIC FUNCTIONS

- SECTION 5.8 - EXPONENTIAL GROWTH AND DECAY MODELS; NEWTON'S LAW; LOGISTIC GROWTH AND DECAY MODELS

Objectives:

- Find equations of populations that obey uninhibited growth
- Find equations of populations that obey the law of decay
- Use Newton's law of cooling
- Use logistic models

A model that gives the number N of cells in a culture after a time t has passed (in the early stages of growth) is

$$N(t) = N_0 e^{kt} \qquad k > 0 \qquad (2)$$

where N_0 is the initial number of cells and k is a positive constant that represents the growth rate of the cells.

A colony of bacteria that grows according to the law of uninhibited growth is modeled by the function $N(t) = 100e^{0.045t}$, where t is measured in days and N is measured in grams.

a. Determine the initial amount of bacteria.
b. What is the growth rate of the bacteria?
c. What is the population after 5 days?
d. How long will it take for the population to reach 140 grams?
e. What is doubling time for the population?

A colony of bacteria increases according to the law of uninhibited growth.

a. If N is the number of cells and t is the time in hours, express N as a function of t.

b. If the number of bacteria doubles in three hours, find the function that gives the number of cells in the culture.

c. How long will it take for the colony to triple in size?

d. How long will it take for the population to double a second time?

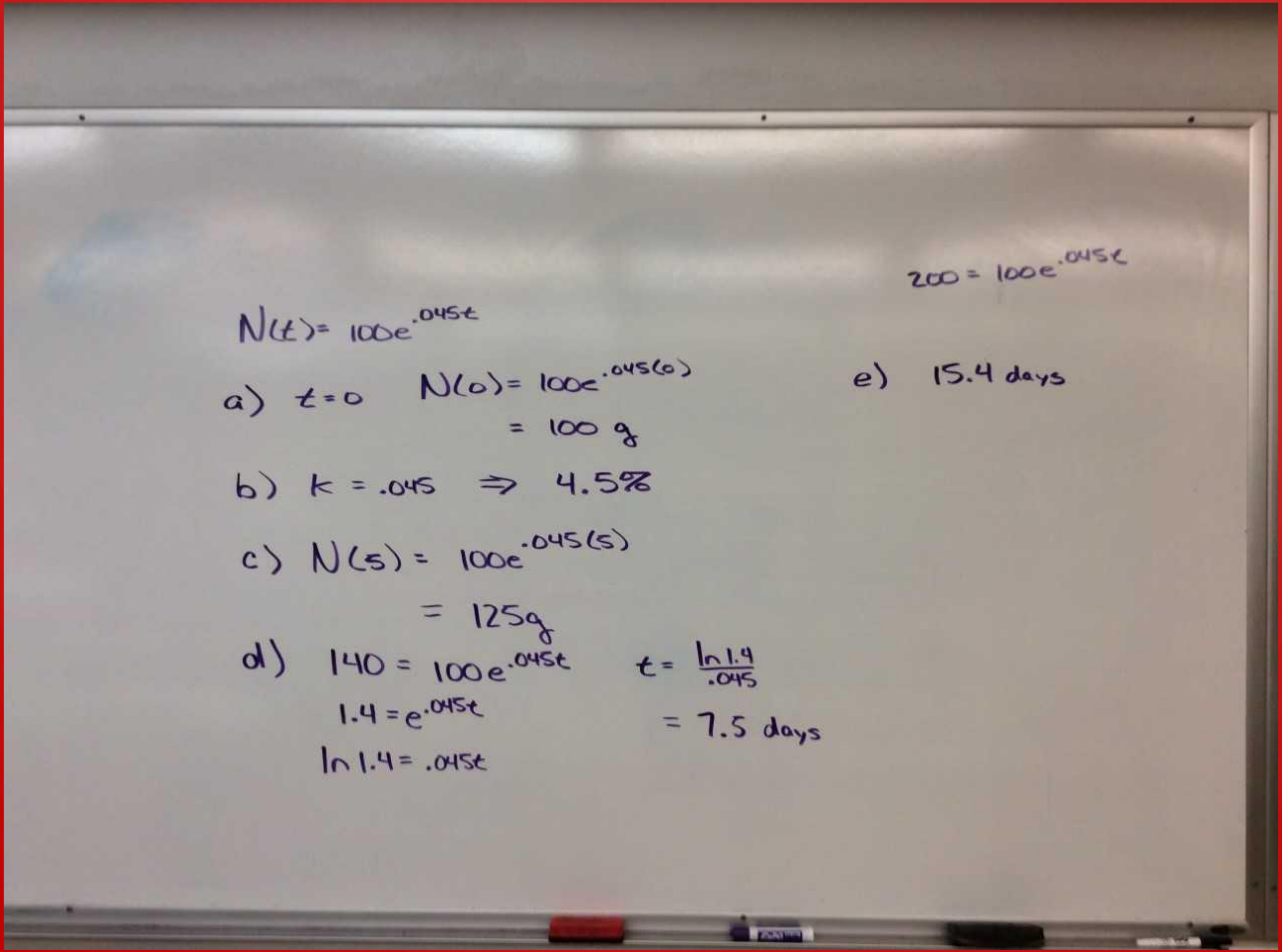
Uninhibited Radioactive Decay

The amount A of a radioactive material present at time t is given by

$$A(t) = A_0 e^{kt} \qquad k < 0 \qquad (3)$$

where A_0 is the original amount of radioactive material and k is a negative number that represents the rate of decay.

Traces of burned wood along with ancient stone tools in an archeological dig in Chile were found to contain approximately 1.67% of the original amount of carbon-14. If the half-life of carbon-14 is 5730 years, approximately when was the tree cut and burned?


200 = 100e^.045t
N(t) = 100e^.045t
a) t = 0   N(0) = 100e^.045(0)
= 100 g
b) k = .045 ⇒ 4.5%
c) N(5) = 100e^.045(5)
= 125g
d) 140 = 100e^.045t
1.4 = e^.045t
ln 1.4 = .045t
t = ln 1.4 / .045
= 7.5 days
e) 15.4 days

a) $N(t) = N_0 e^{rt}$

b) $2N_0 = N_0 e^{r(3)}$

$2 = e^{3r}$

$\ln 2 = 3r$

$r = \frac{\ln 2}{3}$

$= .231$

$N(t) = N_0 e^{.231t}$

c) $3N_0 = N_0 e^{.231t}$

$t = 4.755$ hrs

or

4h 45m

d) 6 h

$A = A_0 e^{rt}$

$\Rightarrow \frac{A_0}{2} = A_0 e^{r(5730)}$

$r = \frac{\ln \frac{1}{2}}{5730} = -.000121$

$.0167 A_0 = A_0 e^{-.000121}$

$t = \frac{\ln .0167}{-.000121}$

$= 33{,}800$ yrs

Newton's Law of Cooling

The temperature u of a heated object at a given time t can be modeled by the following function:

$$u(t) = T + (u_0 - T)e^{kt} \qquad k < 0 \qquad \textbf{(4)}$$

where T is the constant temperature of the surrounding medium, u_0 is the initial temperature of the heated object, and k is a negative constant.

An object is heated to 100°C and is then allowed to cool in a room whose air temperature is 30°C.

a. If the temperature of the object is 80°C after 5 minutes, when will its temperature be 50°C?

b. Determine the elapsed time before the temperature of the object is 35°C.

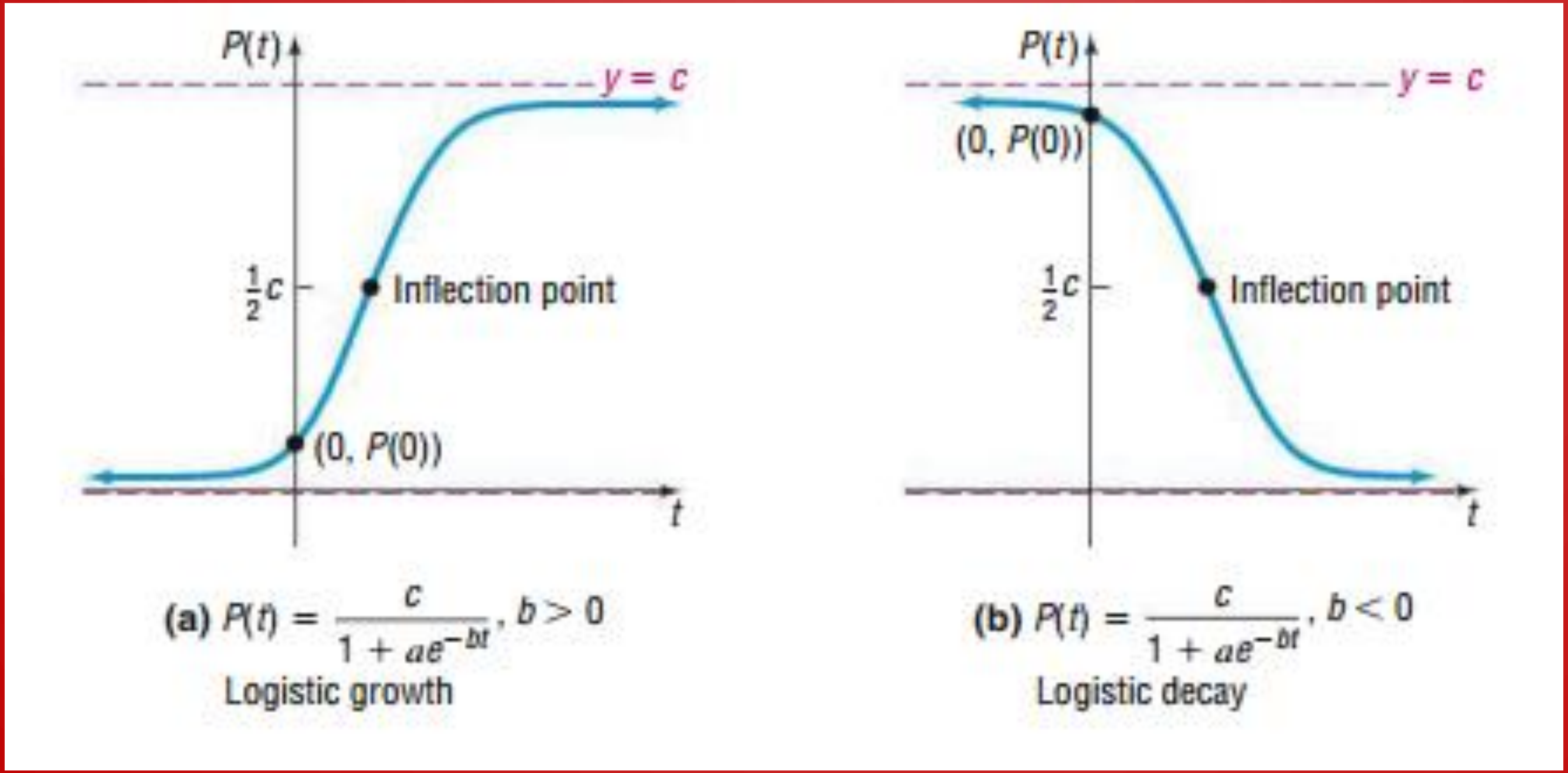
c. What do you notice about the temperature as time passes?

Logistic Model

In a logistic model, the population P after time t is given by the function

$$P(t) = \frac{c}{1 + ae^{-bt}} \qquad (7)$$

where a, b, and c are constants with $a > 0$ and $c > 0$. The model is a growth model if $b > 0$; the model is a decay model if $b < 0$.



P(t)
y = c
1/2 c
Inflection point
(0, P(0))
t
(a) P(t) = c / (1 + ae^{−bt}), b > 0
Logistic growth
P(t)
y = c
(0, P(0))
1/2 c
Inflection point
t
(b) P(t) = c / (1 + ae^{−bt}), b < 0
Logistic decay

Fruit flies are placed in a half-pint milk bottle with a banana (for food) and yeast plants (for food and to provide a stimulus to lay eggs). Suppose the fruit fly population after t days is given by

$$P(t) = \frac{230}{1 + 56.5e^{-0.37t}}$$

a. State the carrying capacity and the growth rate.
b. Determine the initial population.
c. What is the population after 5 days?
d. How long does it take for the population to reach 180?
e. How long does it take for the population to reach one-half of the carrying population?