BOARD NOTES

8 NOVEMBER 2018

CC PRECALCULUS
CHAPTER 5 –
EXPONENTIAL AND LOGARITHMIC FUNCTIONS

- SECTION 5.8 - EXPONENTIAL GROWTH AND DECAY MODELS; NEWTON'S LAW; LOGISTIC GROWTH AND DECAY MODELS

Objectives:

- Find equations of populations that obey uninhibited growth
- Find equations of populations that obey the law of decay
- Use Newton's law of cooling
- Use logistic models

Newton's Law of Cooling

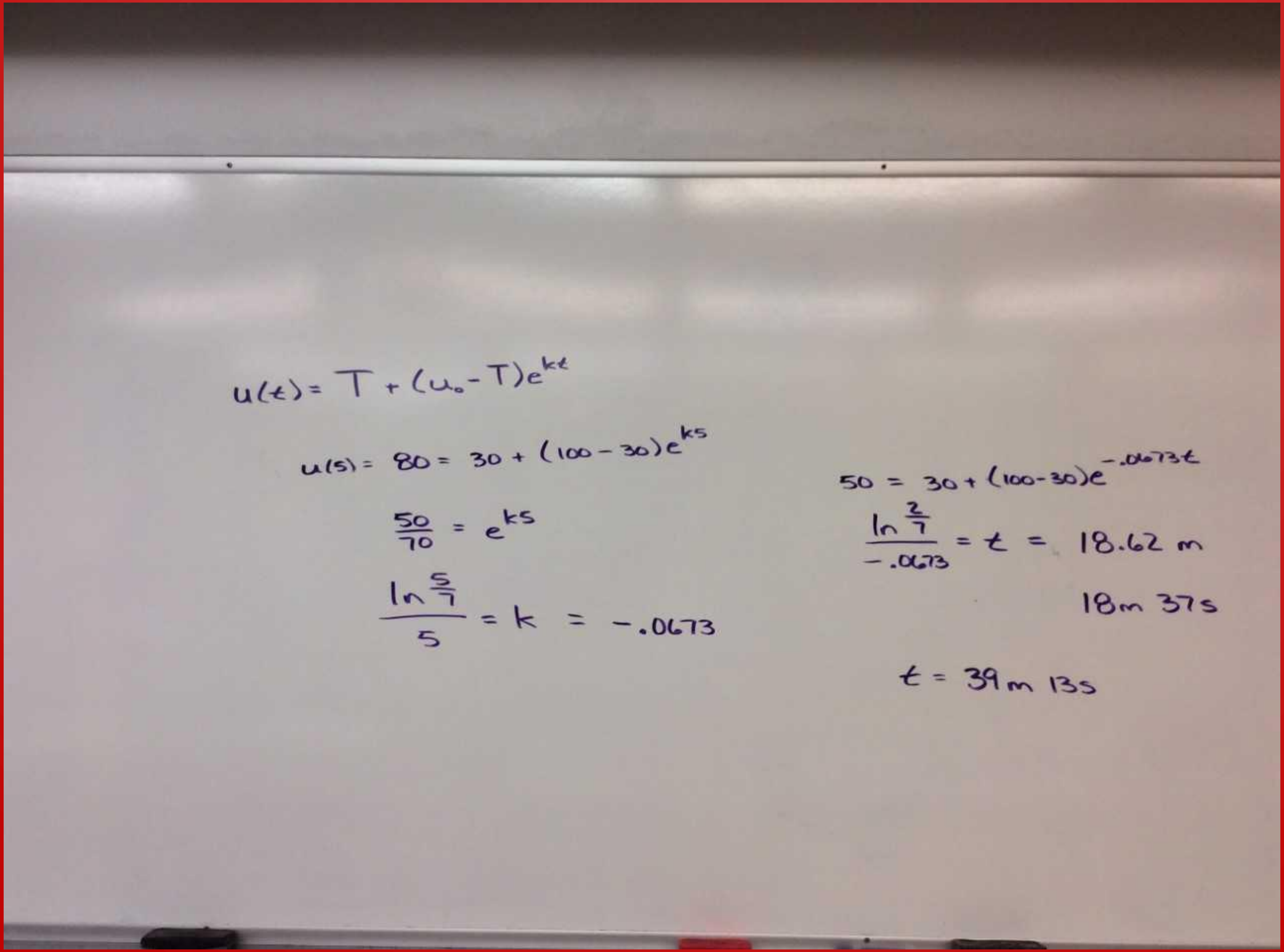
The temperature u of a heated object at a given time t can be modeled by the following function:

$$u(t) = T + (u_0 - T)e^{kt} \qquad k < 0 \qquad \textbf{(4)}$$

where T is the constant temperature of the surrounding medium, u_0 is the initial temperature of the heated object, and k is a negative constant.

An object is heated to 100°C and is then allowed to cool in a room whose air temperature is 30°C.

a. If the temperature of the object is 80°C after 5 minutes, when will its temperature be 50°C?
b. Determine the elapsed time before the temperature of the object is 35°C.
c. What do you notice about the temperature as time passes?

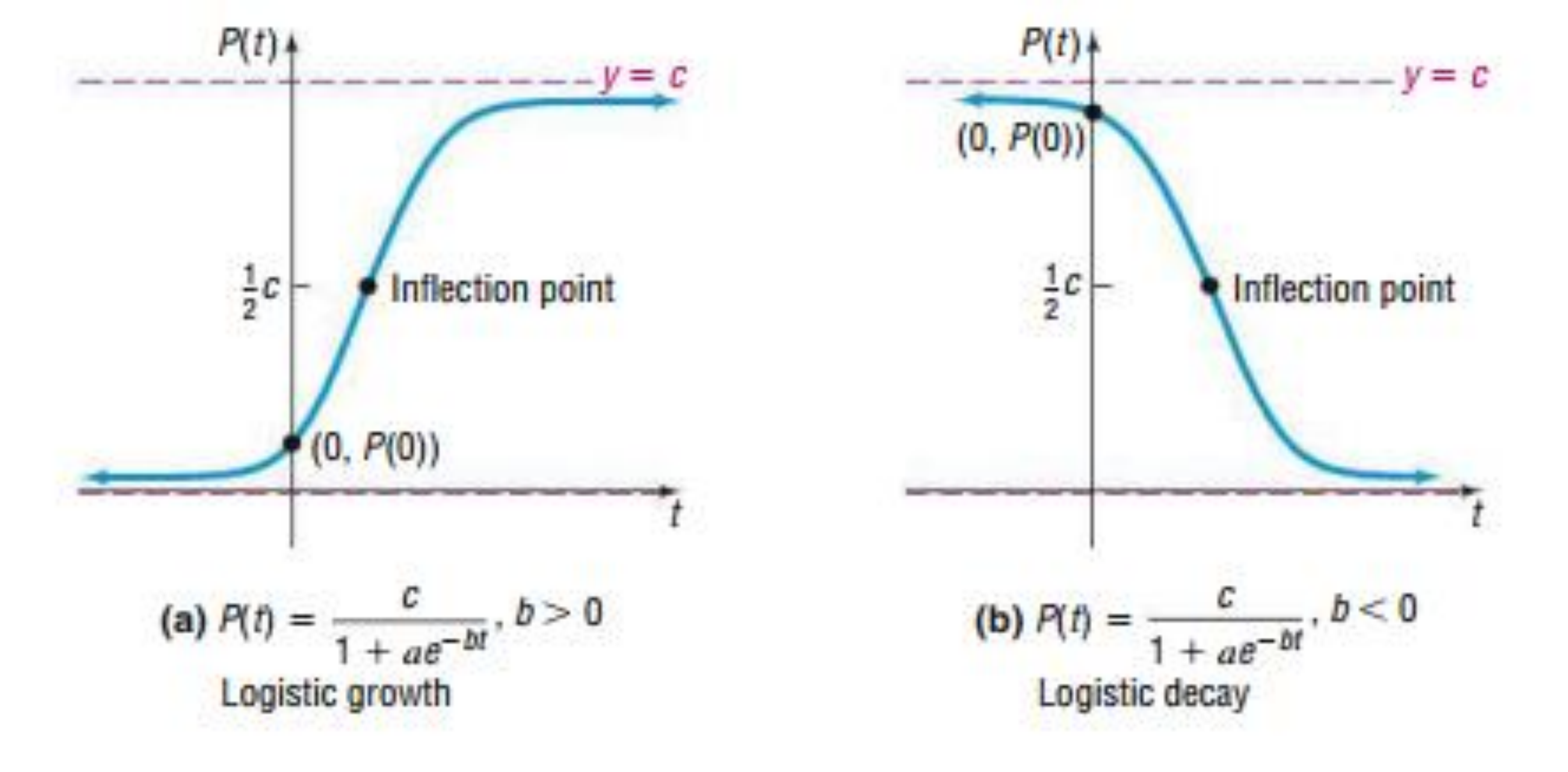

u(t) = T + (u₀ − T)e^{kt}
u(5) = 80 = 30 + (100 − 30)e^{k5}
50/70 = e^{k5}
ln(5/7) / 5 = k = −.0673
50 = 30 + (100−30)e^{−.0673t}
ln(2/7) / −.0673 = t = 18.62 m
18m 37s
t = 39m 13s

Logistic Model

In a logistic model, the population P after time t is given by the function

$$P(t) = \frac{c}{1 + ae^{-bt}} \qquad (7)$$

where a, b, and c are constants with $a > 0$ and $c > 0$. The model is a growth model if $b > 0$; the model is a decay model if $b < 0$.


P(t)
y = c
$\frac{1}{2}c$
Inflection point
(0, P(0))
t
(a) $P(t) = \dfrac{c}{1 + ae^{-bt}}, b > 0$
Logistic growth
P(t)
y = c
(0, P(0))
$\frac{1}{2}c$
Inflection point
t
(b) $P(t) = \dfrac{c}{1 + ae^{-bt}}, b < 0$
Logistic decay

Fruit flies are placed in a half-pint milk bottle with a banana (for food) and yeast plants (for food and to provide a stimulus to lay eggs). Suppose the fruit fly population after t days is given by

$$P(t) = \frac{230}{1 + 56.5e^{-0.37t}}$$

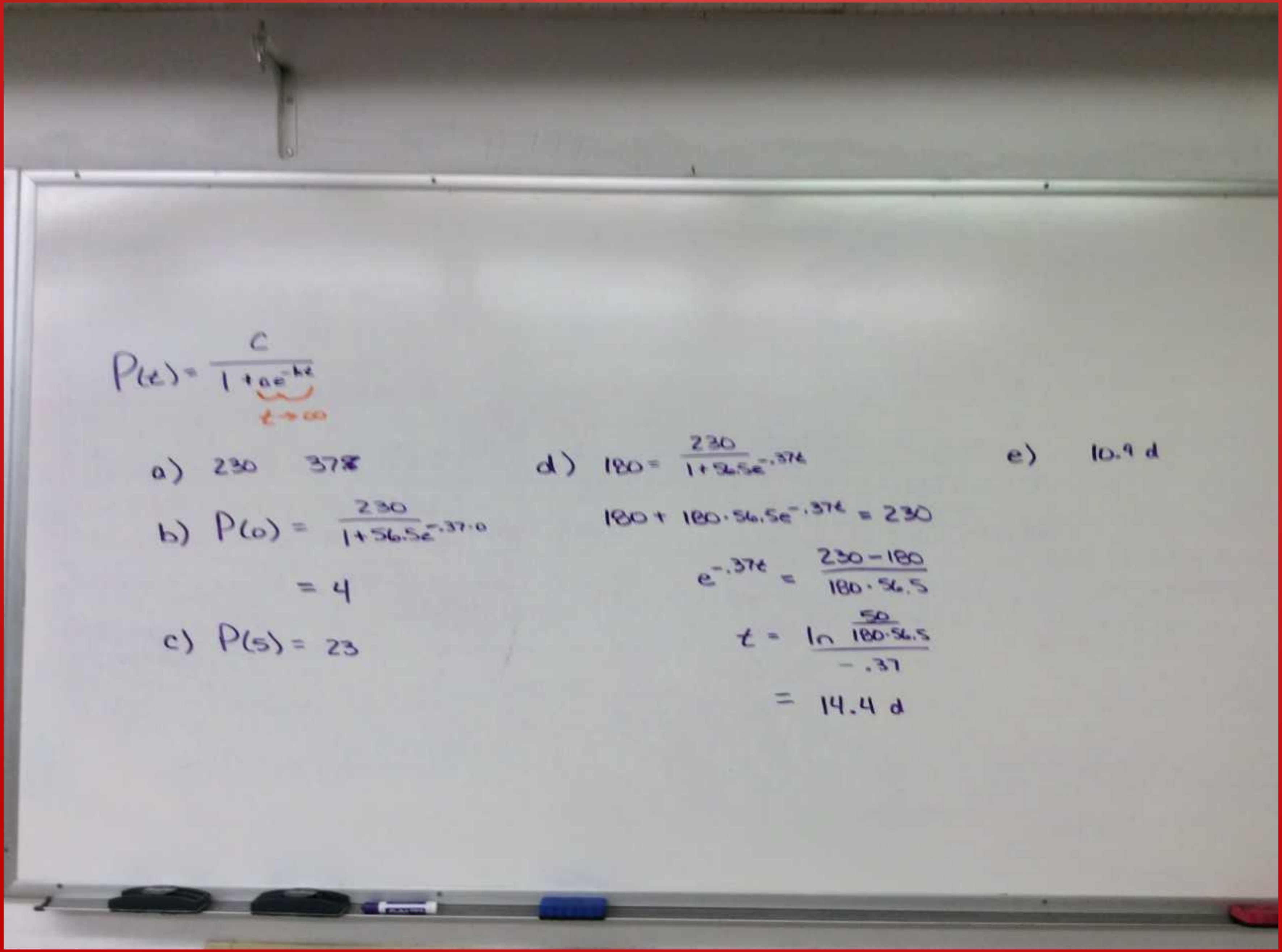
a. State the carrying capacity and the growth rate.

b. Determine the initial population.

c. What is the population after 5 days?

d. How long does it take for the population to reach 180?

e. How long does it take for the population to reach one-half of the carrying population?



FRANK PHILLIPS COLLEGE Plainsmen

BORGER INDEPENDENT SCHOOL DISTRICT 1926

CC PRECALCULUS
CHAPTER 5 –
EXPONENTIAL AND LOGARITHMIC FUNCTIONS

- SECTION 5.9 BUILDING EXPONENTIAL, LOGARITHMIC, LOGISTIC MODELS FROM DATA

Objectives:

- Build an exponential model from data
- Build a logarithmic model from data
- Build a logistic model from data

Mariah deposited $20,000 in a well-diversified mutual fund 6 years ago. The data in the table represents the value of the account at the beginning of each year for the last 7 years.

Year, x	Account Value, y
0	20,000
1	21,516
2	23,355
3	24,885
4	27,484
5	30,053
6	32,622

a. Draw a scatter plot
b. Build an exponential model
c. Graph the exponential model
d. What is value of the account in 10 years?

Jodi, a meteorologist, is interested in finding a function that explains the relation between the height of a weather balloon (in kilometers) and the atmospheric pressure (measured in millimeters of mercury) on the balloon. She collects the following data:

Atmospheric Pressure, p	Height, h
760	0
740	0.184
725	0.328
700	0.565
650	1.079
630	1.291
600	1.634
580	1.862
550	2.235

a. Draw a scatter plot

b. Build an logarithmic model

c. Graph the logarithmic model

d. Predict the height of the weather balloon if the pressure is 560 millimeters of mercury.

Time (hours)	Yeast Biomass
0	9.6
1	18.3
2	29.0
3	47.2
4	71.1
5	119.1
6	174.6
7	257.3
8	350.7
9	441.0
10	513.3
11	559.7
12	594.8
13	629.4
14	640.8
15	651.1
16	655.9
17	659.6
18	661.8

The table represents the amount of yeast biomass in a culture after t hours:

a. Draw a scatter plot

b. Build an logistic model

c. Graph the logistic model

d. What is predicted carrying capacity?

e. What is the predicted population at $t = 19$ hours?