BOARD NOTES

22 JANUARY 2019

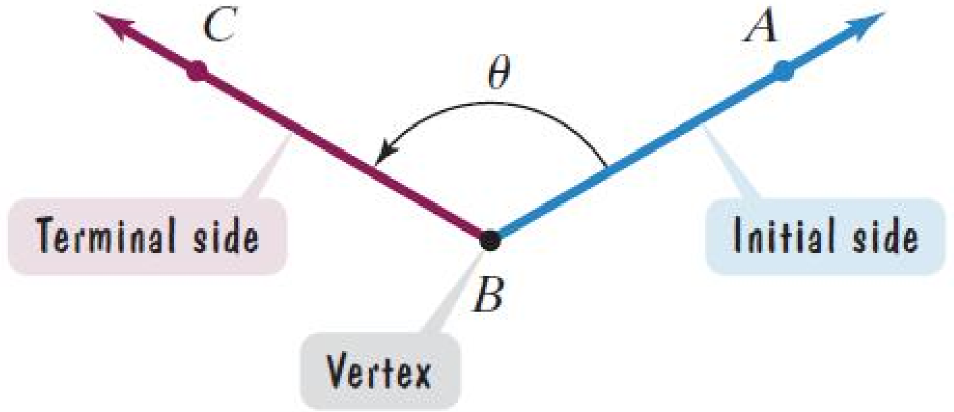
CC TRIGONOMETRY
CHAPTER 1 ANGLES AND TRIGONOMETRIC FUNCTIONS

SECTION 1.1 - Angles and Radian Measure

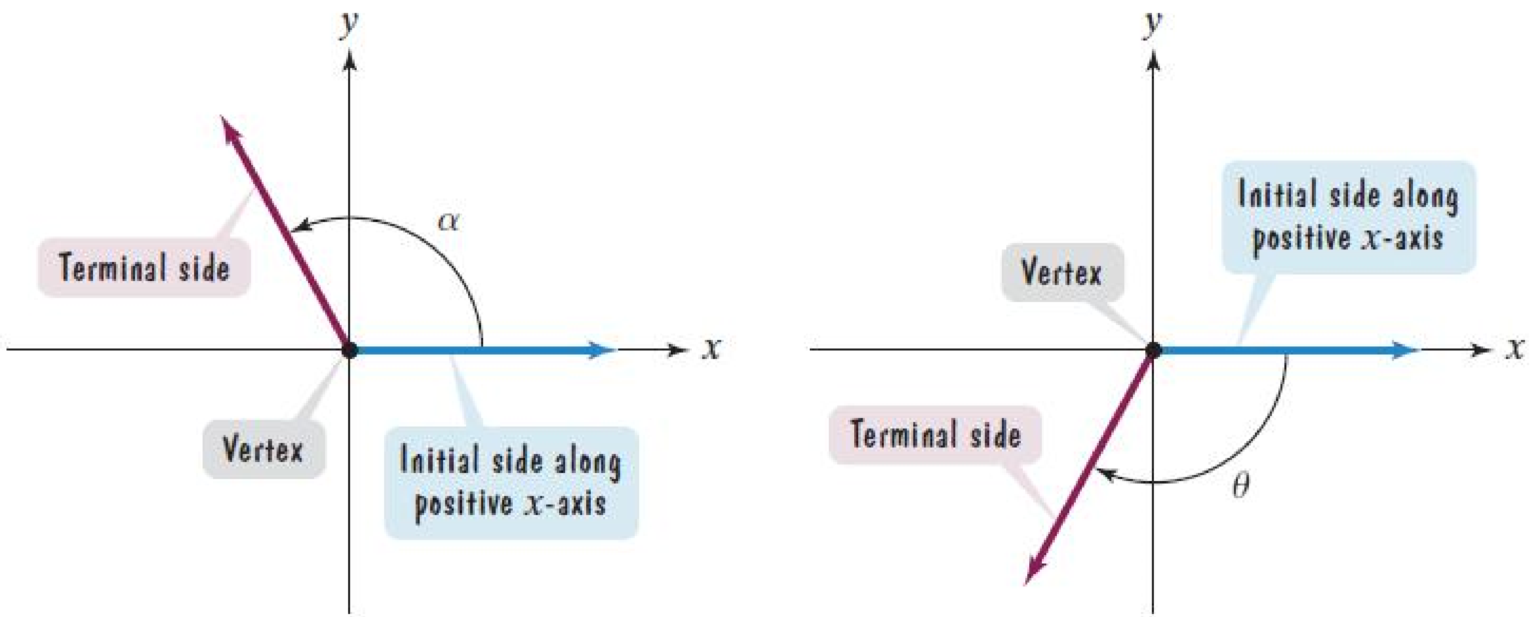
Objectives:

- Learn the vocabulary
- Use degree measure
- Use radian measure
- Convert between the two
- Draw angles in standard position
- Find coterminal angles
- Arc length
- Area of sector
- Linear and angular speed

An **angle** is formed by two rays that have a common endpoint. One ray is called the **initial side** and the other the **terminal side**.

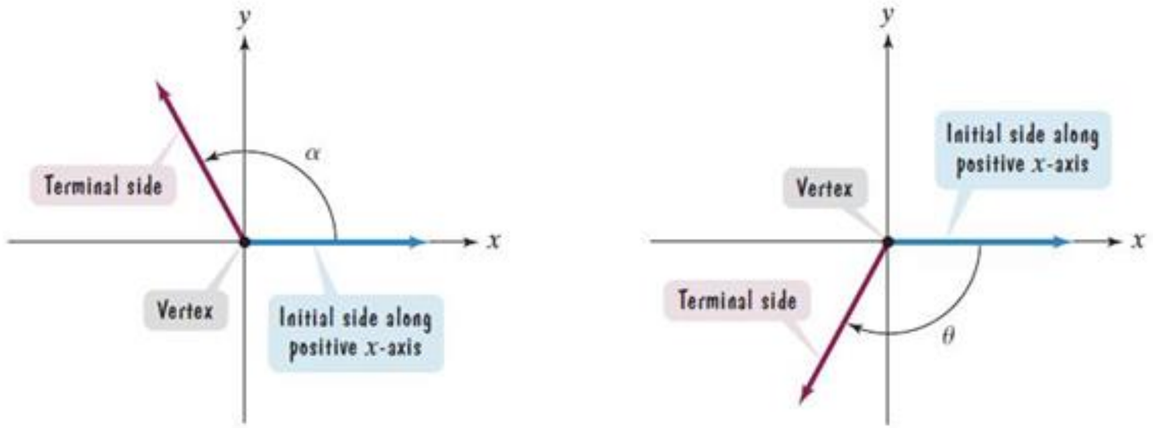


An angle is in standard position if its vertex is at the origin of a rectangular coordinate system and its initial side lies along the positive x-axis.



Positive angles are generated by counterclockwise rotation. Thus, angle

by clockwise rotation. Thus, angle θ is negative.



Angles are measured by determining the amount of rotation from the initial side to the terminal side. A complete rotation of the circle is 360 degrees, or 360°.

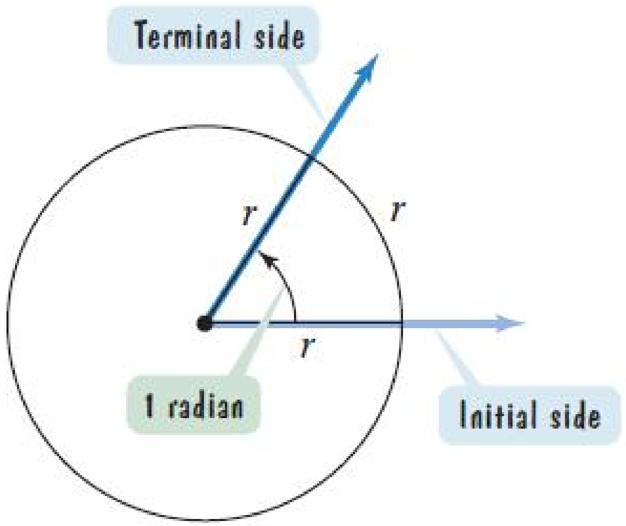
An **acute angle** measures less than 90°.

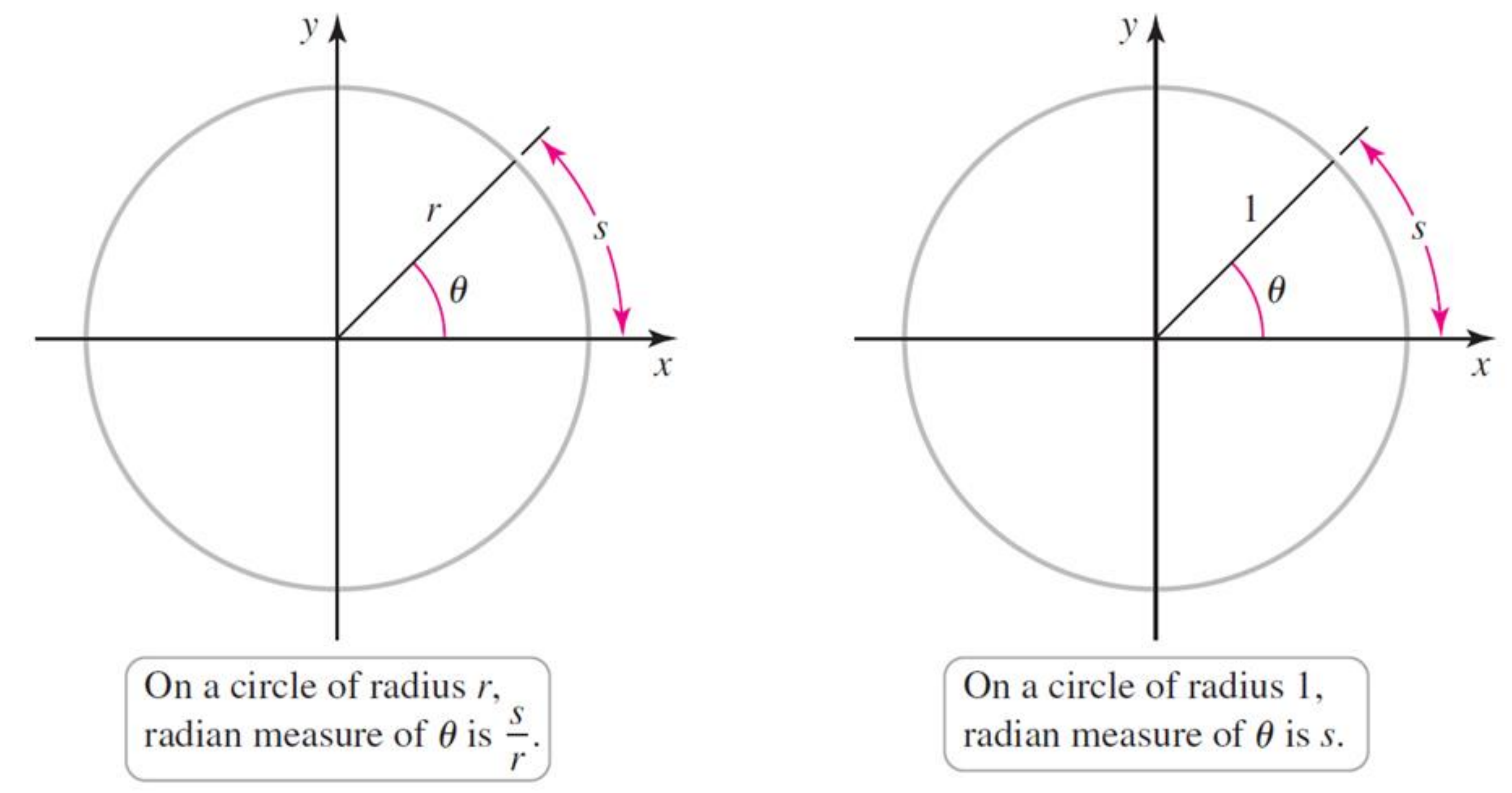
A **right angle** measures 90°.

An **obtuse angle** measures more than 90° but less than 180°.

A **straight angle** measures 180°.

One **radian** is the measure of the central angle of a circle that intercepts an arc equal in length to the radius of the circle.




y
r
s
θ
x
On a circle of radius r, radian measure of θ is s/r.
y
1
s
θ
x
On a circle of radius 1, radian measure of θ is s.

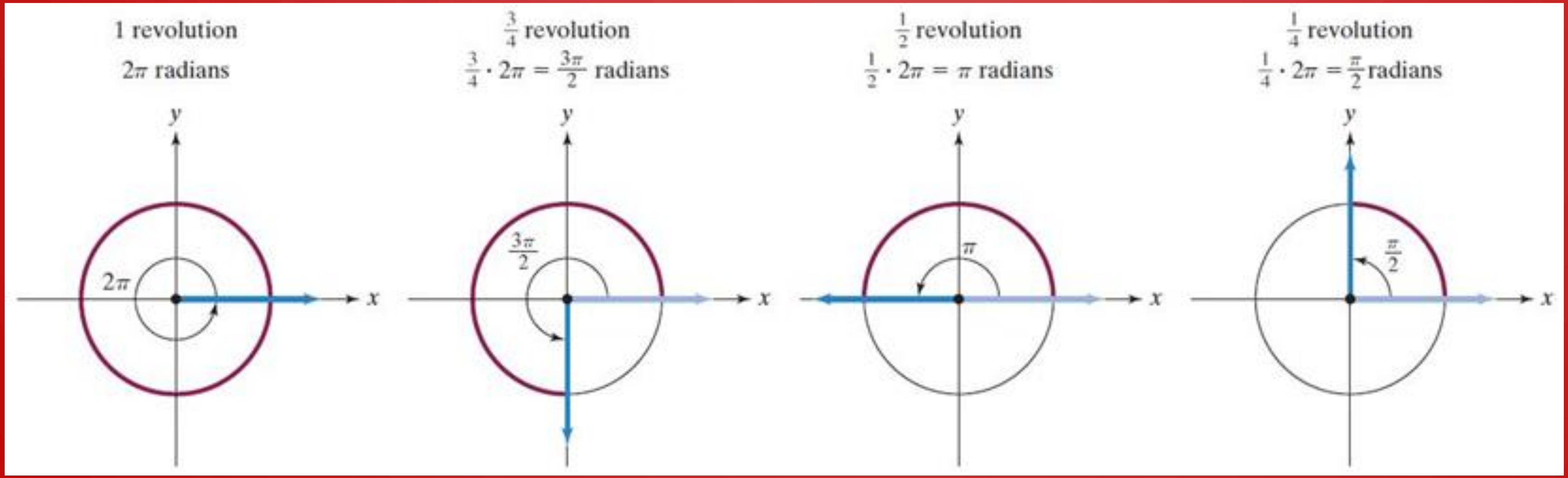
Using the basic relationship π radians $= 180^\circ$,

1. To convert degrees to radians, multiply degrees by

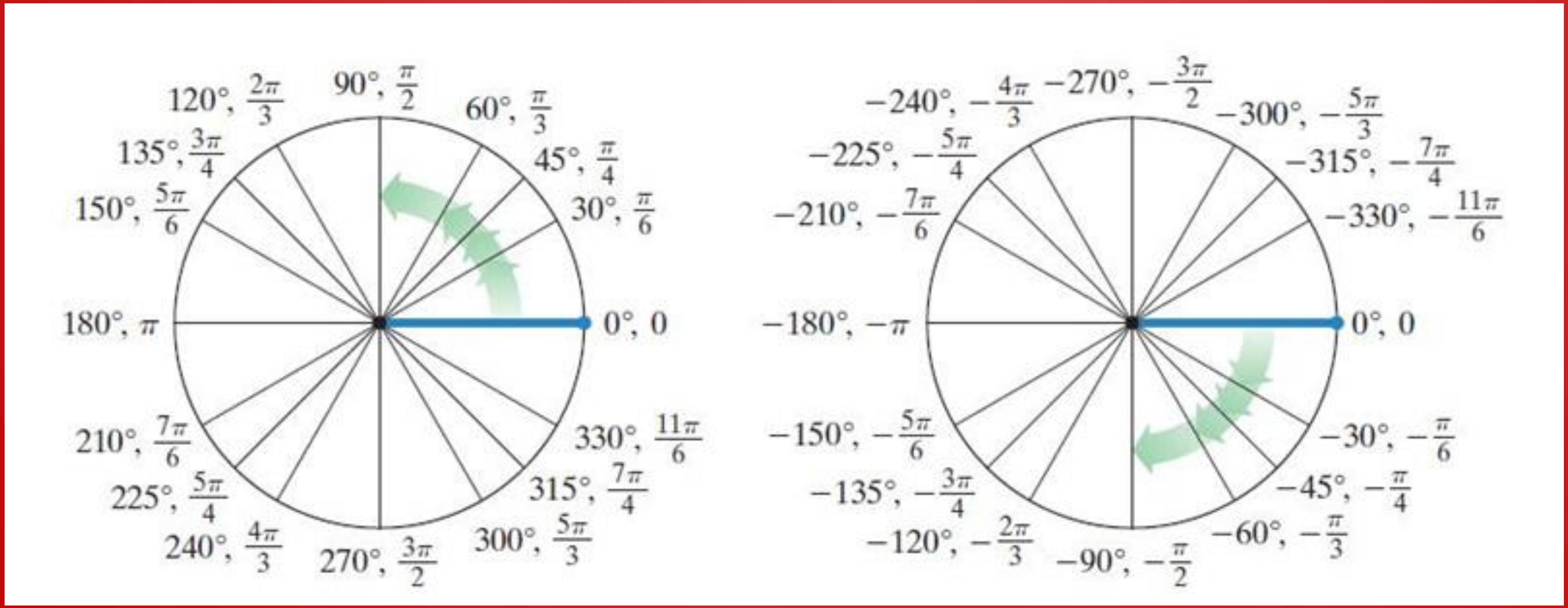
$$\frac{\pi \text{ radians}}{180^\circ}.$$

2. To convert radians to degrees, multiply radians by

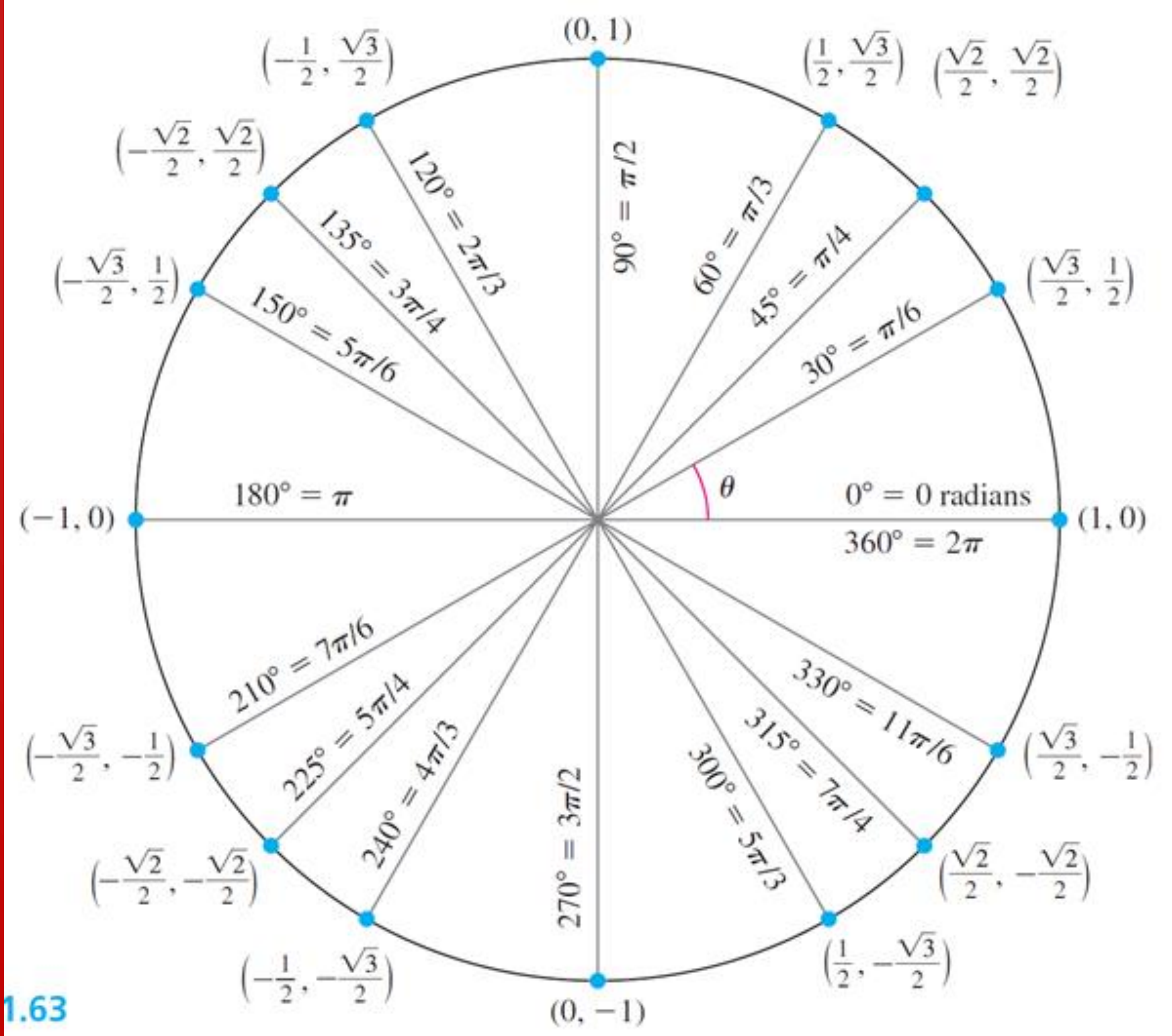
$$\frac{180^\circ}{\pi \text{ radians}}.$$



1 revolution
2π radians
3/4 revolution
3/4 · 2π = 3π/2 radians
1/2 revolution
1/2 · 2π = π radians
1/4 revolution
1/4 · 2π = π/2 radians



120°, $\frac{2\pi}{3}$
90°, $\frac{\pi}{2}$
60°, $\frac{\pi}{3}$
135°, $\frac{3\pi}{4}$
45°, $\frac{\pi}{4}$
150°, $\frac{5\pi}{6}$
30°, $\frac{\pi}{6}$
180°, π
0°, 0
210°, $\frac{7\pi}{6}$
330°, $\frac{11\pi}{6}$
225°, $\frac{5\pi}{4}$
315°, $\frac{7\pi}{4}$
240°, $\frac{4\pi}{3}$
270°, $\frac{3\pi}{2}$
300°, $\frac{5\pi}{3}$
−240°, $-\frac{4\pi}{3}$
−270°, $-\frac{3\pi}{2}$
−300°, $-\frac{5\pi}{3}$
−225°, $-\frac{5\pi}{4}$
−315°, $-\frac{7\pi}{4}$
−210°, $-\frac{7\pi}{6}$
−330°, $-\frac{11\pi}{6}$
−180°, $-\pi$
0°, 0
−150°, $-\frac{5\pi}{6}$
−30°, $-\frac{\pi}{6}$
−135°, $-\frac{3\pi}{4}$
−45°, $-\frac{\pi}{4}$
−120°, $-\frac{2\pi}{3}$
−60°, $-\frac{\pi}{3}$
−90°, $-\frac{\pi}{2}$



(0, 1)
$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$120° = 2\pi/3$
$90° = \pi/2$
$60° = \pi/3$
$135° = 3\pi/4$
$45° = \pi/4$
$150° = 5\pi/6$
$30° = \pi/6$
θ
$180° = \pi$
$0° = 0$ radians
$360° = 2\pi$
(−1, 0)
(1, 0)
$210° = 7\pi/6$
$225° = 5\pi/4$
$240° = 4\pi/3$
$270° = 3\pi/2$
$300° = 5\pi/3$
$315° = 7\pi/4$
$330° = 11\pi/6$
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
(0, −1)
1.63

Terminal Side	Radian Measure of Angle	Degree Measure of Angle
$\frac{1}{12}$ revolution	$\frac{1}{12} \cdot 2\pi = \frac{\pi}{6}$	$\frac{1}{12} \cdot 360^\circ = 30^\circ$
$\frac{1}{8}$ revolution	$\frac{1}{8} \cdot 2\pi = \frac{\pi}{4}$	$\frac{1}{8} \cdot 360^\circ = 45^\circ$
$\frac{1}{6}$ revolution	$\frac{1}{6} \cdot 2\pi = \frac{\pi}{3}$	$\frac{1}{6} \cdot 360^\circ = 60^\circ$
$\frac{1}{4}$ revolution	$\frac{1}{4} \cdot 2\pi = \frac{\pi}{2}$	$\frac{1}{4} \cdot 360^\circ = 90^\circ$
$\frac{1}{3}$ revolution	$\frac{1}{3} \cdot 2\pi = \frac{2\pi}{3}$	$\frac{1}{3} \cdot 360^\circ = 120^\circ$

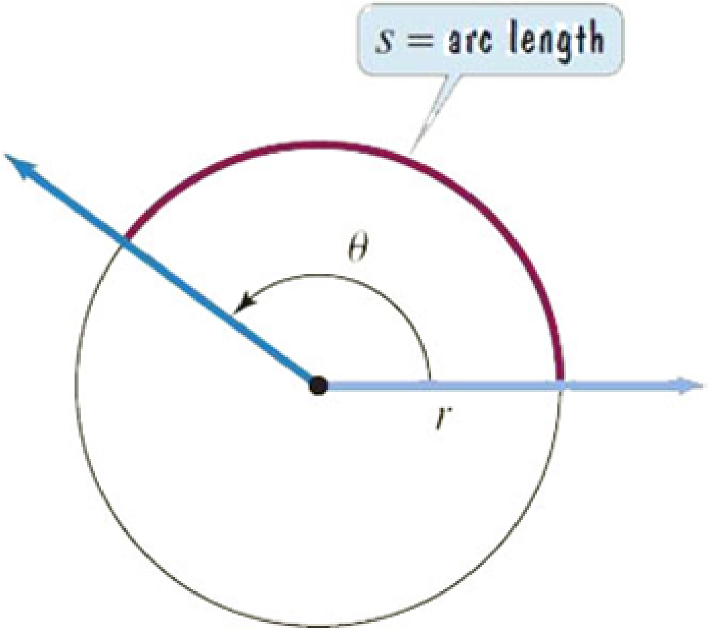
Terminal Side	Radian Measure of Angle	Degree Measure of Angle
$\frac{1}{2}$ revolution	$\frac{1}{2} \cdot 2\pi = \pi$	$\frac{1}{2} \cdot 360^\circ = 180^\circ$
$\frac{2}{3}$ revolution	$\frac{2}{3} \cdot 2\pi = \frac{4\pi}{3}$	$\frac{2}{3} \cdot 360^\circ = 240^\circ$
$\frac{3}{4}$ revolution	$\frac{3}{4} \cdot 2\pi = \frac{3\pi}{2}$	$\frac{3}{4} \cdot 360^\circ = 270^\circ$
$\frac{7}{8}$ revolution	$\frac{7}{8} \cdot 2\pi = \frac{7\pi}{4}$	$\frac{7}{8} \cdot 360^\circ = 315^\circ$
1 revolution	$1 \cdot 2\pi = 2\pi$	$1 \cdot 360^\circ = 360^\circ$

Two angles with the same initial and terminal sides but possibly different rotations are called **coterminal angles**.

Let r be the radius of a circle and θ the nonnegative radian measure of a central angle of the circle.

The length of the arc intercepted by the central angle is

$$s = r\theta.$$



Let r be the radius of a circle and θ the nonnegative radian measure of a central angle of the circle. The area of the sector formed by the central angle is

$$A = \frac{1}{2}r^2\theta.$$

If a point is in motion on a circle of radius r through an angle of θ radians in time t, then its **linear speed** is

$$v = \frac{s}{t},$$

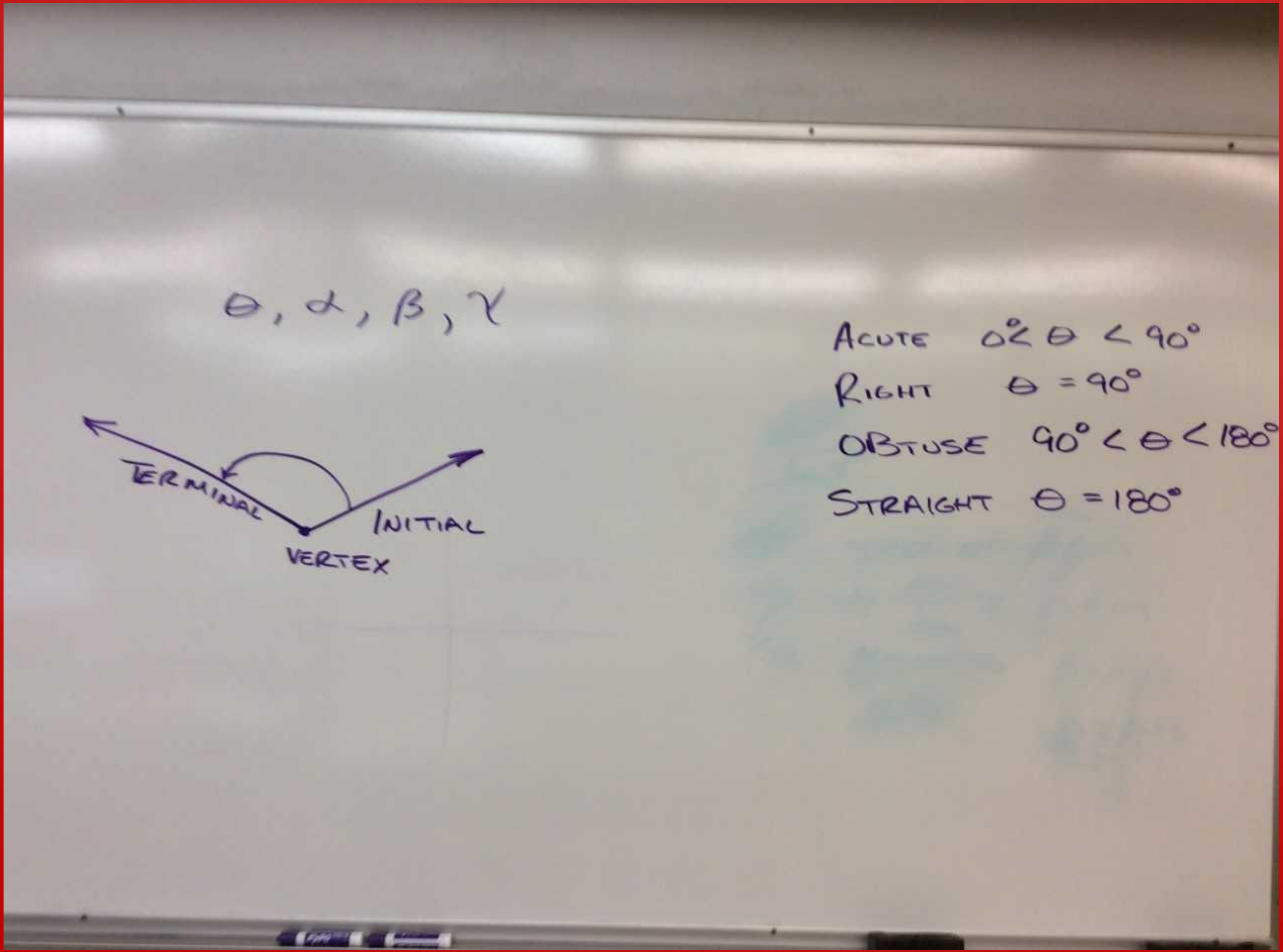
where s is the arc length given by $s = r\theta$, and its **angular speed** is

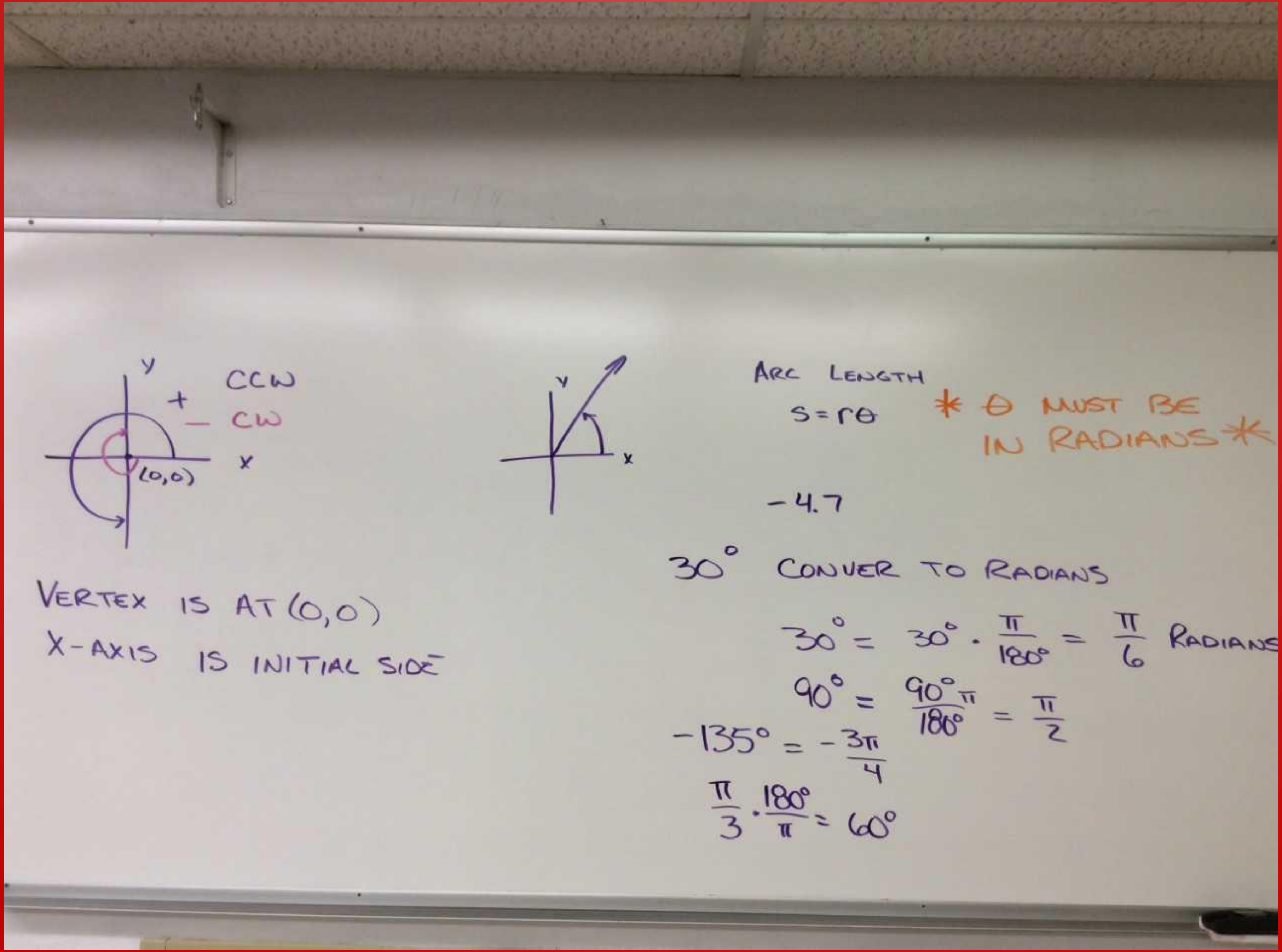
$$\omega = \frac{\theta}{t}.$$

The linear speed, v, of a point a distance r from the center of rotation is given by

$$v = r\omega,$$

Where ω is the angular speed in radians per unit of time.


θ, α, β, γ
TERMINAL
INITIAL
VERTEX
ACUTE 0° < θ < 90°
RIGHT θ = 90°
OBTUSE 90° < θ < 180°
STRAIGHT θ = 180°



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