BOARD NOTES

23 JANUARY 2019

CC TRIGONOMETRY
CHAPTER 1 ANGLES AND TRIGONOMETRIC FUNCTIONS

SECTION 1.1 - Angles and Radian Measure

Objectives:

- Learn the vocabulary
- Use degree measure
- Use radian measure
- Convert between the two
- Draw angles in standard position
- Find coterminal angles
- Arc length
- Area of sector
- Linear and angular speed

Arc Length

For a circle of radius r, a central angle of θ radians subtends an arc whose length s is

$$s = r\theta$$

Area of a Sector

The area A of the sector of a circle of radius r formed by a central angle of θ radians is

$$A = \frac{1}{2}r^2\theta$$

If a point is in motion on a circle of radius r through an angle of θ radians in time t, then its **linear speed** is

$$v = \frac{s}{t},$$

where s is the arc length given by $s = r\theta$, and its **angular speed** is

$$\omega = \frac{\theta}{t}.$$

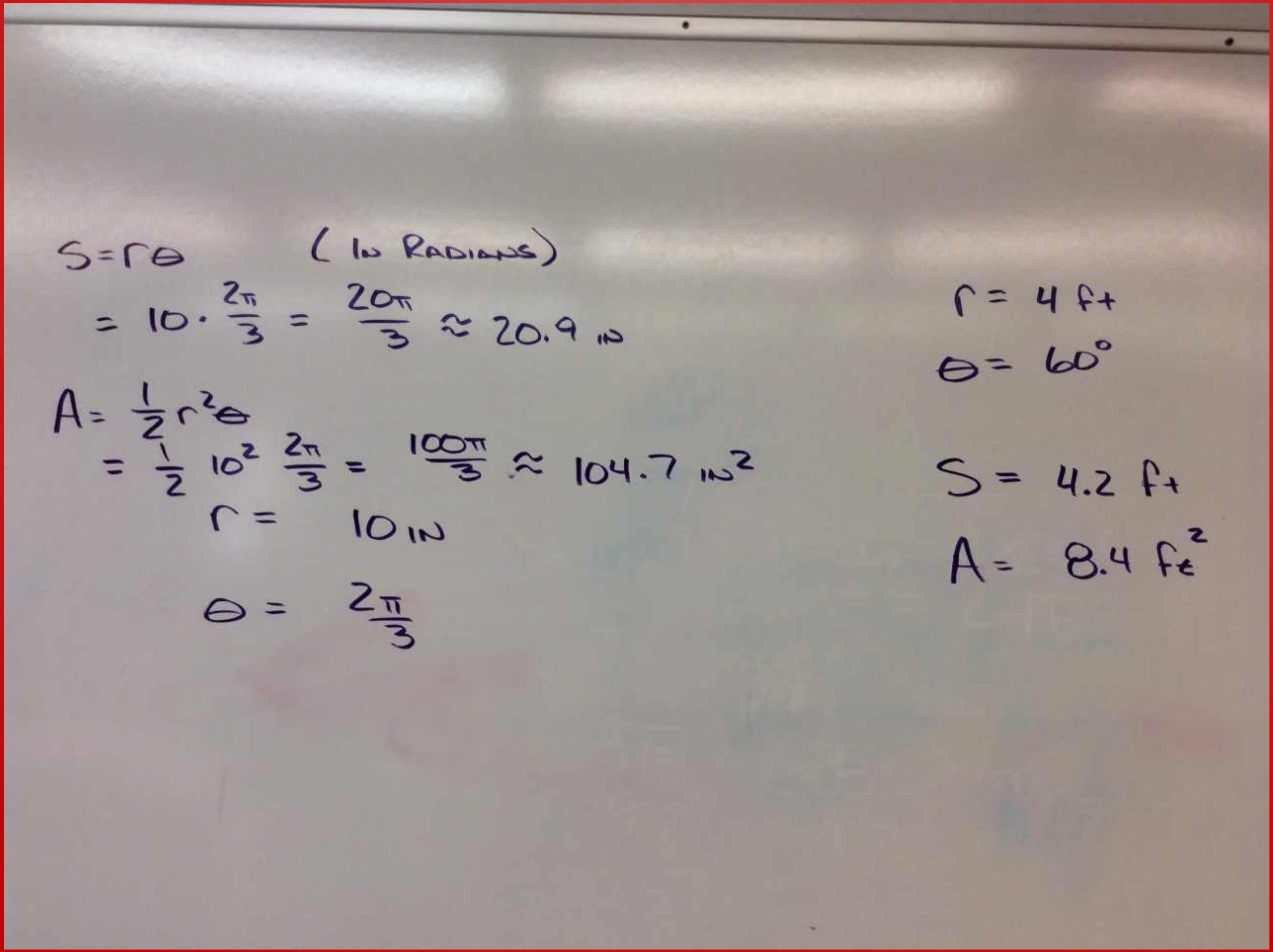
The linear speed, v, of a point a distance r from the center of rotation is given by

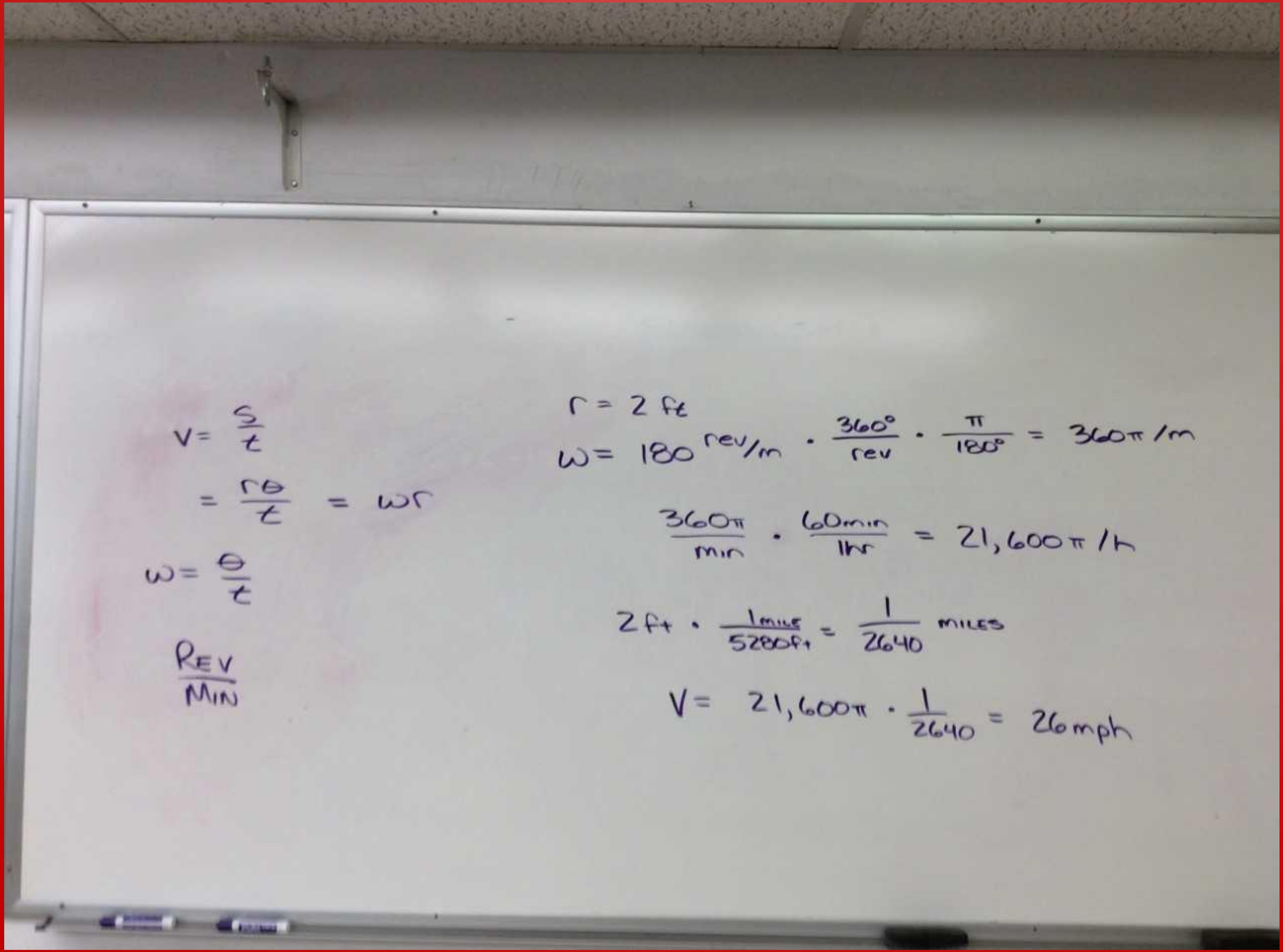
$$v = r\omega,$$

Where ω is the angular speed in radians per unit of time.

Finding Linear Speed

A child is spinning a rock at the end of a 2-foot rope at the rate of 180 revolutions per minute (rpm). Find the linear speed of the rock when it is released.


S=rθ (In Radians)
= 10 · 2π/3 = 20π/3 ≈ 20.9 in
A = 1/2 r²θ
= 1/2 10² 2π/3 = 100π/3 ≈ 104.7 in²
r = 10 in
θ = 2π/3
r = 4 ft
θ = 60°
S = 4.2 ft
A = 8.4 ft²


V = s/t
= rθ/t = ωr
ω = θ/t
Rev/Min
r = 2 ft
ω = 180 rev/m · 360°/rev · π/180° = 360π/m
360π/min · 60min/1hr = 21,600π/h
2ft · 1mile/5280ft = 1/2640 miles
V = 21,600π · 1/2640 = 26mph

CC TRIGONOMETRY
CHAPTER 1 ANGLES AND TRIGONOMETRIC FUNCTIONS

SECTION 1.2 - Right Triangle Trigonometry

Objectives:

- Use right triangles to evaluate trigonometric functions
- Find function values for 30°, 45°, and 60°
- Recognize and use fundamental identities
- Use equal cofunctions of complements
- Use a calculator to solve trig functions
- Solve applied problems

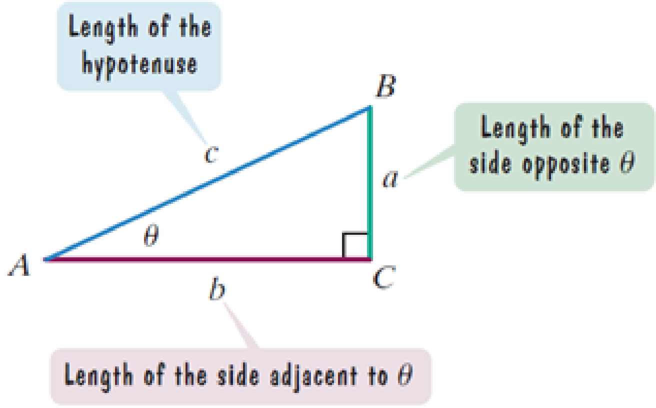
The six trigonometric functions are:

Function	Abbreviation
sine	sin
cosine	cos
tangent	tan
cosecant	csc
secant	sec
cotangent	cot

$$\sin\theta = \frac{\text{length of side opposite angle}\,\theta}{\text{length of hypotenuse}} = \frac{a}{c}$$

$$\cos\theta = \frac{\text{length of side adjacent to angle}\,\theta}{\text{length of hypotenuse}} = \frac{b}{c}$$

$$\tan\theta = \frac{\text{length of side opposite angle}\,\theta}{\text{length of side adjacent to angle}\,\theta} = \frac{a}{c}$$

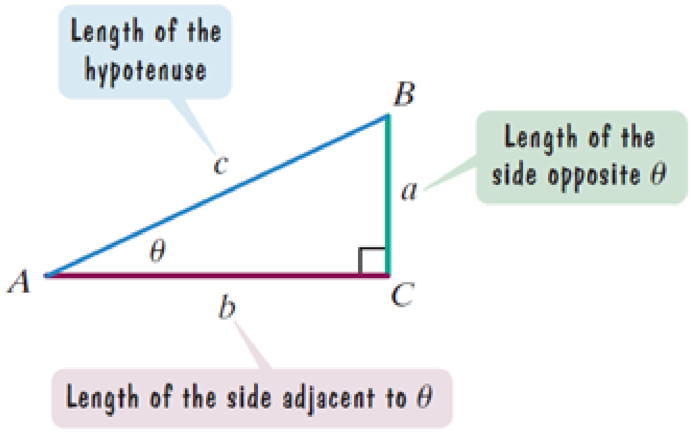


In general, **the trigonometric functions of *θ* depend only on the size of angle *θ* and not on the size of the triangle.**

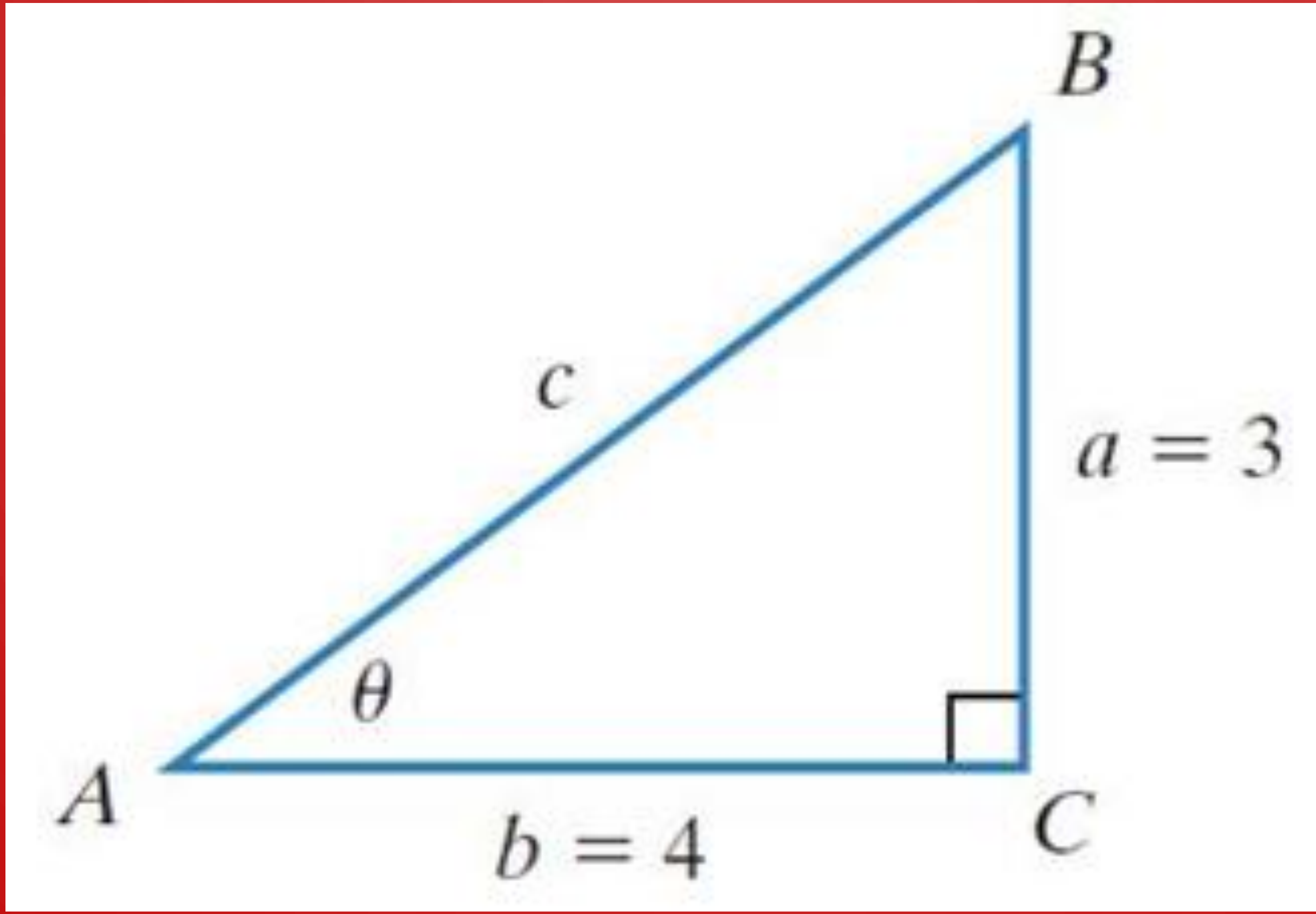
$$\csc\theta = \frac{\text{length of hypotenuse}}{\text{length of side opposite angle}\,\theta} = \frac{c}{a}$$

$$\sec\theta = \frac{\text{length of hypotenuse}}{\text{length of side adjacent to angle}\,\theta} = \frac{c}{b}$$

$$\cot\theta = \frac{\text{length of side adjacent to angle}\,\theta}{\text{length of side opposite angle}\,\theta} = \frac{b}{a}$$



In general, **the trigonometric functions of θ depend only on the size of angle θ and not on the size of the triangle.**


B
c
a = 3
θ
A
b = 4
C

θ	$30^\circ = \frac{\pi}{6}$	$45^\circ = \frac{\pi}{4}$	$60^\circ = \frac{\pi}{3}$
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan\theta$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

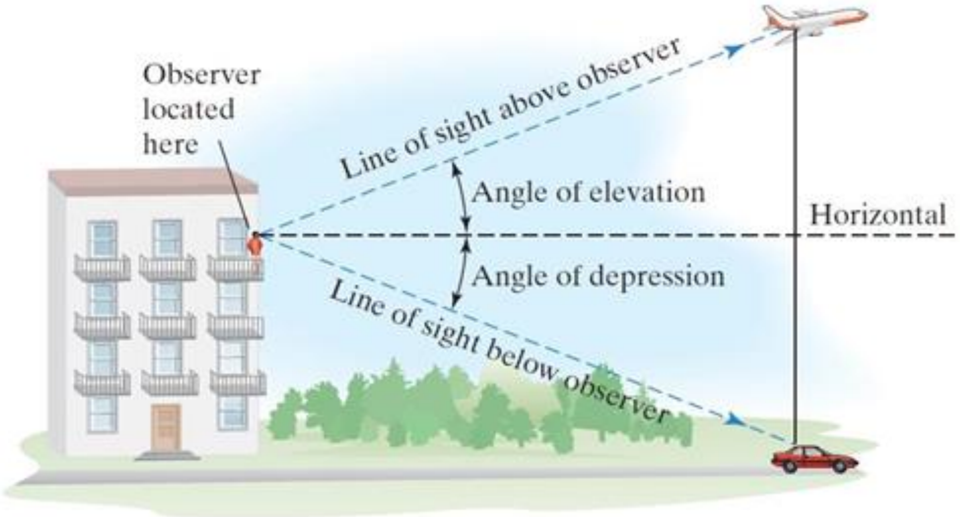
The value of a trigonometric function of θ is equal to the cofunction of the complement of θ. Cofunctions of complementary angles are equal.

$$\sin\theta = \cos(90° - \theta) \qquad \cos\theta = \sin(90° - \theta)$$

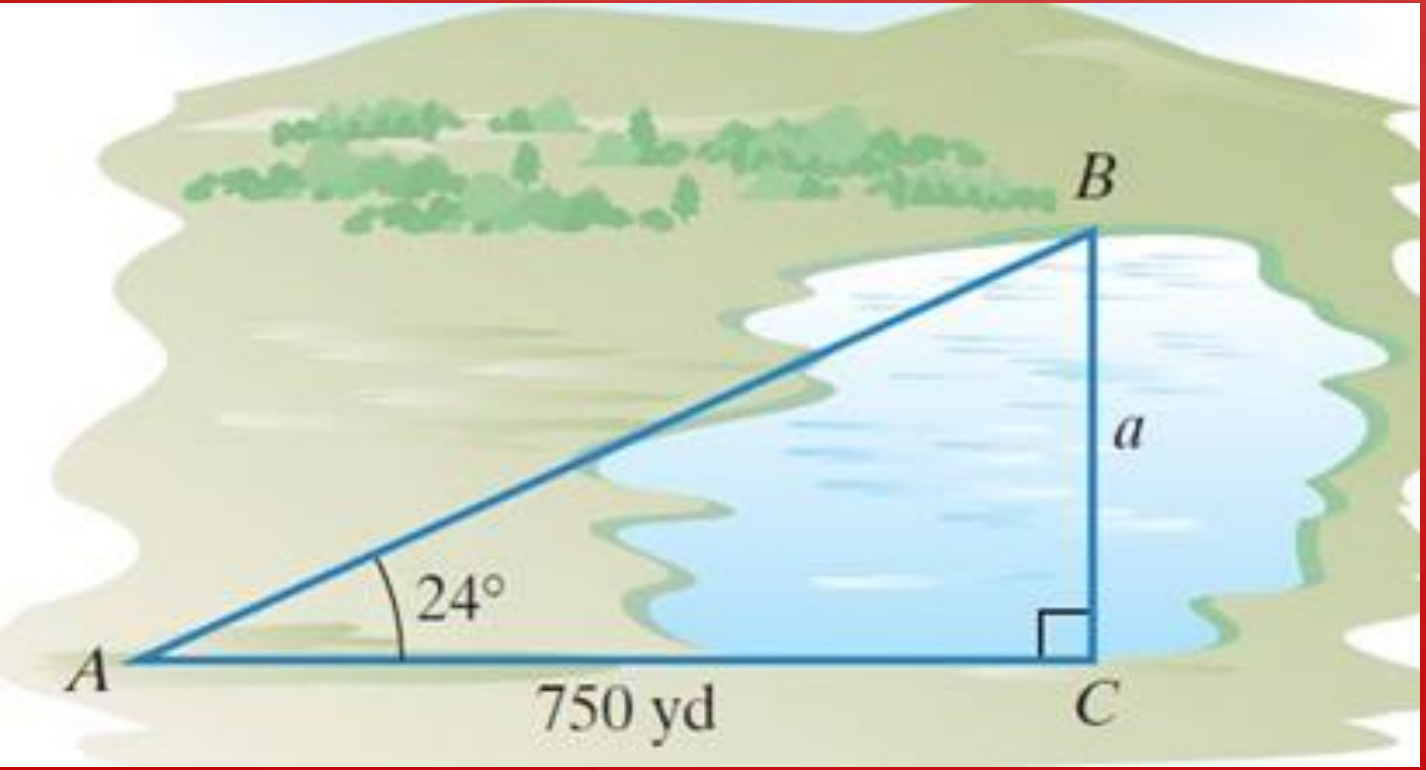
$$\tan\theta = \cot(90° - \theta) \qquad \cot\theta = \tan(90° - \theta)$$

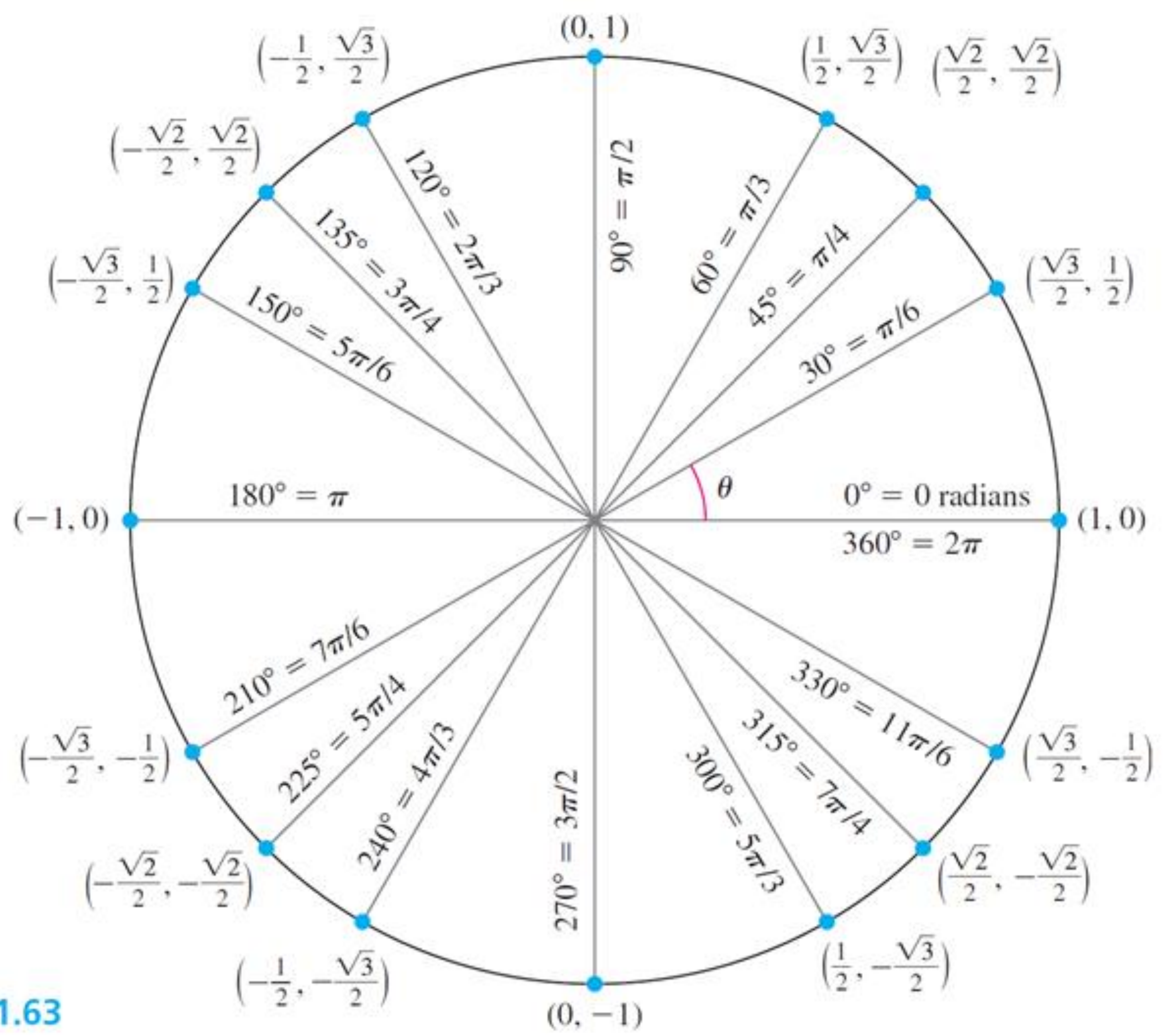
$$\sec\theta = \csc(90° - \theta) \qquad \csc\theta = \sec(90° - \theta)$$

An angle formed by a horizontal line and the line of sight to an object that is above the horizontal line is called the **angle of elevation**. The angle formed by the horizontal line and the line of sight to an object that is below the horizontal line is called the **angle of depression**.



The irregular blue shape in the figure represents a lake. The distance across the lake, *a*, is unknown. To find this distance, a surveyor took the measurements shown in the figure. What is the distance across the lake?



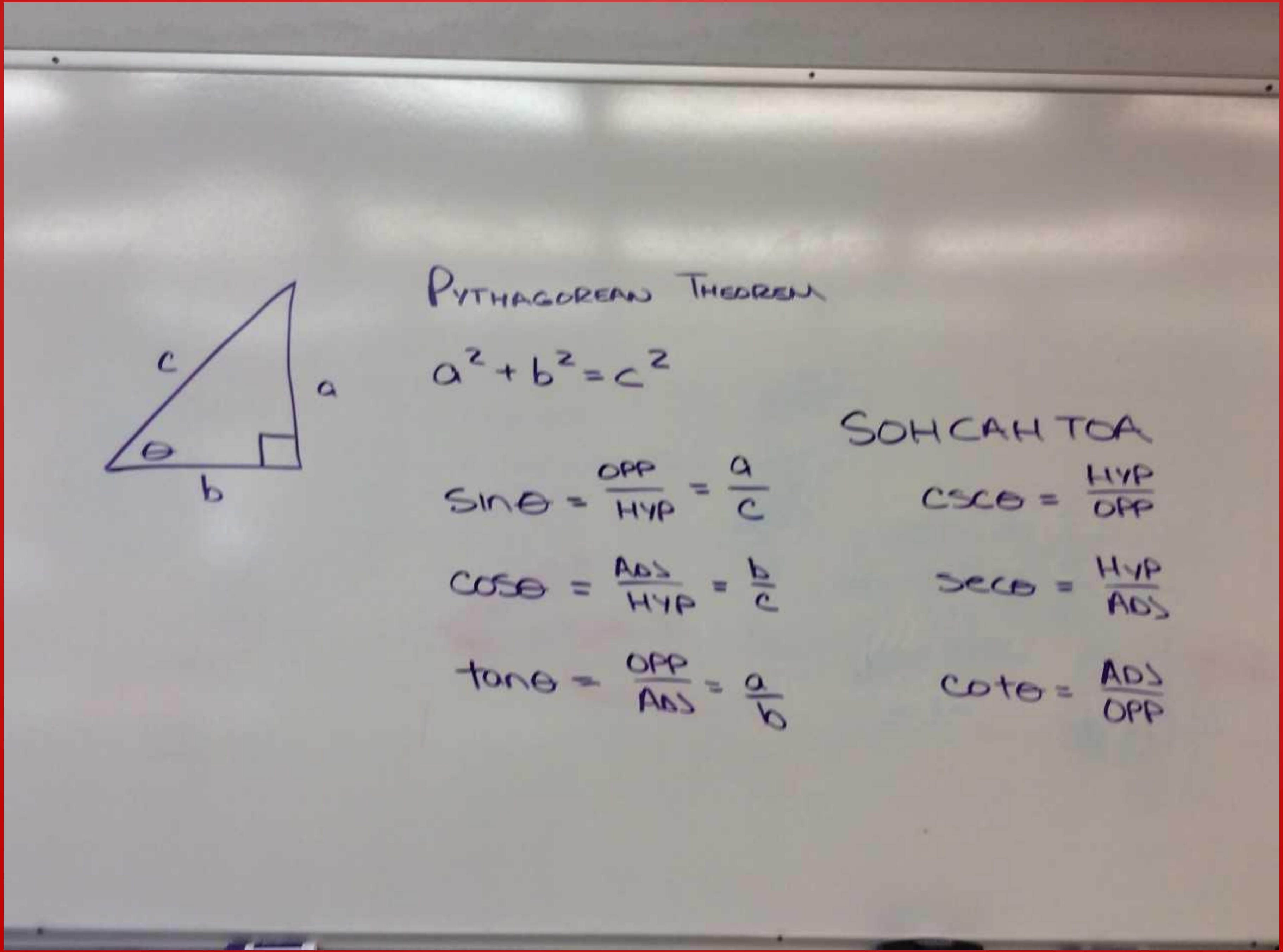


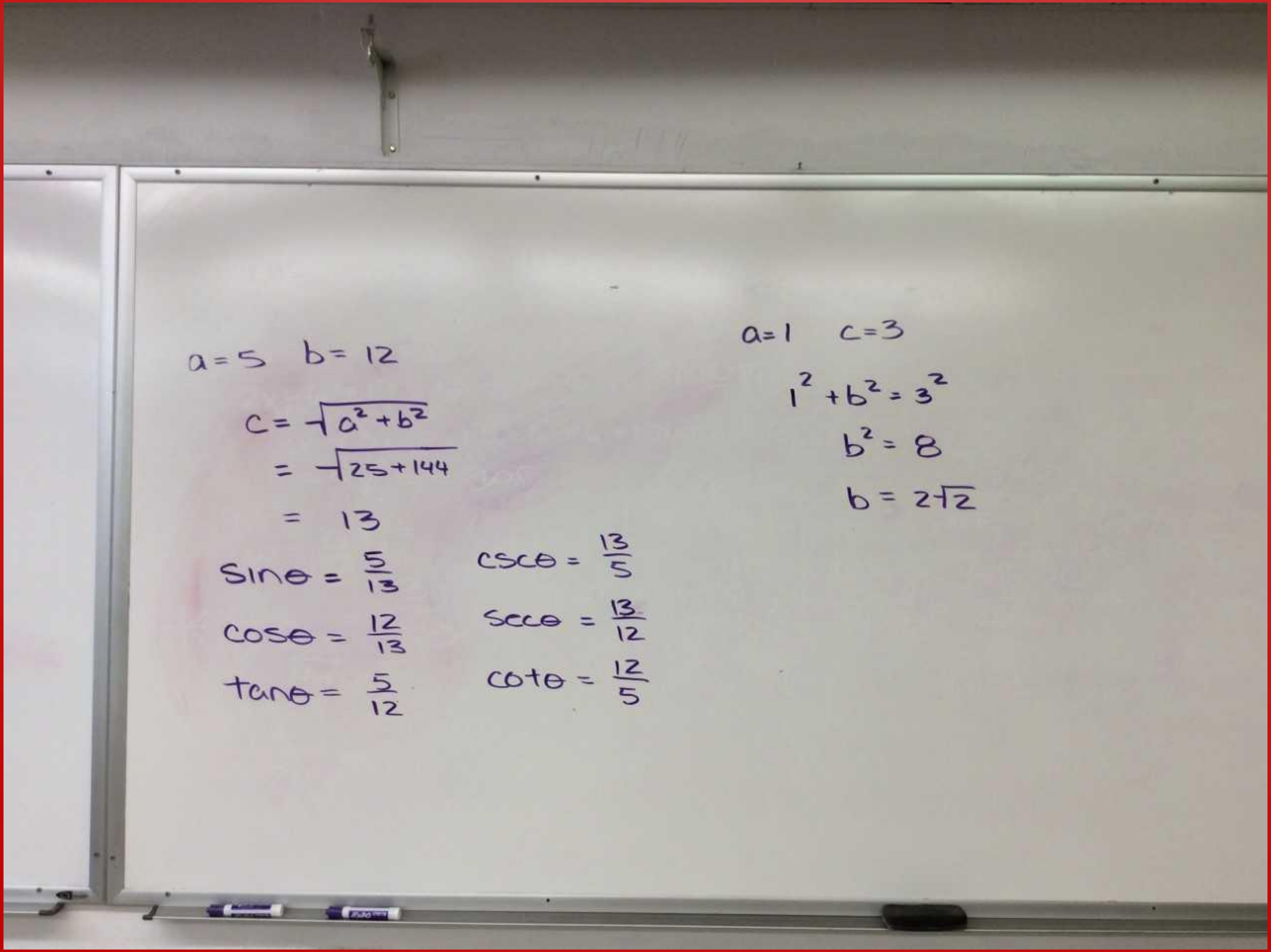
(0, 1)
$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$120° = 2\pi/3$
$90° = \pi/2$
$60° = \pi/3$
$135° = 3\pi/4$
$45° = \pi/4$
$150° = 5\pi/6$
$30° = \pi/6$
θ
$180° = \pi$
$0° = 0$ radians
$360° = 2\pi$
(−1, 0)
(1, 0)
$210° = 7\pi/6$
$225° = 5\pi/4$
$240° = 4\pi/3$
$270° = 3\pi/2$
$300° = 5\pi/3$
$315° = 7\pi/4$
$330° = 11\pi/6$
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
(0, −1)

1.63

Terminal Side	Radian Measure of Angle	Degree Measure of Angle
$\frac{1}{12}$ revolution	$\frac{1}{12} \cdot 2\pi = \frac{\pi}{6}$	$\frac{1}{12} \cdot 360^\circ = 30^\circ$
$\frac{1}{8}$ revolution	$\frac{1}{8} \cdot 2\pi = \frac{\pi}{4}$	$\frac{1}{8} \cdot 360^\circ = 45^\circ$
$\frac{1}{6}$ revolution	$\frac{1}{6} \cdot 2\pi = \frac{\pi}{3}$	$\frac{1}{6} \cdot 360^\circ = 60^\circ$
$\frac{1}{4}$ revolution	$\frac{1}{4} \cdot 2\pi = \frac{\pi}{2}$	$\frac{1}{4} \cdot 360^\circ = 90^\circ$
$\frac{1}{3}$ revolution	$\frac{1}{3} \cdot 2\pi = \frac{2\pi}{3}$	$\frac{1}{3} \cdot 360^\circ = 120^\circ$

Terminal Side	Radian Measure of Angle	Degree Measure of Angle
$\frac{1}{2}$ revolution	$\frac{1}{2} \cdot 2\pi = \pi$	$\frac{1}{2} \cdot 360^\circ = 180^\circ$
$\frac{2}{3}$ revolution	$\frac{2}{3} \cdot 2\pi = \frac{4\pi}{3}$	$\frac{2}{3} \cdot 360^\circ = 240^\circ$
$\frac{3}{4}$ revolution	$\frac{3}{4} \cdot 2\pi = \frac{3\pi}{2}$	$\frac{3}{4} \cdot 360^\circ = 270^\circ$
$\frac{7}{8}$ revolution	$\frac{7}{8} \cdot 2\pi = \frac{7\pi}{4}$	$\frac{7}{8} \cdot 360^\circ = 315^\circ$
1 revolution	$1 \cdot 2\pi = 2\pi$	$1 \cdot 360^\circ = 360^\circ$


c
a
θ
b
PYTHAGOREAN THEOREM
$a^2 + b^2 = c^2$
SOHCAHTOA
sinθ = OPP/HYP = a/c
cosθ = ADJ/HYP = b/c
tanθ = OPP/ADJ = a/b
cscθ = HYP/OPP
secθ = HYP/ADJ
cotθ = ADJ/OPP



a=5 b=12
c = √(a² + b²)
= √(25+144)
= 13
sinθ = 5/13
cosθ = 12/13
tanθ = 5/12
cscθ = 13/5
secθ = 13/12
cotθ = 12/5
a=1 c=3
1² + b² = 3²
b² = 8
b = 2√2