BOARD NOTES

24 JANUARY 2019

CC TRIGONOMETRY
CHAPTER 1 ANGLES AND TRIGONOMETRIC FUNCTIONS

SECTION 1.2 - Right Triangle Trigonometry

Objectives:

- Use right triangles to evaluate trigonometric functions
- Find function values for 30°, 45°, and 60°
- Recognize and use fundamental identities
- Use equal cofunctions of complements
- Use a calculator to solve trig functions
- Solve applied problems

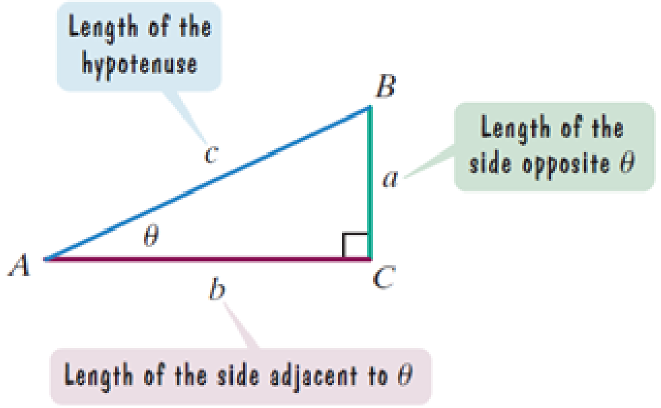
The six trigonometric functions are:

Function	Abbreviation
sine	sin
cosine	cos
tangent	tan
cosecant	csc
secant	sec
cotangent	cot

$$\sin\theta = \frac{\text{length of side opposite angle}\,\theta}{\text{length of hypotenuse}} = \frac{a}{c}$$

$$\cos\theta = \frac{\text{length of side adjacent to angle}\,\theta}{\text{length of hypotenuse}} = \frac{b}{c}$$

$$\tan\theta = \frac{\text{length of side opposite angle}\,\theta}{\text{length of side adjacent to angle}\,\theta} = \frac{a}{c}$$

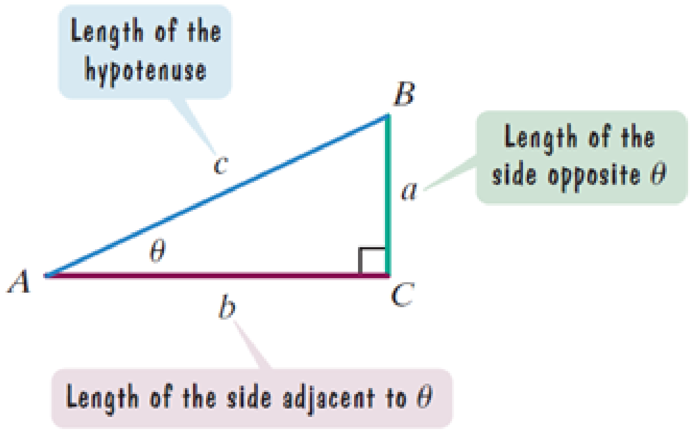


In general, **the trigonometric functions of** $\boldsymbol{\theta}$ **depend only on the size of angle** $\boldsymbol{\theta}$ **and not on the size of the triangle.**

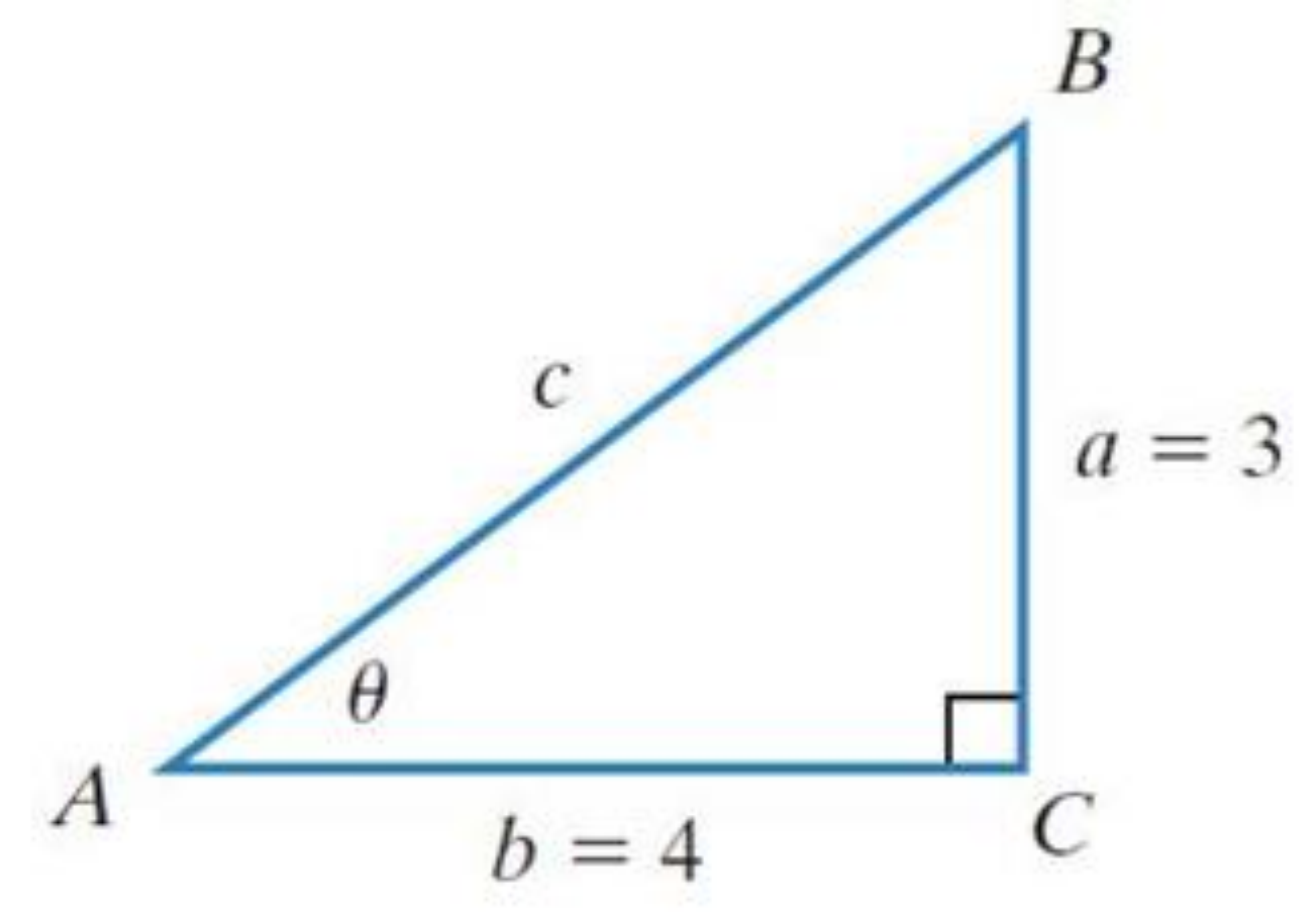
$$\csc\theta = \frac{\text{length of hypotenuse}}{\text{length of side opposite angle}\,\theta} = \frac{c}{a}$$

$$\sec\theta = \frac{\text{length of hypotenuse}}{\text{length of side adjacent to angle}\,\theta} = \frac{c}{b}$$

$$\cot\theta = \frac{\text{length of side adjacent to angle}\,\theta}{\text{length of side opposite angle}\,\theta} = \frac{b}{a}$$



In general, **the trigonometric functions of θ depend only on the size of angle θ and not on the size of the triangle.**


B
c
a = 3
θ
A
b = 4
C

θ	$30^\circ = \frac{\pi}{6}$	$45^\circ = \frac{\pi}{4}$	$60^\circ = \frac{\pi}{3}$
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan\theta$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

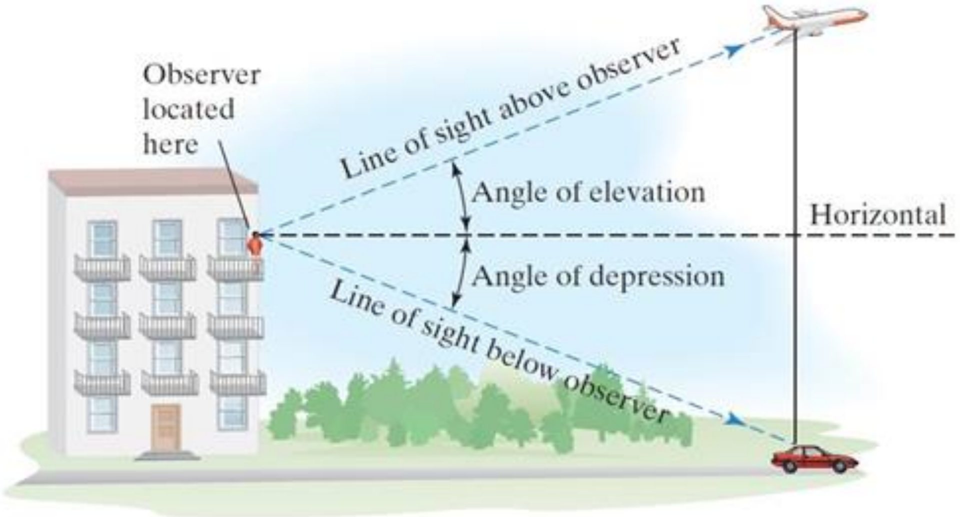
The value of a trigonometric function of θ is equal to the cofunction of the complement of θ. Cofunctions of complementary angles are equal.

$$\sin\theta = \cos(90^\circ - \theta) \qquad \cos\theta = \sin(90^\circ - \theta)$$

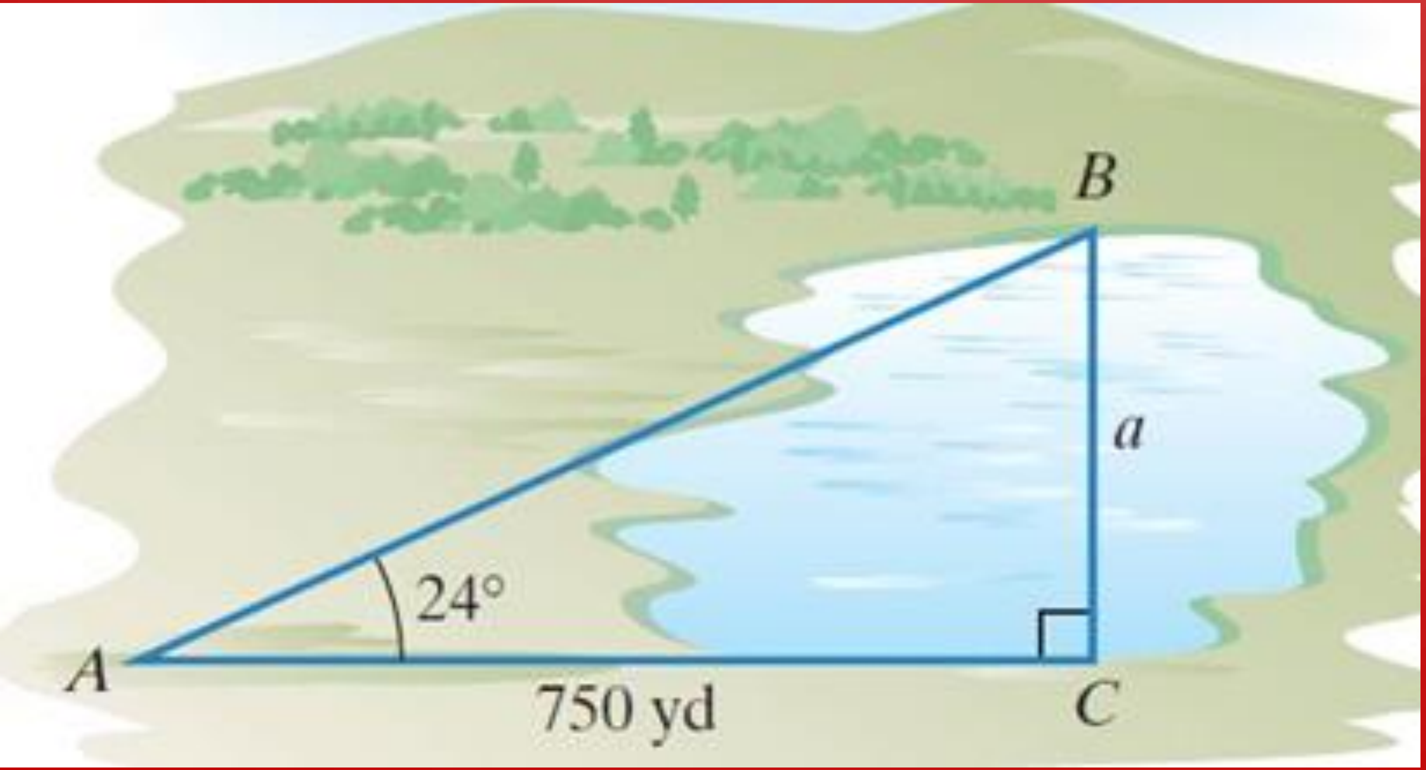
$$\tan\theta = \cot(90^\circ - \theta) \qquad \cot\theta = \tan(90^\circ - \theta)$$

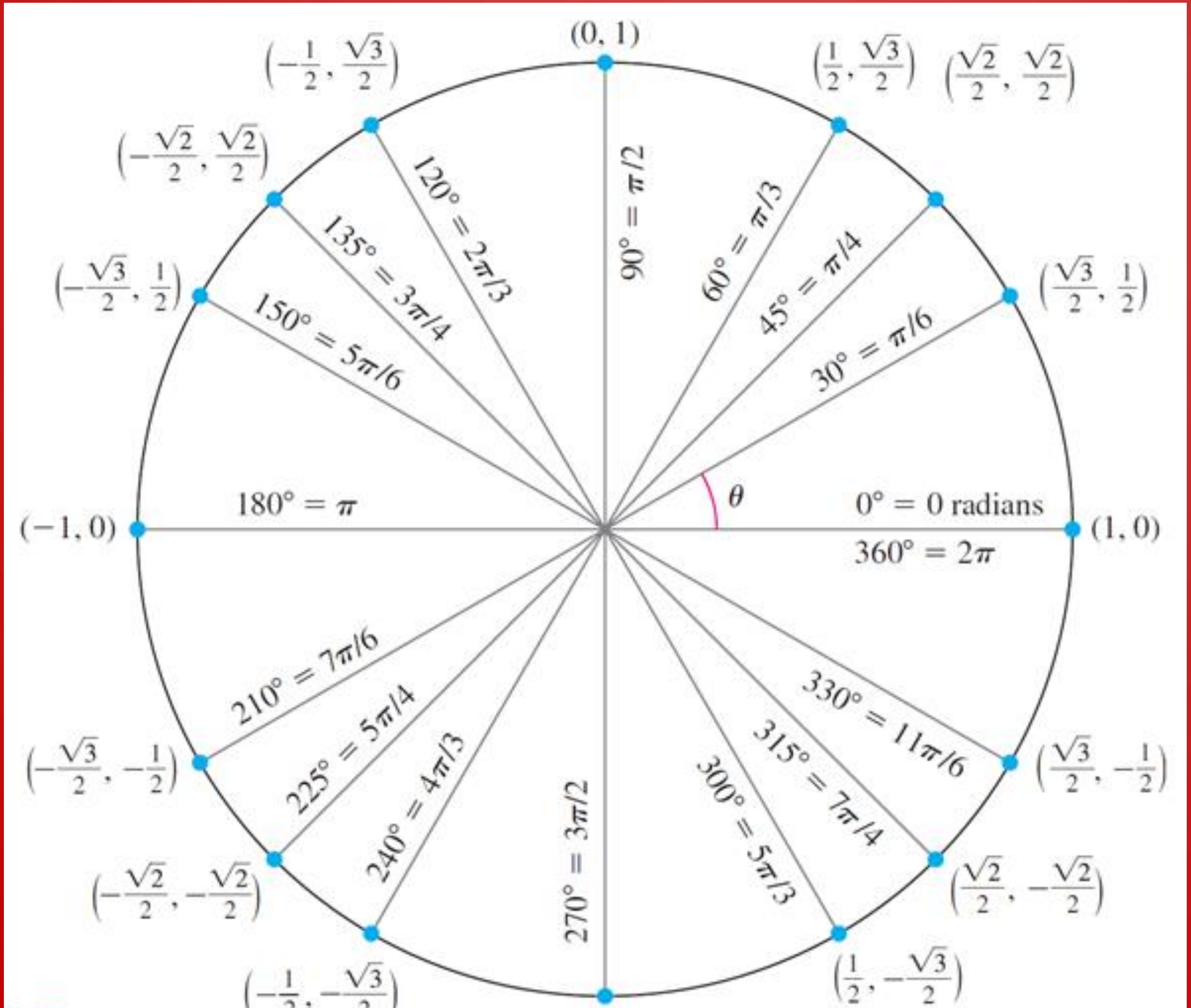
$$\sec\theta = \csc(90^\circ - \theta) \qquad \csc\theta = \sec(90^\circ - \theta)$$

An angle formed by a horizontal line and the line of sight to an object that is above the horizontal line is called the **angle of elevation**. The angle formed by the horizontal line and the line of sight to an object that is below the horizontal line is called the **angle of depression**.



The irregular blue shape in the figure represents a lake. The distance across the lake, *a*, is unknown. To find this distance, a surveyor took the measurements shown in the figure. What is the distance across the lake?

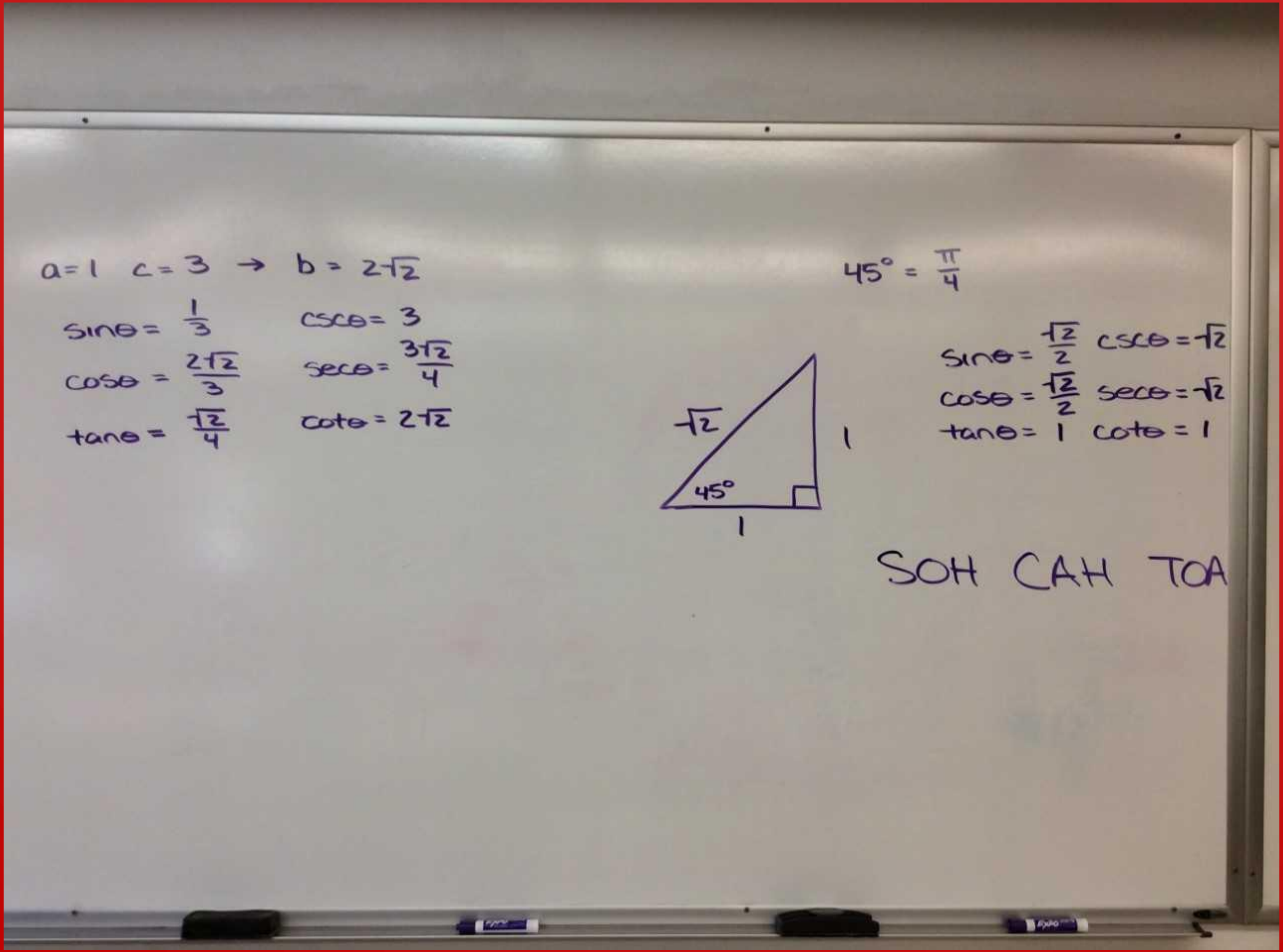


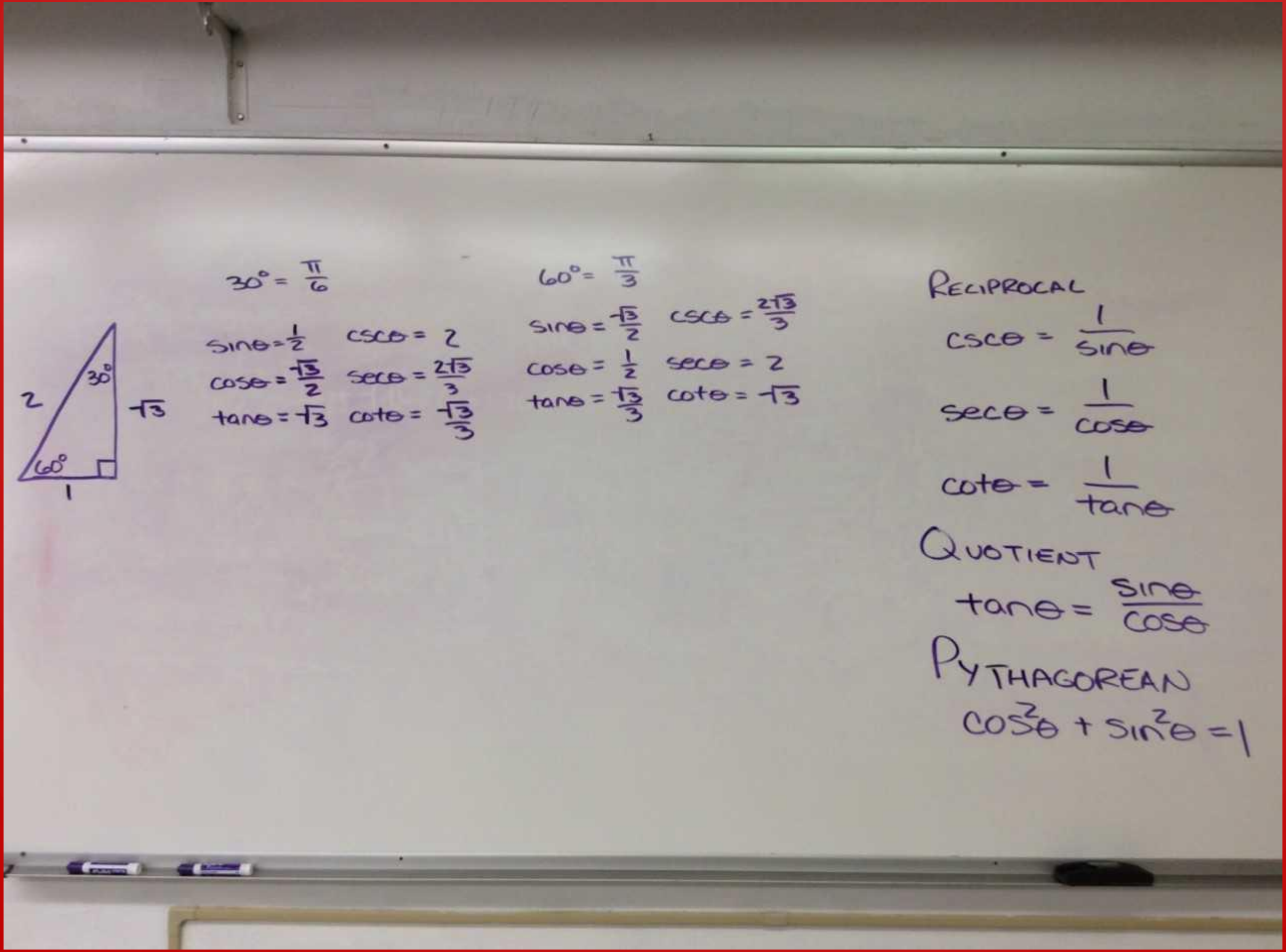

(0, 1)
(−1/2, √3/2)
(1/2, √3/2)
(√2/2, √2/2)
(−√2/2, √2/2)
(−√3/2, 1/2)
(√3/2, 1/2)
120° = 2π/3
90° = π/2
60° = π/3
135° = 3π/4
45° = π/4
150° = 5π/6
30° = π/6
θ
180° = π
0° = 0 radians
360° = 2π
(−1, 0)
(1, 0)
210° = 7π/6
330° = 11π/6
225° = 5π/4
315° = 7π/4
240° = 4π/3
300° = 5π/3
270° = 3π/2
(−√3/2, −1/2)
(√3/2, −1/2)
(−√2/2, −√2/2)
(√2/2, −√2/2)
(−1/2, −√3/2)
(1/2, −√3/2)
(0, −1)

1.63

Terminal Side	Radian Measure of Angle	Degree Measure of Angle
$\frac{1}{12}$ revolution	$\frac{1}{12} \cdot 2\pi = \frac{\pi}{6}$	$\frac{1}{12} \cdot 360^\circ = 30^\circ$
$\frac{1}{8}$ revolution	$\frac{1}{8} \cdot 2\pi = \frac{\pi}{4}$	$\frac{1}{8} \cdot 360^\circ = 45^\circ$
$\frac{1}{6}$ revolution	$\frac{1}{6} \cdot 2\pi = \frac{\pi}{3}$	$\frac{1}{6} \cdot 360^\circ = 60^\circ$
$\frac{1}{4}$ revolution	$\frac{1}{4} \cdot 2\pi = \frac{\pi}{2}$	$\frac{1}{4} \cdot 360^\circ = 90^\circ$
$\frac{1}{3}$ revolution	$\frac{1}{3} \cdot 2\pi = \frac{2\pi}{3}$	$\frac{1}{3} \cdot 360^\circ = 120^\circ$

Terminal Side	Radian Measure of Angle	Degree Measure of Angle
$\frac{1}{2}$ revolution	$\frac{1}{2} \cdot 2\pi = \pi$	$\frac{1}{2} \cdot 360^\circ = 180^\circ$
$\frac{2}{3}$ revolution	$\frac{2}{3} \cdot 2\pi = \frac{4\pi}{3}$	$\frac{2}{3} \cdot 360^\circ = 240^\circ$
$\frac{3}{4}$ revolution	$\frac{3}{4} \cdot 2\pi = \frac{3\pi}{2}$	$\frac{3}{4} \cdot 360^\circ = 270^\circ$
$\frac{7}{8}$ revolution	$\frac{7}{8} \cdot 2\pi = \frac{7\pi}{4}$	$\frac{7}{8} \cdot 360^\circ = 315^\circ$
1 revolution	$1 \cdot 2\pi = 2\pi$	$1 \cdot 360^\circ = 360^\circ$


a=1 c=3 → b = 2√2
sinθ = 1/3
cscθ = 3
cosθ = 2√2/3
secθ = 3√2/4
tanθ = √2/4
cotθ = 2√2
45° = π/4
√2
1
45°
1
sinθ = √2/2
cscθ = √2
cosθ = √2/2
secθ = √2
tanθ = 1
cotθ = 1
SOH CAH TOA


30° = π/6
2
30°
√3
60°
1
sinθ = 1/2
cscθ = 2
cosθ = √3/2
secθ = 2√3/3
tanθ = √3
cotθ = √3/3
60° = π/3
sinθ = √3/2
cscθ = 2√3/3
cosθ = 1/2
secθ = 2
tanθ = √3/3
cotθ = √3
RECIPROCAL
cscθ = 1/sinθ
secθ = 1/cosθ
cotθ = 1/tanθ
QUOTIENT
tanθ = sinθ/cosθ
PYTHAGOREAN
cos²θ + sin²θ = 1

$\sin\theta = \frac{2}{5}$ What is $\cot\theta$?

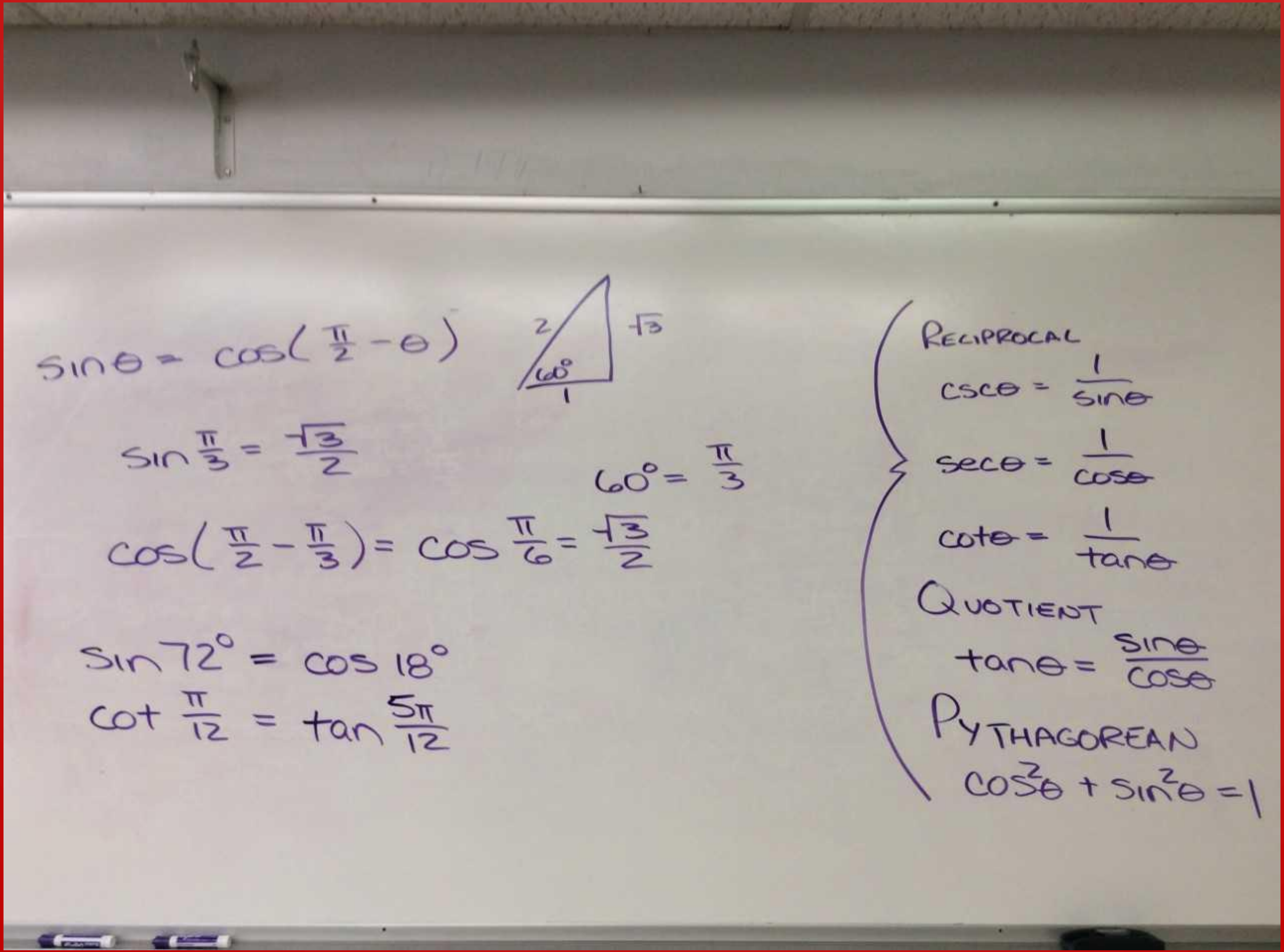
$\cos\theta = \frac{\sqrt{21}}{5}$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{\sin\theta}{\cos\theta}} = \frac{\cos\theta}{\sin\theta} = \frac{\frac{\sqrt{21}}{5}}{\frac{2}{5}} = \frac{\sqrt{21}}{2}$$

$\sin\theta = \frac{3}{5}$ QI

$$\cos^2\theta + \left(\frac{3}{5}\right)^2 = 1 \qquad \cos\theta = \frac{4}{5}$$

$$\cos^2\theta = \frac{16}{25}$$


sinθ = cos(π/2 − θ)
2
√3
60°
1
sin π/3 = √3/2
60° = π/3
cos(π/2 − π/3) = cos π/6 = √3/2
sin 72° = cos 18°
cot π/12 = tan 5π/12
RECIPROCAL
cscθ = 1/sinθ
secθ = 1/cosθ
cotθ = 1/tanθ
QUOTIENT
tanθ = sinθ/cosθ
PYTHAGOREAN
cos²θ + sin²θ = 1