BOARD NOTES

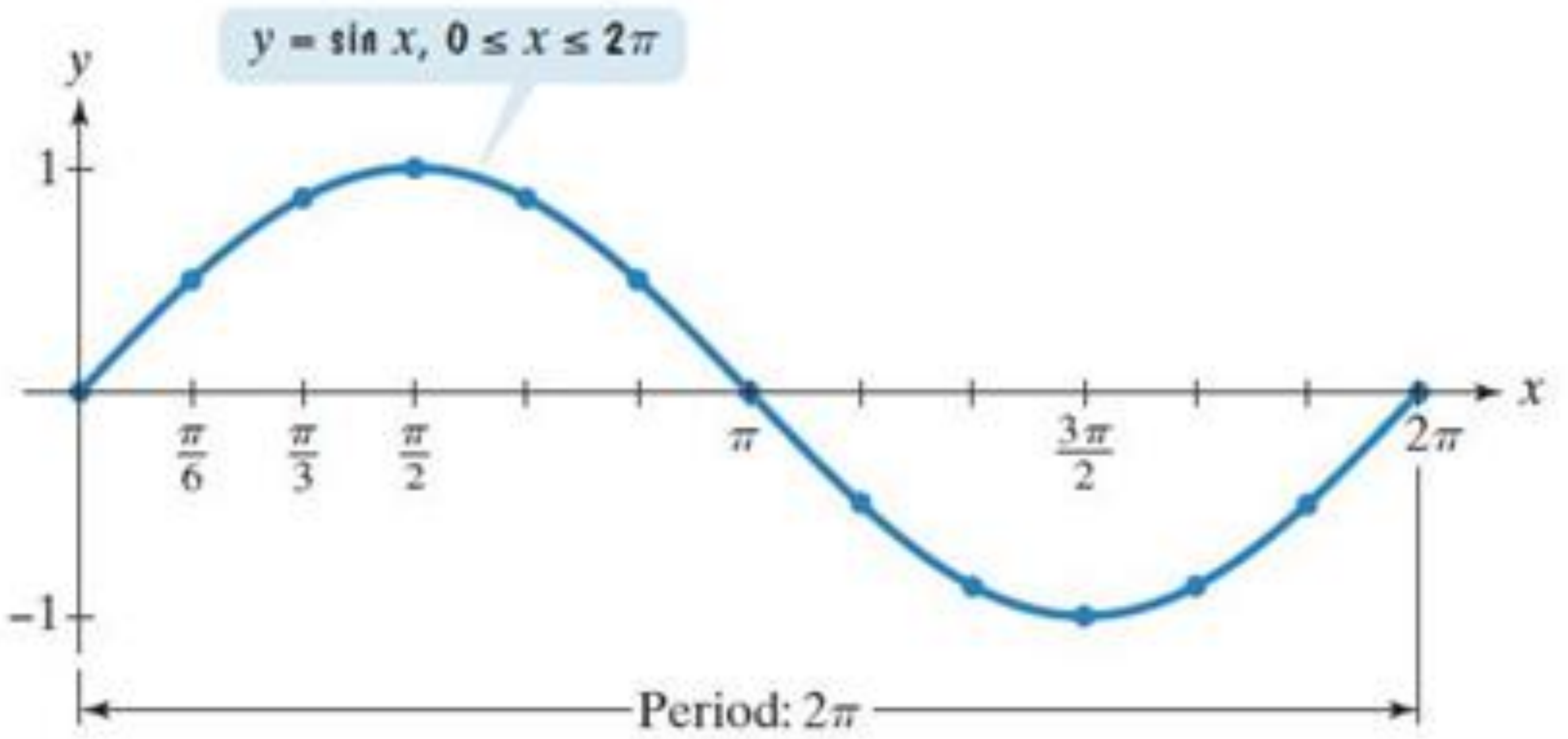
7 FEBRUARY 2019

CC TRIGONOMETRY
CHAPTER 2 – GRAPHS OF THE TRIGONOMETRIC FUNCTIONS; INVERSE TRIGONOMETRIC FUNCTIONS

SECTION 2.1 - Graphs of Sine and Cosine Functions

Objectives:

- Understand the graph of $y = \sin x$.
- Graph variations of $y = \sin x$.
- Understand the graph of $y = \cos x$.
- Graph variations of $y = \cos x$.
- Use vertical shifts of sine and cosine curves.
- Model periodic behavior.



y = sin x, 0 ≤ x ≤ 2π
y
1
-1
x
π/6
π/3
π/2
π
3π/2
2π
Period: 2π

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$y = \sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0

As x increases from 0 to $\frac{\pi}{2}$, y increases from 0 to 1.

As x increases from $\frac{\pi}{2}$ to π, y decreases from 1 to 0.

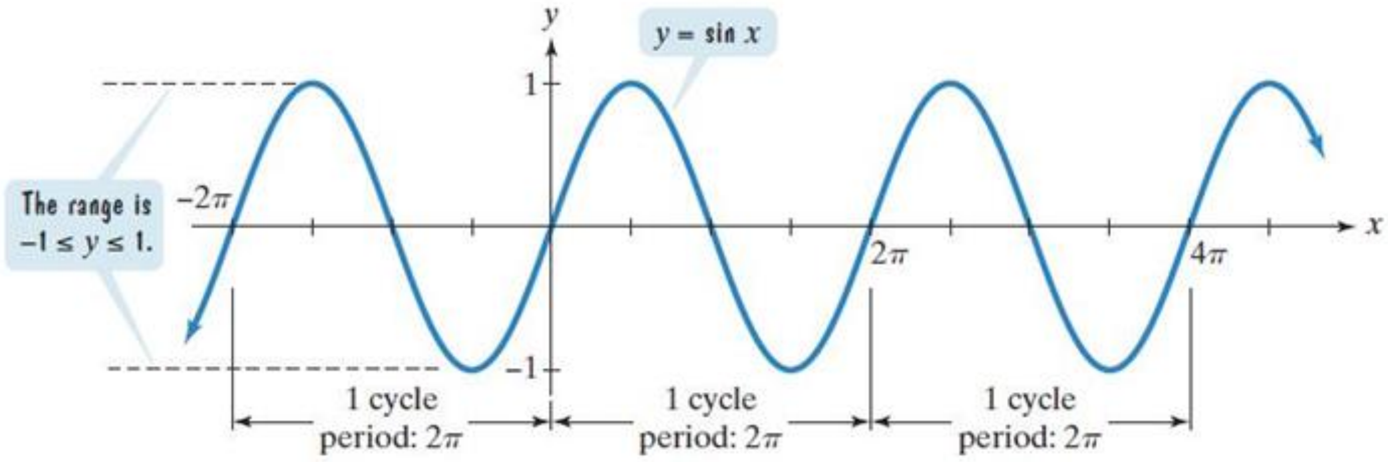
As x increases from π to $\frac{3\pi}{2}$, y decreases from 0 to -1.

As x increases from $\frac{3\pi}{2}$ to 2π, y increases from -1 to 0.

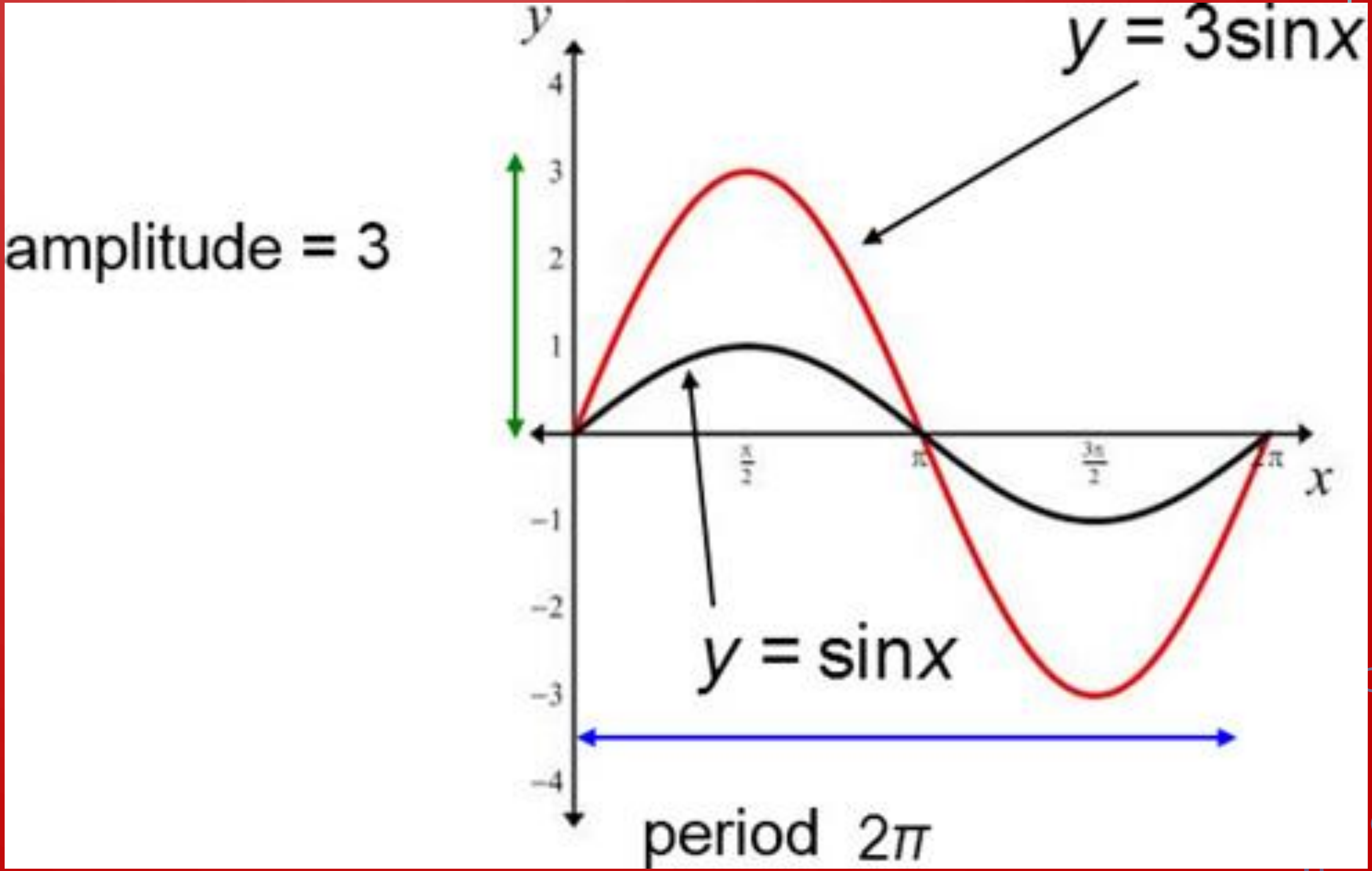
The domain is $(-\infty, \infty)$. The range is $[-1, 1]$.

The period is 2π. The function is an odd function:

$$\sin(-x) = -\sin x.$$



1. Identify the amplitude and the period.
2. Find the values of x for the five key points—the three x-intercepts, the maximum point, and the minimum point. Start with the value of x where the cycle begins and add quarter-periods—that is, $\frac{\text{period}}{4}$—to find successive values of x.
3. Find the values of y for the five key points by evaluating the function at each value of x from step 2.
4. Connect the five key points with a smooth curve and graph one complete cycle of the given function.
5. Extend the graph in step 4 to the left or right as desired.

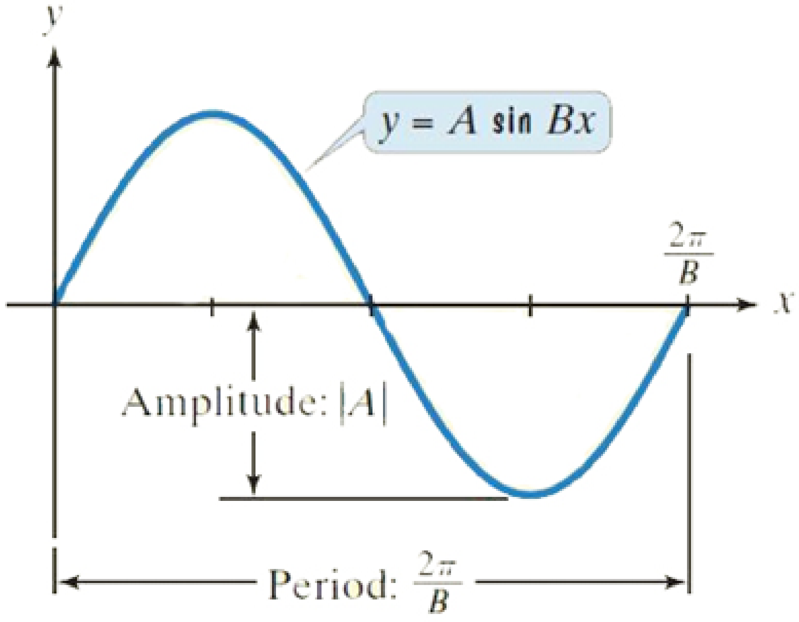

amplitude = 3
y = 3sinx
y = sinx
period 2π
y
x

Amplitudes and periods

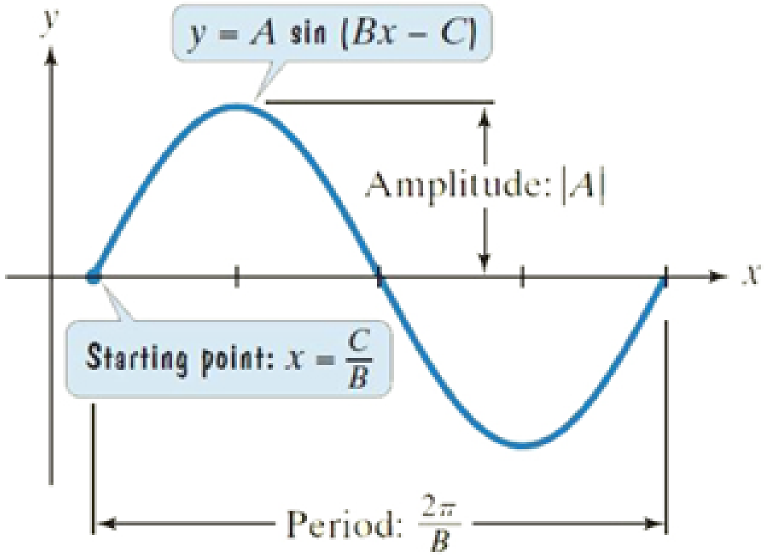
The graph of $y = A \sin Bx$, $B > 0$, has

$$\text{amplitude} = |A|$$

$$\text{period} = \frac{2\pi}{B}.$$

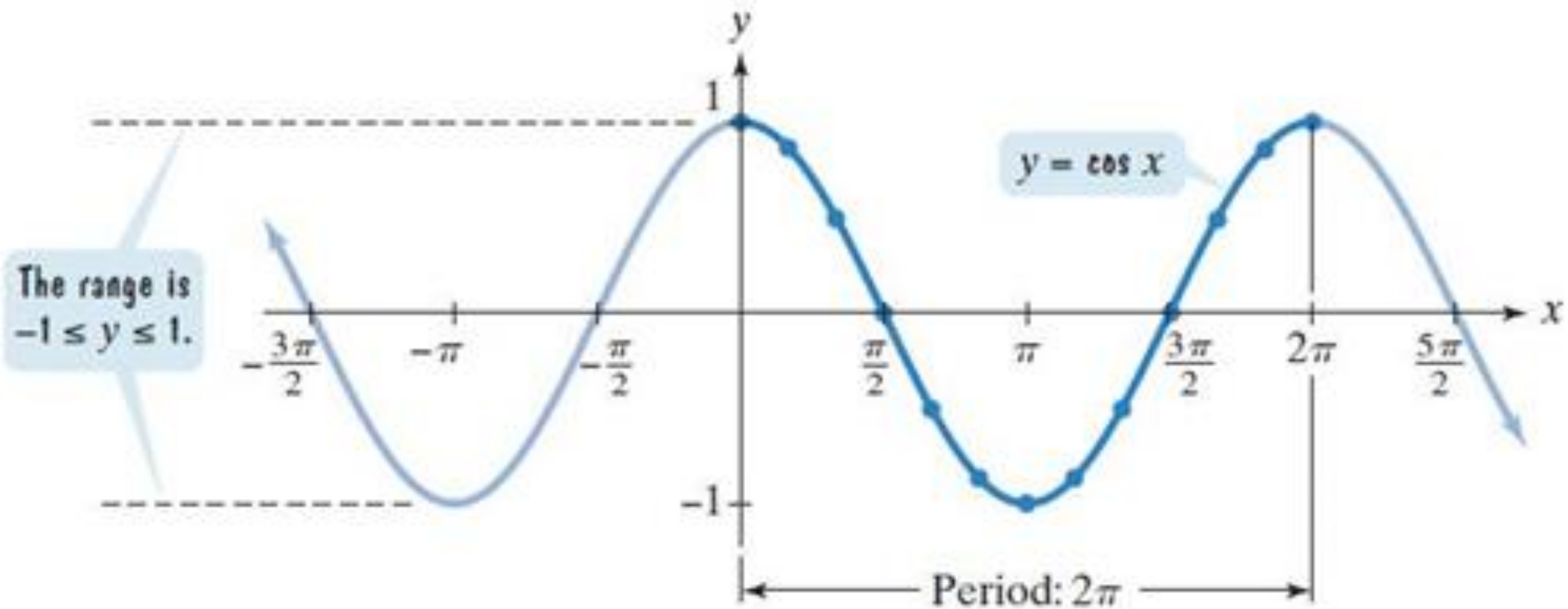


The graph of $y = A\sin(Bx - C)$, $B > 0$, is obtained by horizontally shifting the graph of $y = A \sin Bx$ so that the starting point of the circle is shifted from $x = 0$ to $x = \frac{C}{B}$. If $\frac{C}{B} > 0$, the shift is to the right. If $\frac{C}{B} < 0$, the shift is to the left. The number $\frac{C}{B}$ is called the **phase shift.**



$$\text{amplitude} = |A|$$

$$\text{period} = \frac{2\pi}{B}$$



y
1
-1
x
The range is
−1 ≤ y ≤ 1.
y = cos x
−3π/2
−π
−π/2
π/2
π
3π/2
2π
5π/2
Period: 2π

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$y = \cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1

As x increases from 0 to $\frac{\pi}{2}$, y decreases from 1 to 0.

As x increases from $\frac{\pi}{2}$ to π, y decreases from 0 to −1.

As x increases from π to $\frac{3\pi}{2}$, y increases from −1 to 0.

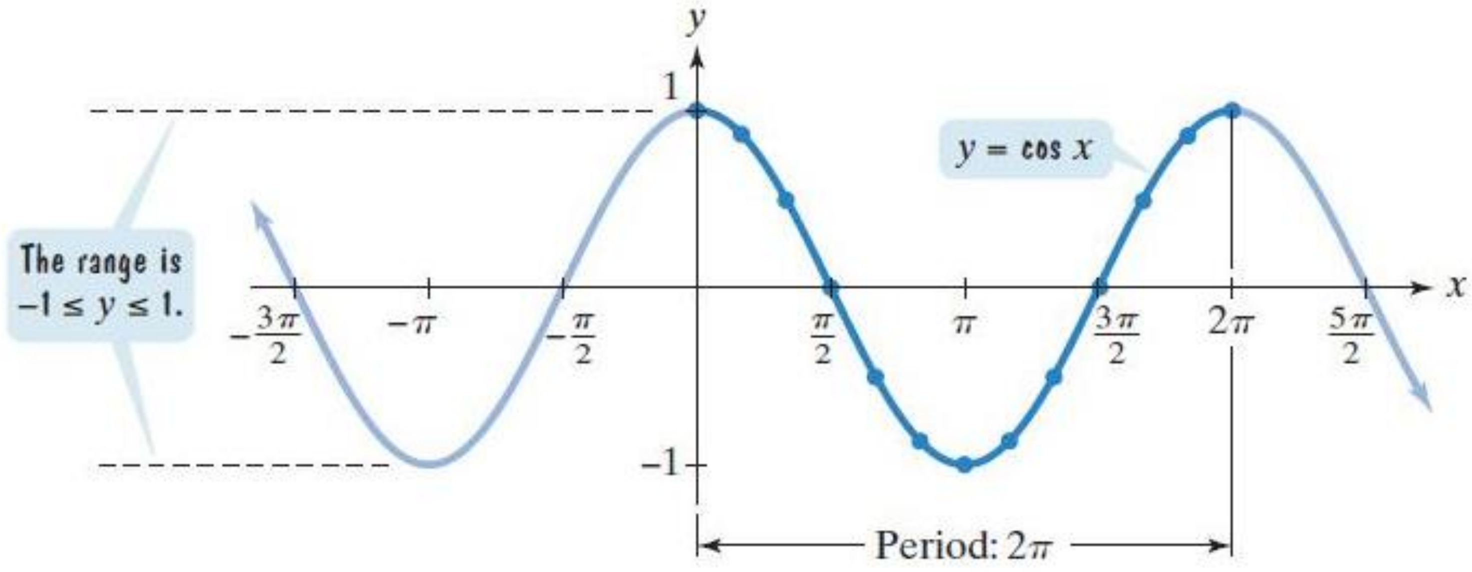
As x increases from $\frac{3\pi}{2}$ to 2π, y increases from 0 to 1.

The domain is $(-\infty, \infty)$.

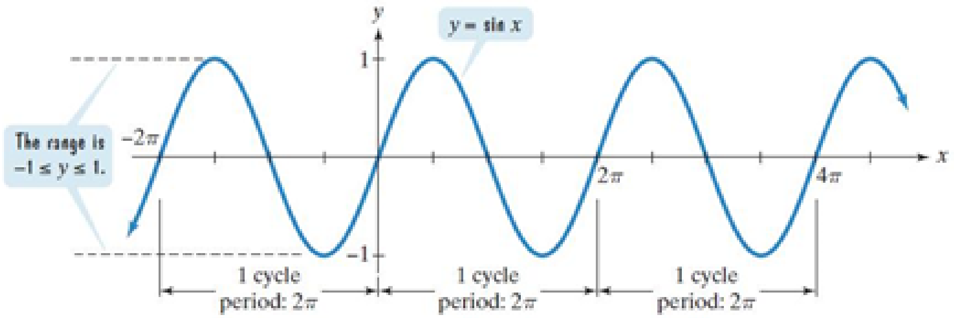
The range is $[-1, 1]$.

The period is 2π.

The function is an even function: $\cos(-x) = \cos x$.



The graphs of sine functions and cosine functions are called **sinusoidal graphs**.

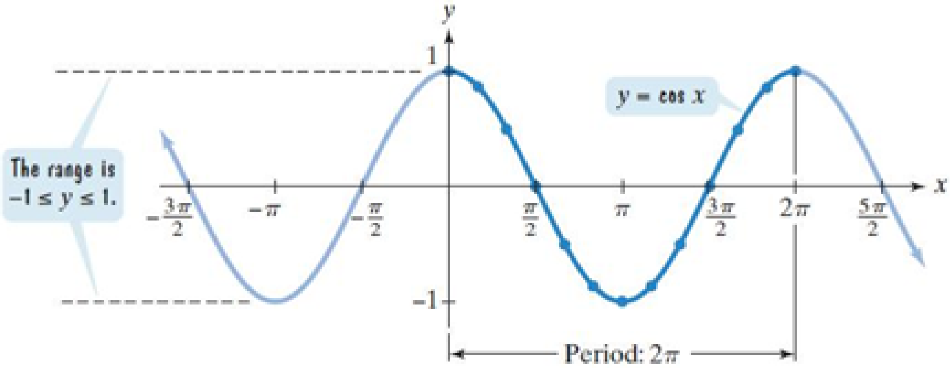


The graph of $y = \cos x$

is the graph of $y = \sin x$

with a phase shift of $\frac{\pi}{2}$.

$$\cos x = \sin\left(x + \frac{\pi}{2}\right)$$



For sinusoidal graphs of the form

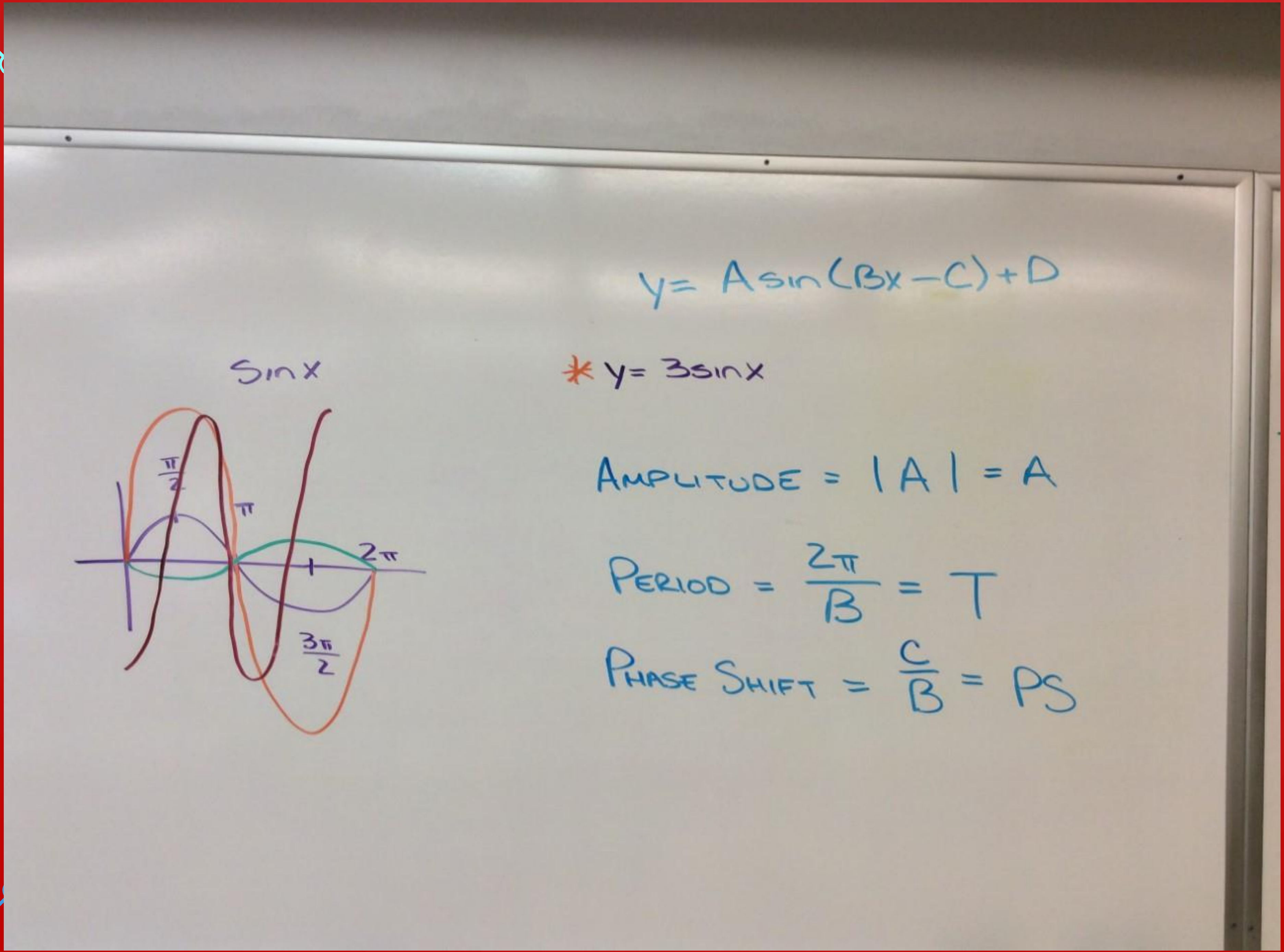
$y = A\sin(Bx - C) + D$ and $y = A\cos(Bx - C) + D$

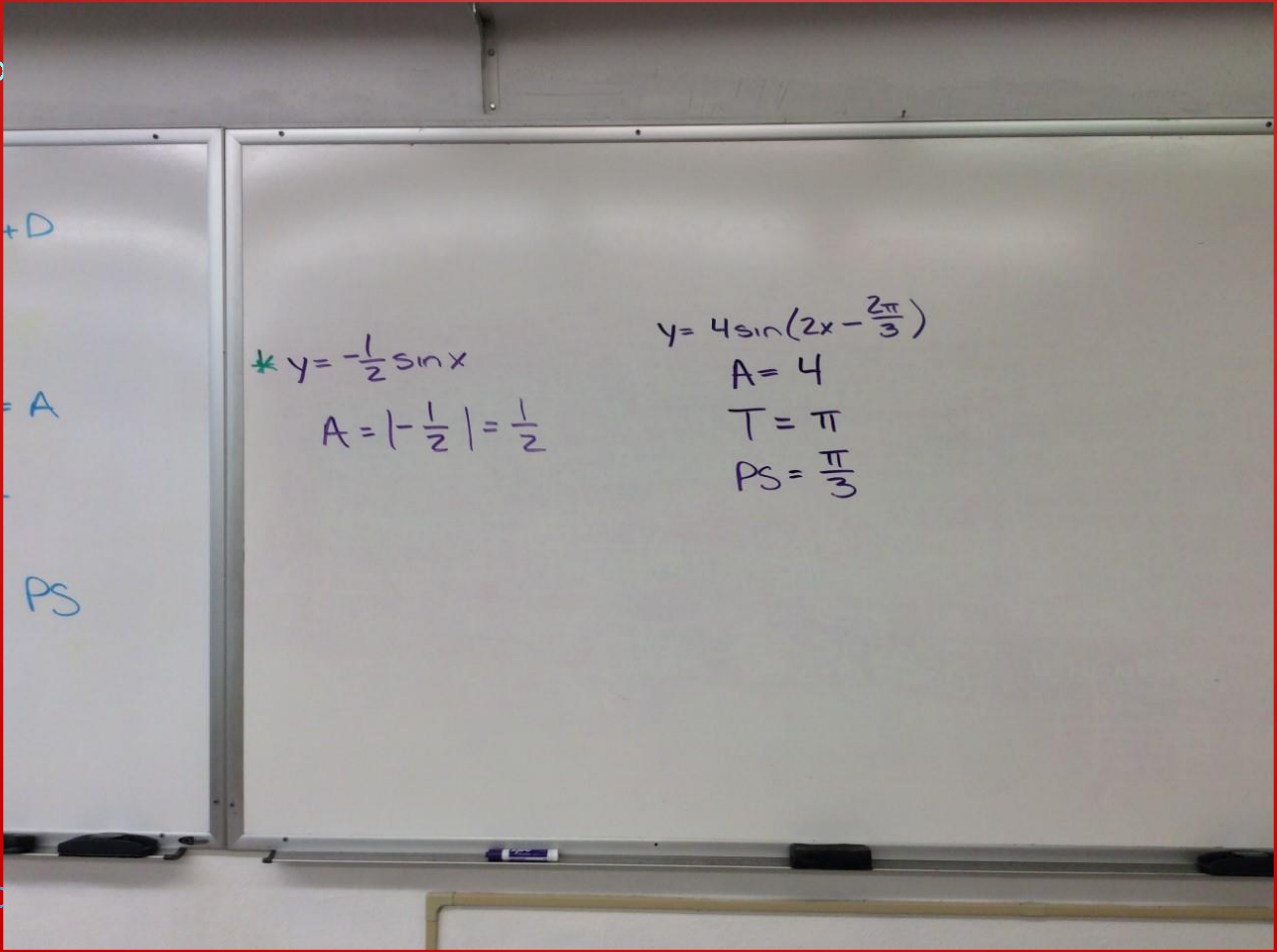
the constant D causes a vertical shift in the graph.

These vertical shifts result in sinusoidal graphs oscillating about the horizontal line $y = D$ rather than about the x-axis.

The maximum value of y is $D + |A|$.

The minimum value of y is $D - |A|$.


y= Asin(Bx−C)+D
sinx
* y= 3sinx
π/2
π
2π
3π/2
AMPLITUDE = |A| = A
PERIOD = 2π/B = T
PHASE SHIFT = C/B = PS


+D
= A
PS
* y = -1/2 sin x
A = |-1/2| = 1/2
y = 4sin(2x - 2π/3)
A = 4
T = π
PS = π/3