BOARD NOTES

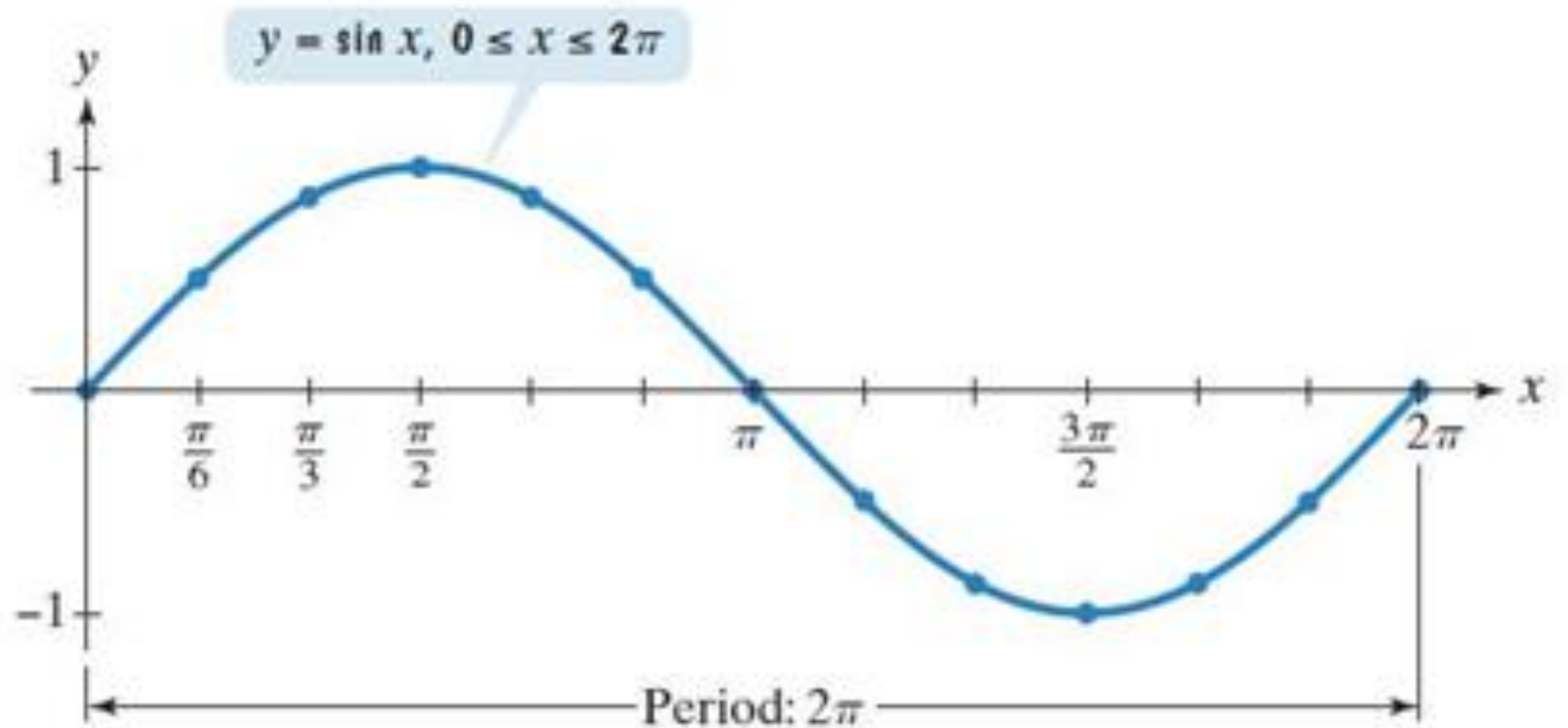
7 FEBRUARY 2019

CC TRIGONOMETRY
CHAPTER 2 – GRAPHS OF THE TRIGONOMETRIC FUNCTIONS; INVERSE TRIGONOMETRIC FUNCTIONS

SECTION 2.1 - Graphs of Sine and Cosine Functions

Objectives:

- Understand the graph of $y = \sin x$.
- Graph variations of $y = \sin x$.
- Understand the graph of $y = \cos x$.
- Graph variations of $y = \cos x$.
- Use vertical shifts of sine and cosine curves.
- Model periodic behavior.



y = sin x, 0 ≤ x ≤ 2π
y
1
−1
x
π/6
π/3
π/2
π
3π/2
2π
Period: 2π

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$y = \sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0

As x increases from 0 to $\frac{\pi}{2}$, y increases from 0 to 1.

As x increases from $\frac{\pi}{2}$ to π, y decreases from 1 to 0.

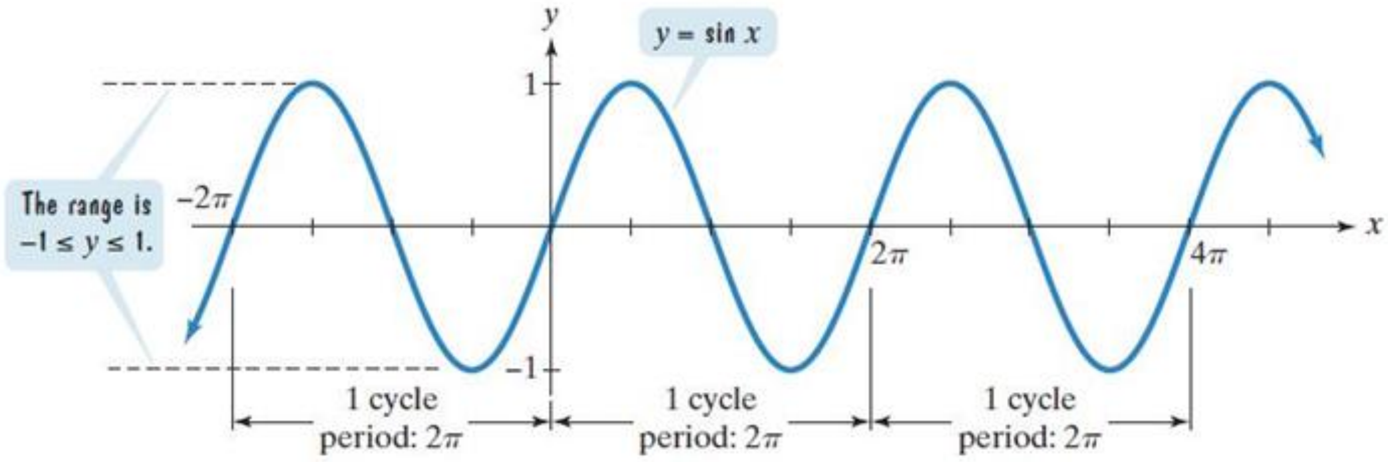
As x increases from π to $\frac{3\pi}{2}$, y decreases from 0 to −1.

As x increases from $\frac{3\pi}{2}$ to 2π, y increases from −1 to 0.

The domain is $(-\infty, \infty)$. The range is $[-1, 1]$.

The period is 2π. The function is an odd function:

$$\sin(-x) = -\sin x.$$



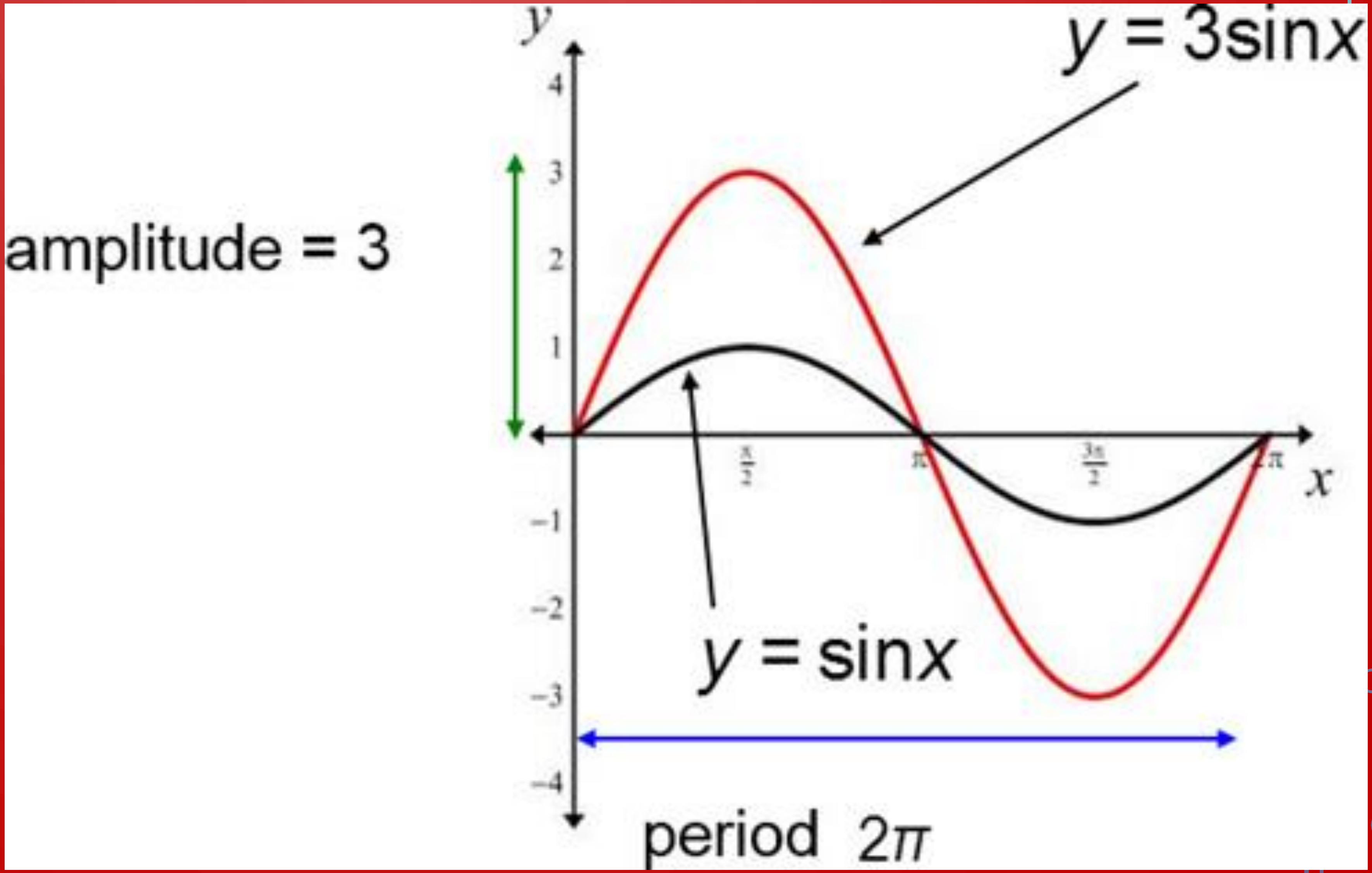
1. Identify the amplitude and the period.

2. Find the values of x for the five key points—the three x-intercepts, the maximum point, and the minimum point. Start with the value of x where the cycle begins and add quarter-periods—that is, $\frac{\text{period}}{4}$—to find successive values of x.

3. Find the values of y for the five key points by evaluating the function at each value of x from step 2.

4. Connect the five key points with a smooth curve and graph one complete cycle of the given function.

5. Extend the graph in step 4 to the left or right as desired.

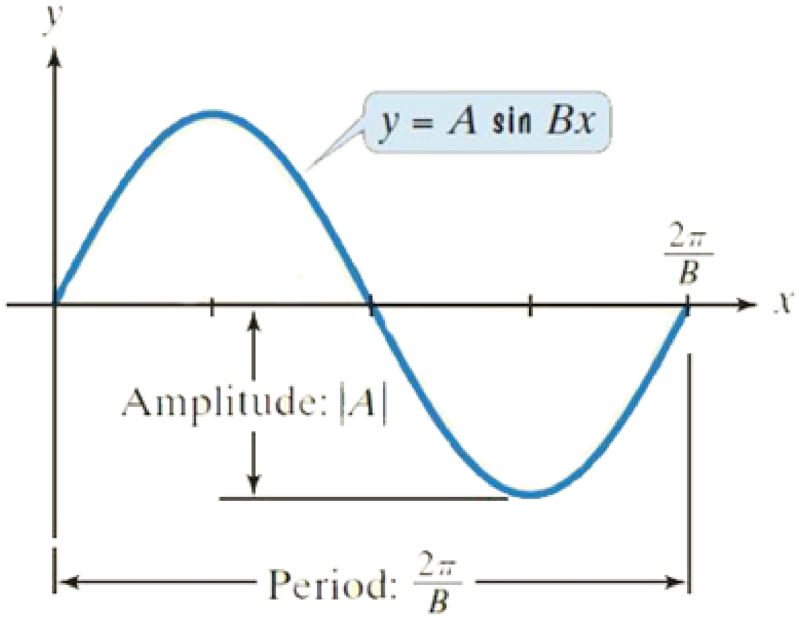

amplitude = 3
y = 3sinx
y = sinx
period 2π
y
x
4
3
2
1
−1
−2
−3
−4

Amplitudes and periods

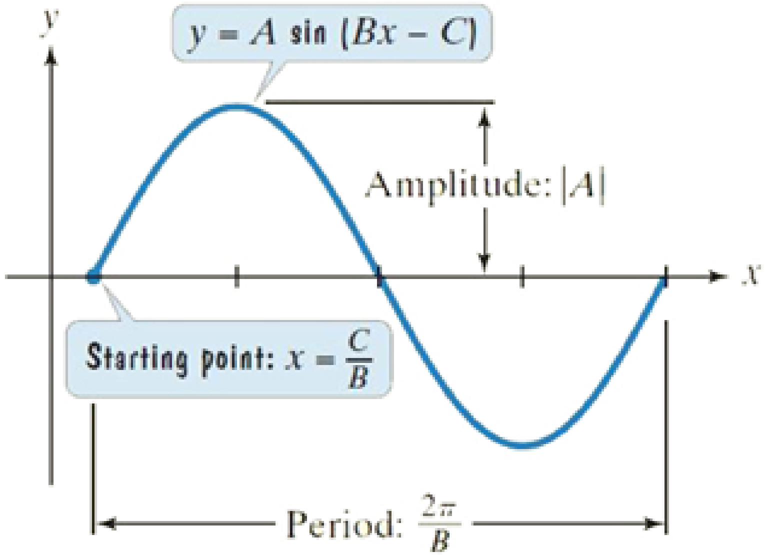
The graph of $y = A \sin Bx$, $B > 0$, has

$$\text{amplitude} = |A|$$

$$\text{period} = \frac{2\pi}{B}.$$

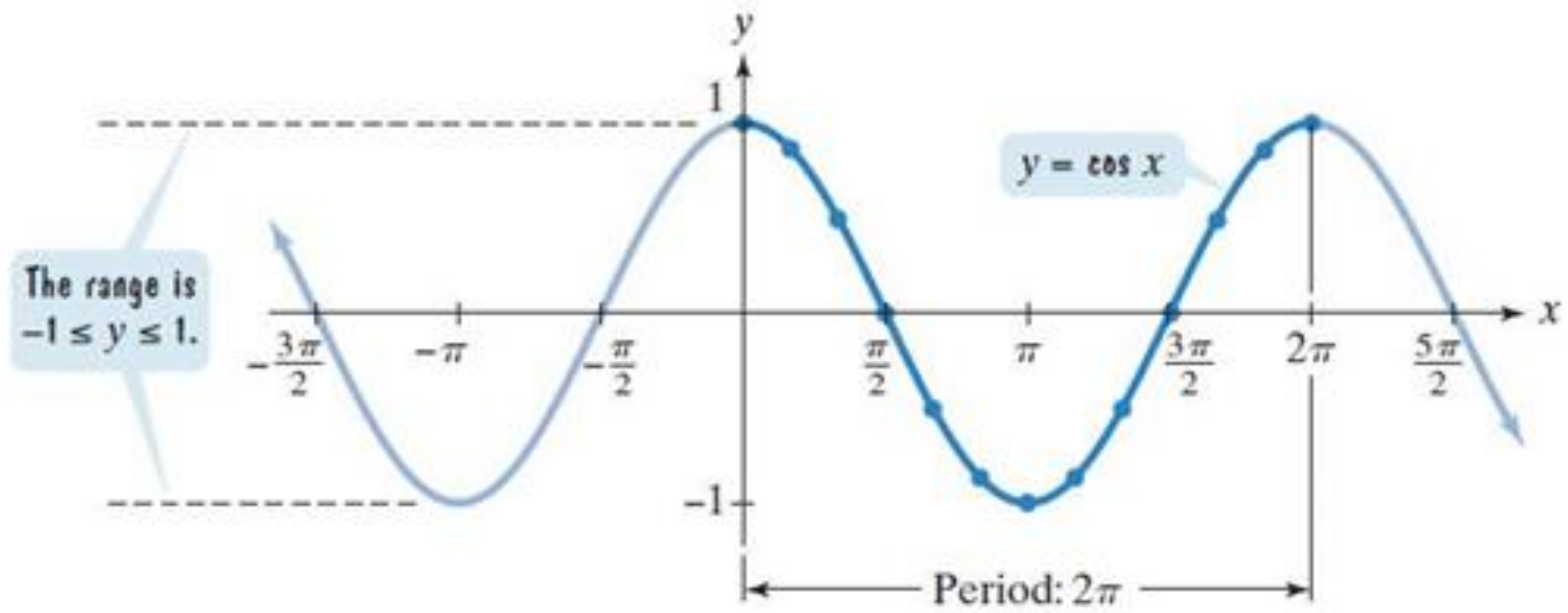


The graph of $y = A\sin(Bx - C)$, $B > 0$, is obtained by horizontally shifting the graph of $y = A \sin Bx$ so that the starting point of the circle is shifted from $x = 0$ to $x = \frac{C}{B}$. If $\frac{C}{B} > 0$, the shift is to the right. If $\frac{C}{B} < 0$, the shift is to the left. The number $\frac{C}{B}$ is called the **phase shift.**



$$\text{amplitude} = |A|$$

$$\text{period} = \frac{2\pi}{B}$$


y
1
-1
x
The range is
-1 ≤ y ≤ 1.
y = cos x
-3π/2
-π
-π/2
π/2
π
3π/2
2π
5π/2
Period: 2π

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$y = \cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1

As x increases from 0 to $\frac{\pi}{2}$, y decreases from 1 to 0.

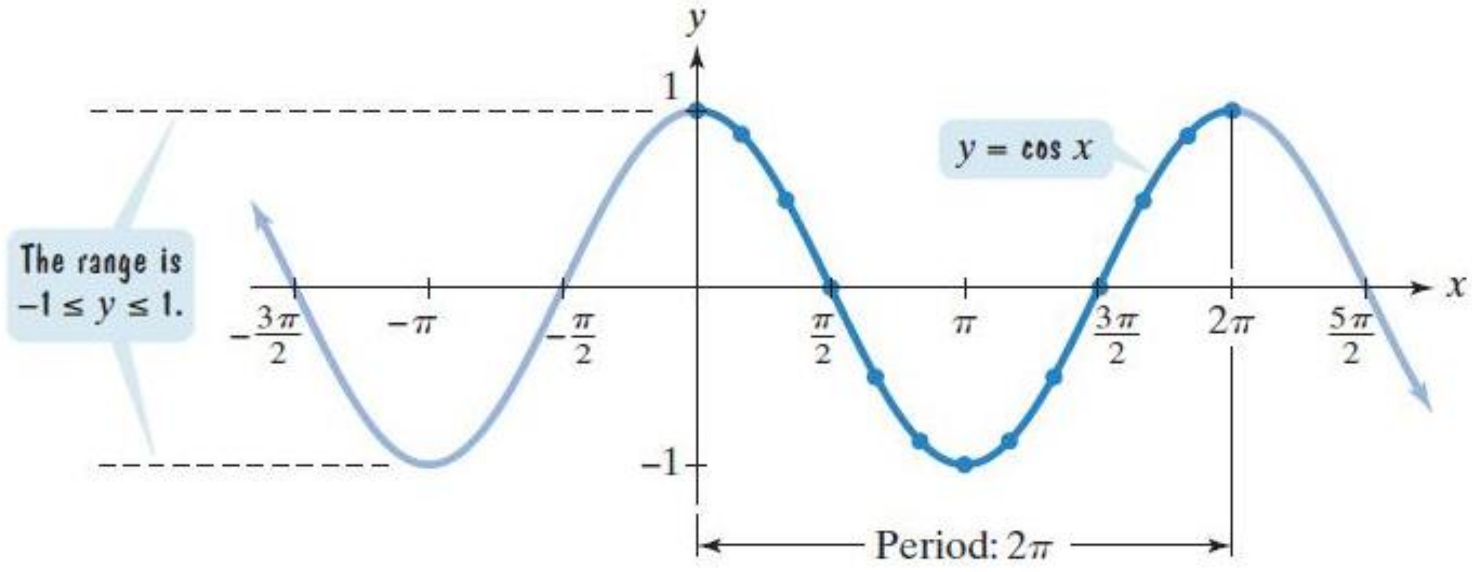
As x increases from $\frac{\pi}{2}$ to π, y decreases from 0 to −1.

As x increases from π to $\frac{3\pi}{2}$, y increases from −1 to 0.

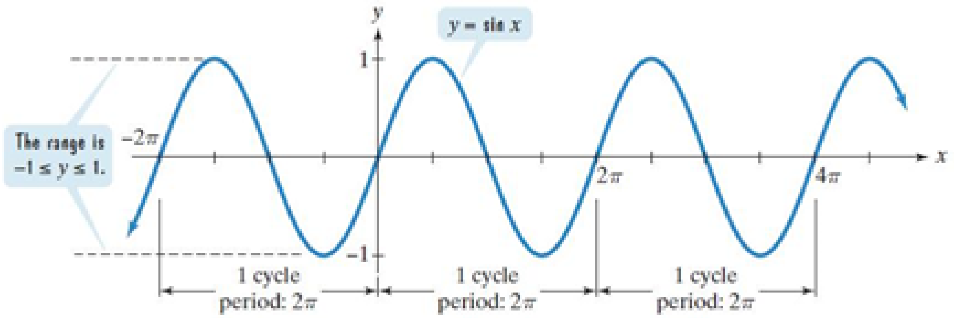
As x increases from $\frac{3\pi}{2}$ to 2π, y increases from 0 to 1.

The domain is $(-\infty, \infty)$. The range is $[-1, 1]$.

The period is 2π. The function is an even function: $\cos(-x) = \cos x$.

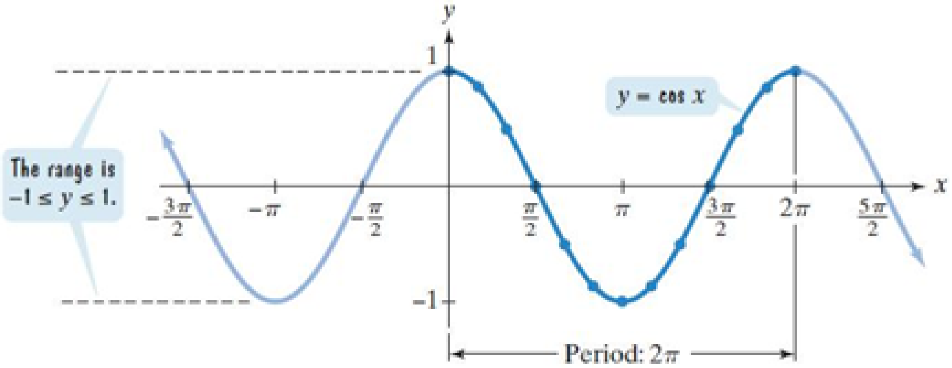


The graphs of sine functions and cosine functions are called **sinusoidal graphs**.



The graph of $y = \cos x$ is the graph of $y = \sin x$ with a phase shift of $\frac{\pi}{2}$.

$$\cos x = \sin\left(x + \frac{\pi}{2}\right)$$



For sinusoidal graphs of the form

$$y = A\sin(Bx - C) + D \text{ and } y = A\cos(Bx - C) + D$$

the constant D causes a vertical shift in the graph.

These vertical shifts result in sinusoidal graphs oscillating about the horizontal line $y = D$ rather than about the x-axis.

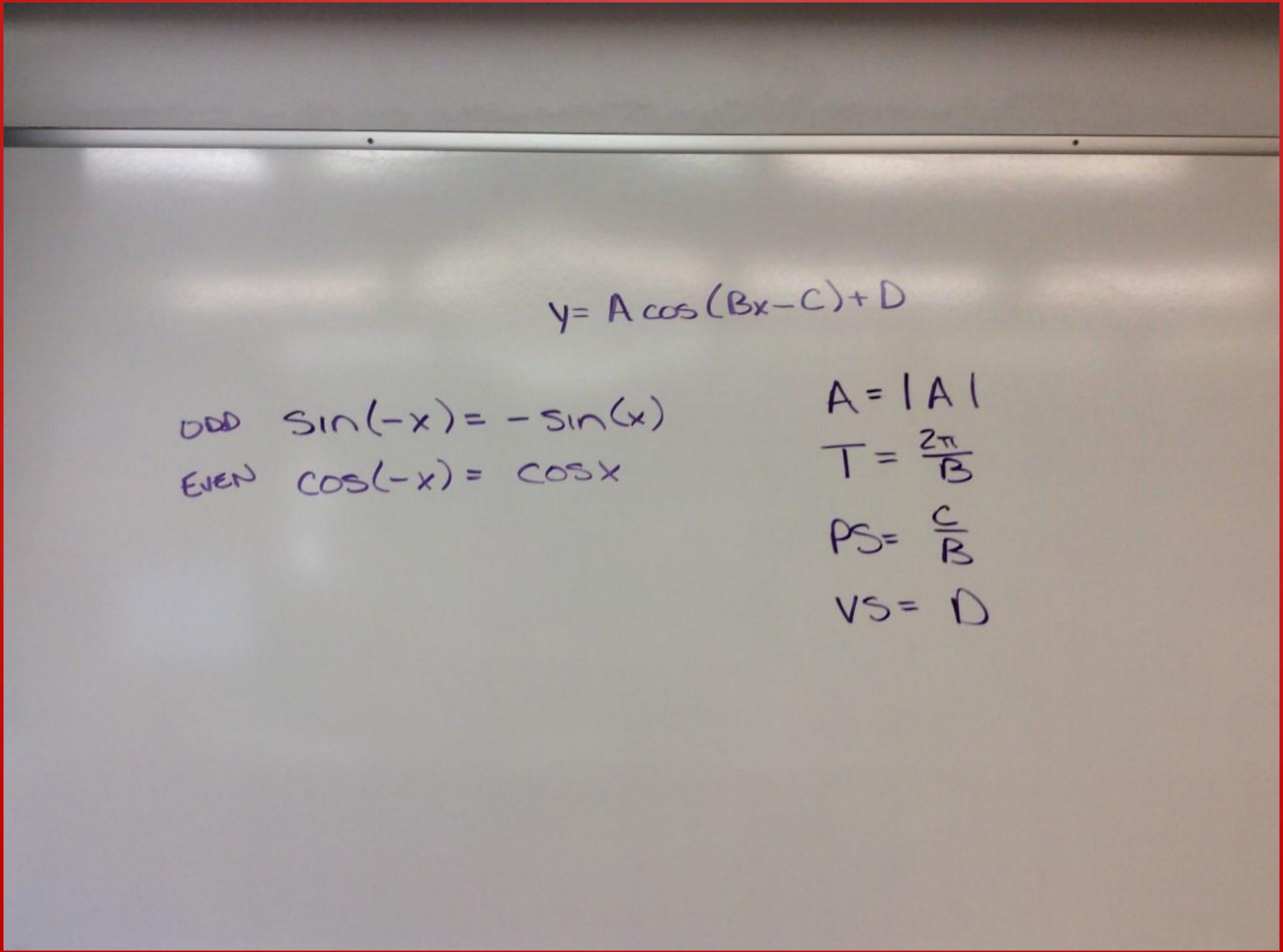
The maximum value of y is $D + |A|$.

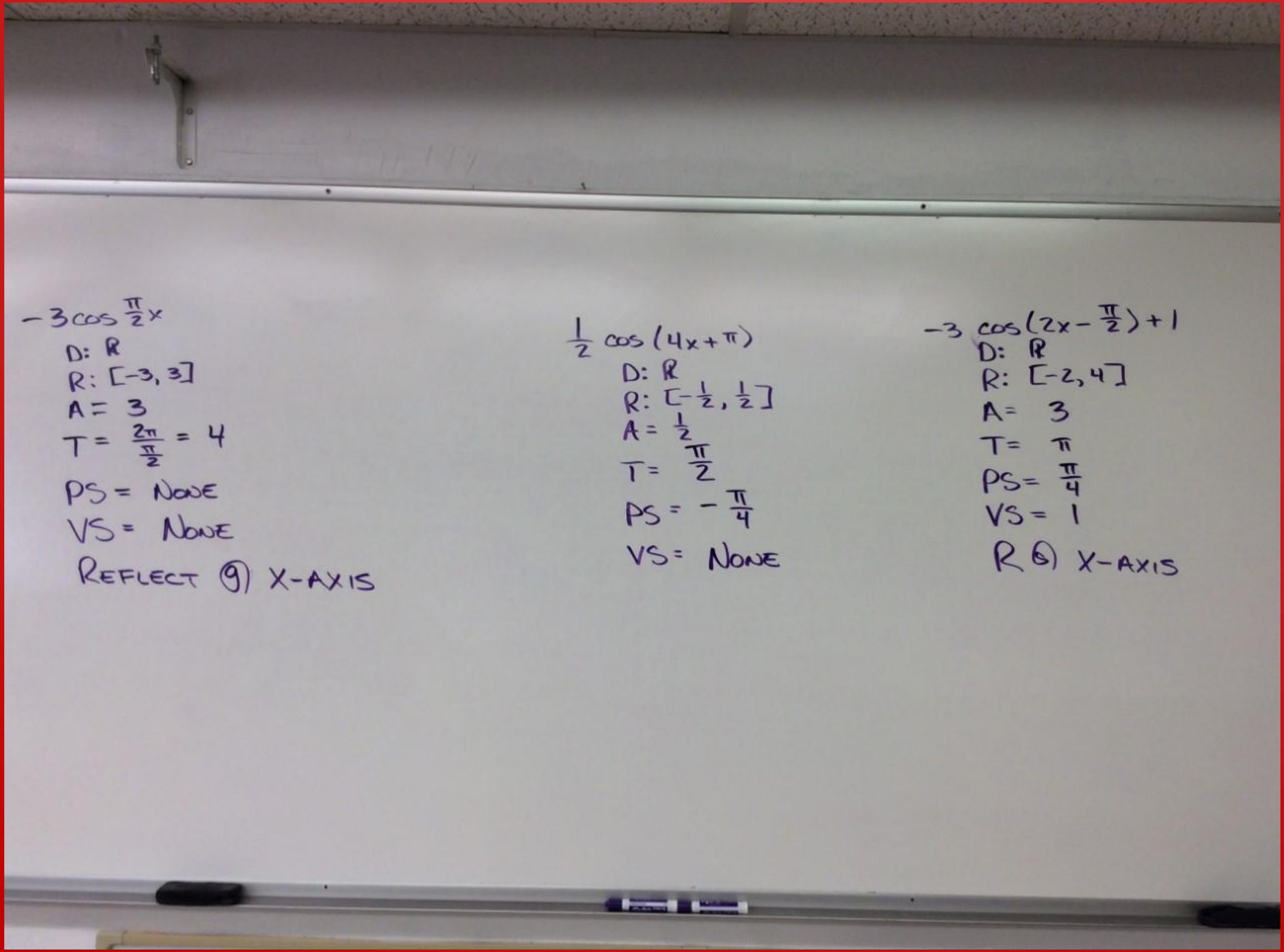
The minimum value of y is $D - |A|$.

FRANK PHILLIPS COLLEGE Plainsmen

BORGER INDEPENDENT SCHOOL DISTRICT 1926

A region that is 30° north of the Equator averages a minimum of 10 hours of daylight in December. Hours of daylight are at a maximum of 14 hours in June. Let x represent the month of the year, with 1 for January, 2 for February, 3 for March, and 12 for December. If y represents the number of hours of daylight in month x, use a sine function of the form $y = A\sin(Bx - C) + D$ to model the hours of daylight.


y = A cos(Bx − C) + D
ODD sin(−x) = −sin(x)
EVEN cos(−x) = cos x
A = |A|
T = 2π/B
PS = C/B
VS = D


$-3\cos\frac{\pi}{2}x$
D: $\mathbb{R}$
R: [-3, 3]
A = 3
$T = \frac{2\pi}{\frac{\pi}{2}} = 4$
PS = NONE
VS = NONE
REFLECT @ X-AXIS
$\frac{1}{2}\cos(4x+\pi)$
D: $\mathbb{R}$
R: $[-\frac{1}{2}, \frac{1}{2}]$
A = $\frac{1}{2}$
T = $\frac{\pi}{2}$
PS = $-\frac{\pi}{4}$
VS = NONE
$-3\cos(2x-\frac{\pi}{2})+1$
D: $\mathbb{R}$
R: [-2, 4]
A = 3
T = π
PS = $\frac{\pi}{4}$
VS = 1
R @ X-AXIS