BOARD NOTES

19 FEBRUARY 2019

CC TRIGONOMETRY
CHAPTER 2 – GRAPHS OF THE TRIGONOMETRIC FUNCTIONS; INVERSE TRIGONOMETRIC FUNCTIONS

SECTION 2.3 - Inverse Trigonometric Functions

Objectives:

- Understand/use inverse sine function
- Understand/use inverse cosine function
- Understand/use inverse tangent function
- Use a calculator to evaluate inverse trigonometric functions
- Find exact values of inverse trig functions

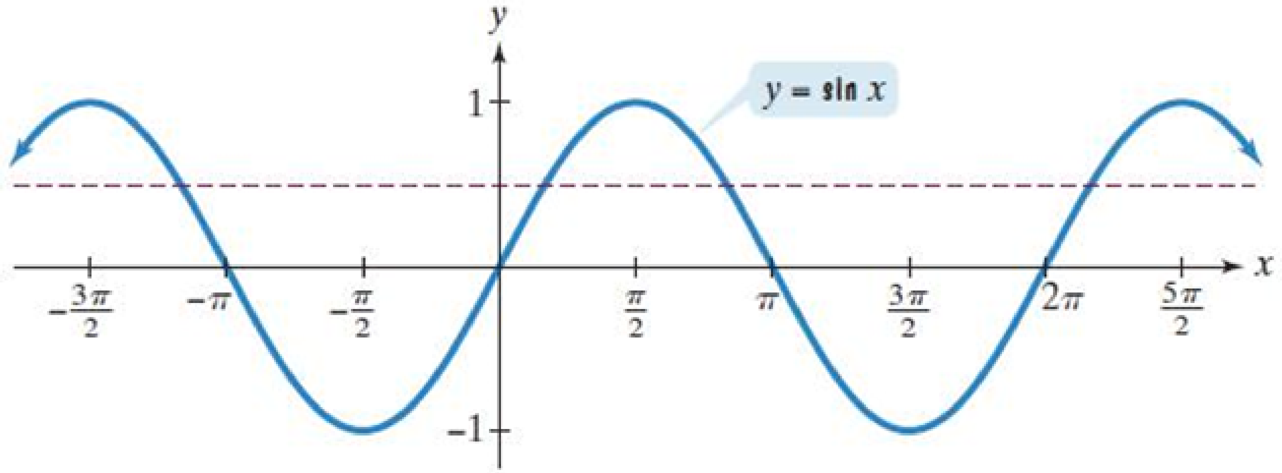
Inverse Functions

Here are some helpful things to remember from our earlier discussion of inverse functions:

If no horizontal line intersects the graph of a function more than once, the function is one-to-one and has an inverse function.

If the point (a, b) is on the graph of f, then the point (b, a) is on the graph of the inverse function, denoted f^{-1}. The graph of f^{-1} is a reflection of the graph of f about the line $y = x$.

The Inverse Sine Function (1 of 2)



The horizontal line test shows that the sine function is not one-to-one; *y* = sin *x* has an inverse function on the restricted domain $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.

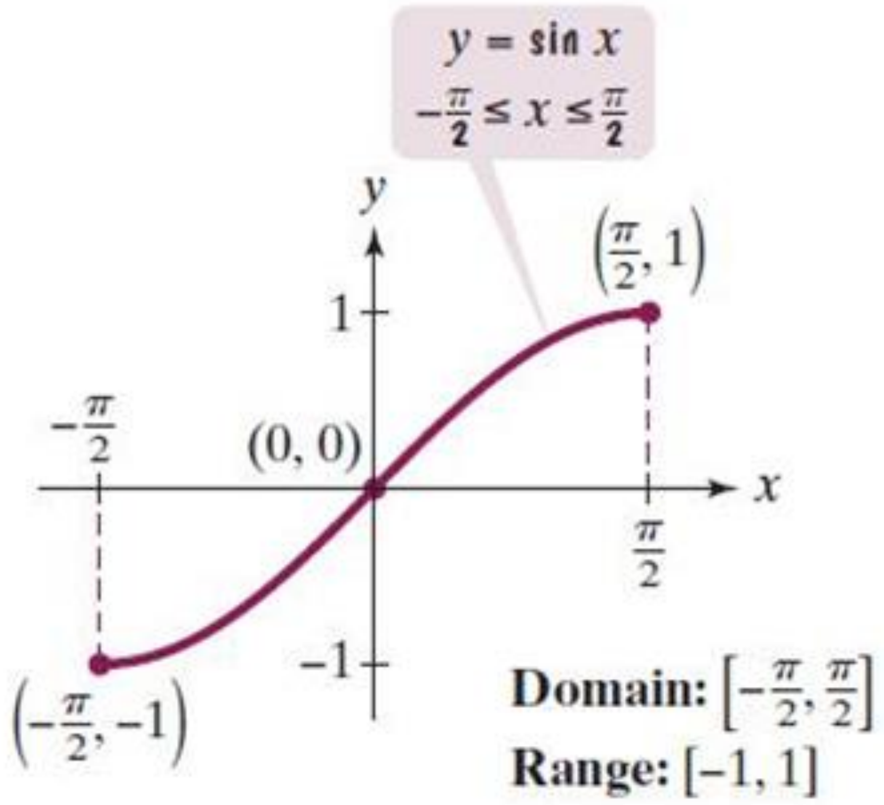
The Inverse Sine Function (2 of 2)

The inverse sine function , denoted by $\sin^{-1}$, is the inverse of the restricted sine function $y = \sin x, -\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

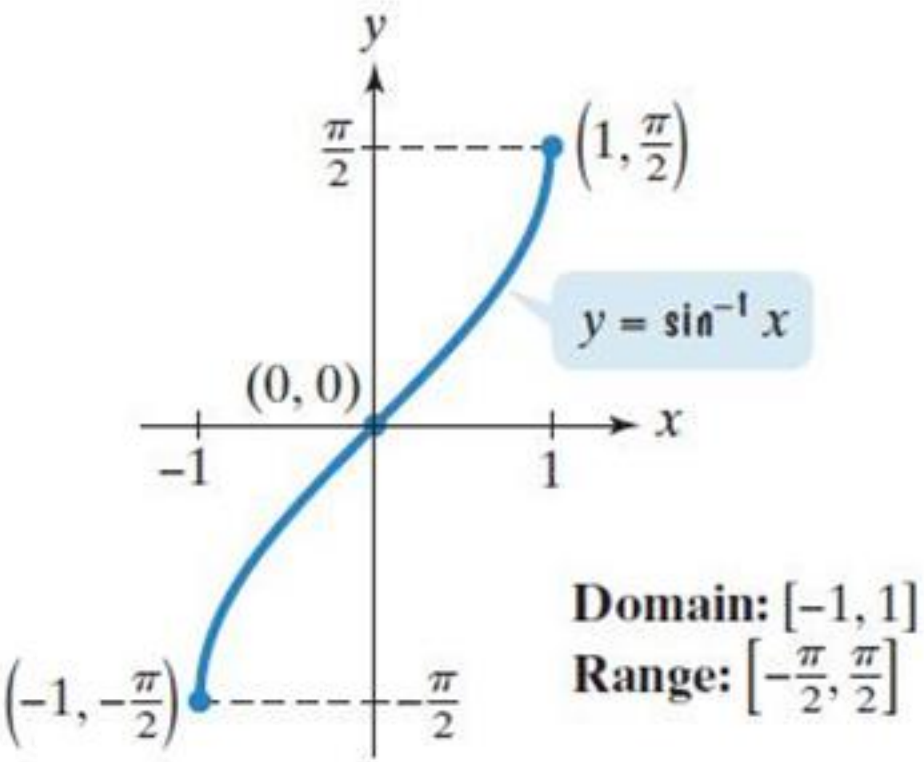
Thus,

$$y = \sin^{-1} x \text{ means } \sin y = x,$$

Where $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ and $-1 \le x \le 1$. We read $y = \sin^{-1} x$ as "y equals the inverse sine at x".

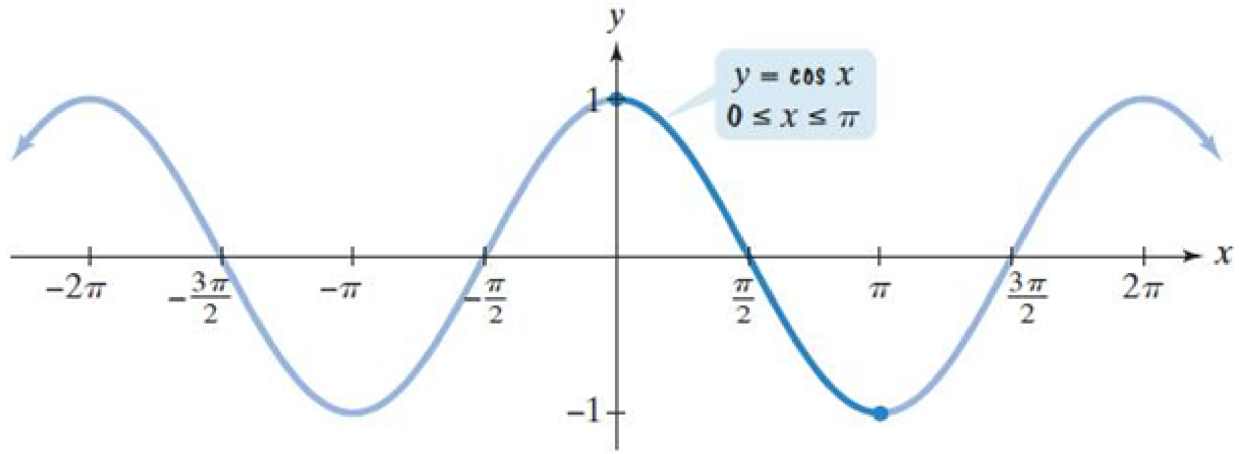


$y = \sin x$
$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$
y
x
$\left(\frac{\pi}{2}, 1\right)$
1
$-\frac{\pi}{2}$
(0, 0)
$\frac{\pi}{2}$
-1
$\left(-\frac{\pi}{2}, -1\right)$
Domain: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
Range: $[-1, 1]$



y
$\frac{\pi}{2}$
$\left(1, \frac{\pi}{2}\right)$
$y = \sin^{-1} x$
(0, 0)
x
-1
1
Domain: $[-1, 1]$
Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\left(-1, -\frac{\pi}{2}\right)$
$-\frac{\pi}{2}$

The Inverse Cosine Function



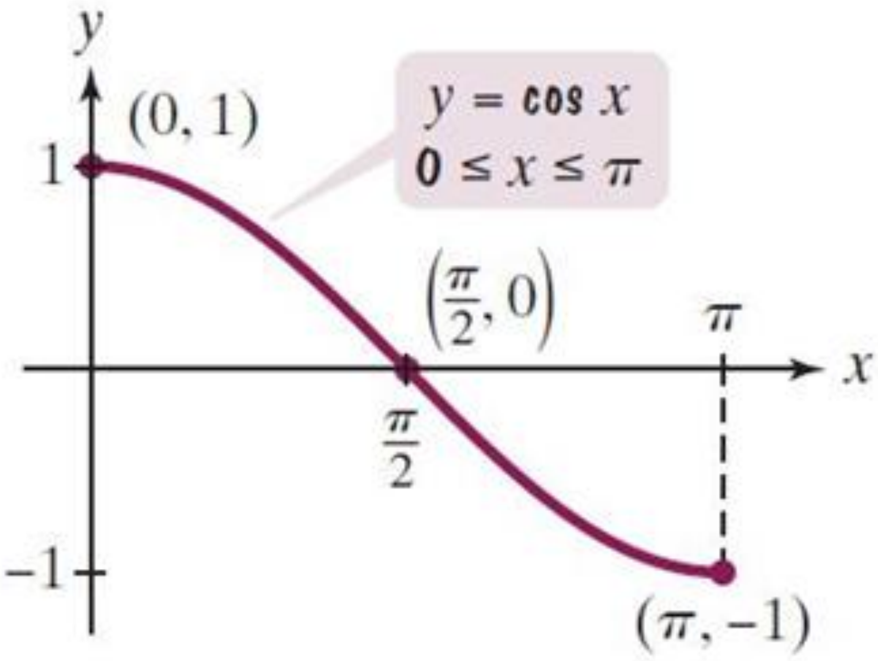
The horizontal line test shows that the cosine function is not one-to-one. $y = \cos x$ has an inverse function on the restricted domain $[0, \pi]$.

The Inverse Cosine Function (2 of 2)

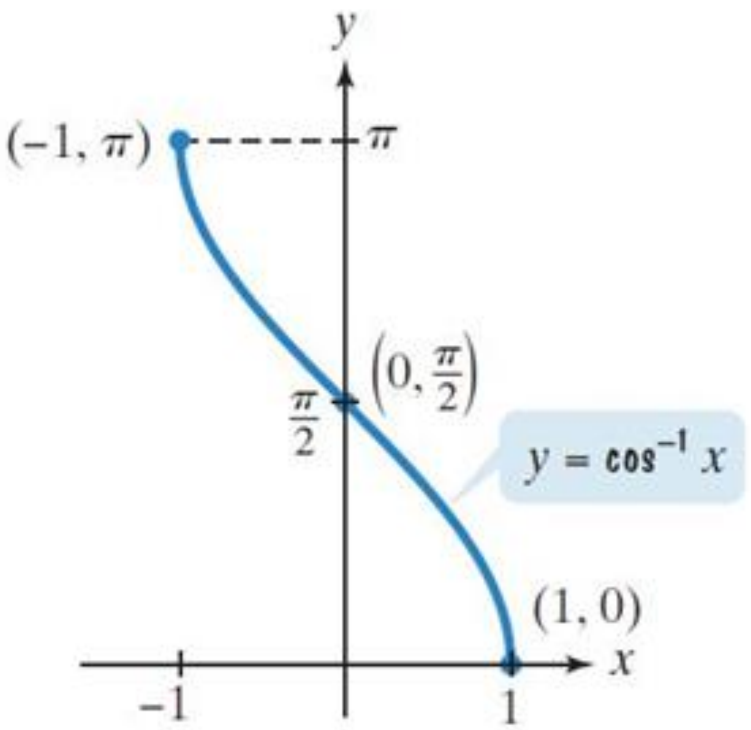
The inverse cosine function, denoted by $\cos^{-1}$,
Is the inverse of the restricted cosine function
$y = \cos x, 0 \le x \le \pi$. Thus

$$y = \cos^{-1} x \text{ means } \cos y = x$$

Where $y = 0 \le y \le \pi$ and $-1 \le x \le 1$.

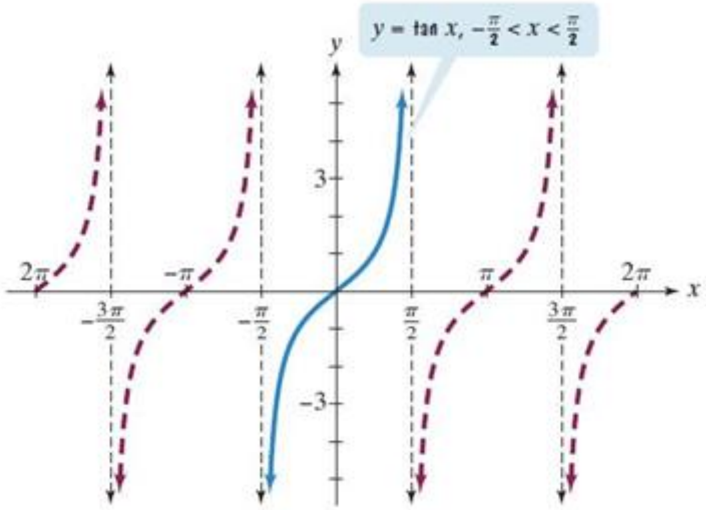


Domain: $[0, \pi]$
Range: $[-1, 1]$



Domain: $[-1, 1]$
Range: $[0, \pi]$

The Inverse Tangent Function (1 of 2)



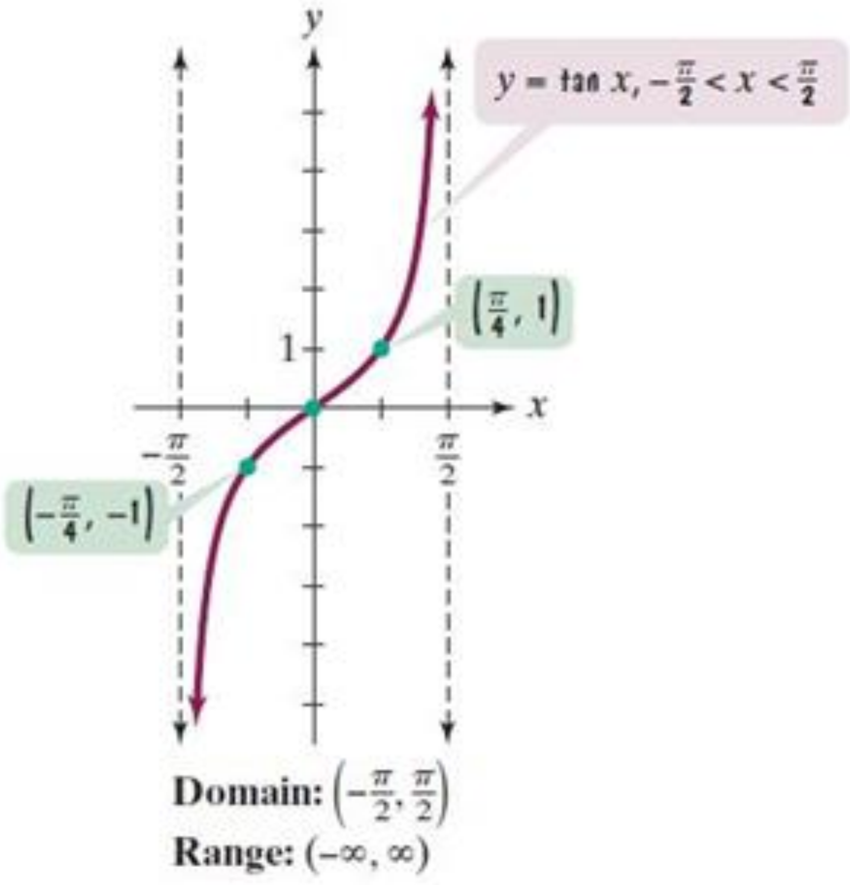
The horizontal line test shows that the tangent function is not one-to-one. $y = \tan x$ has an inverse function on the restricted domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

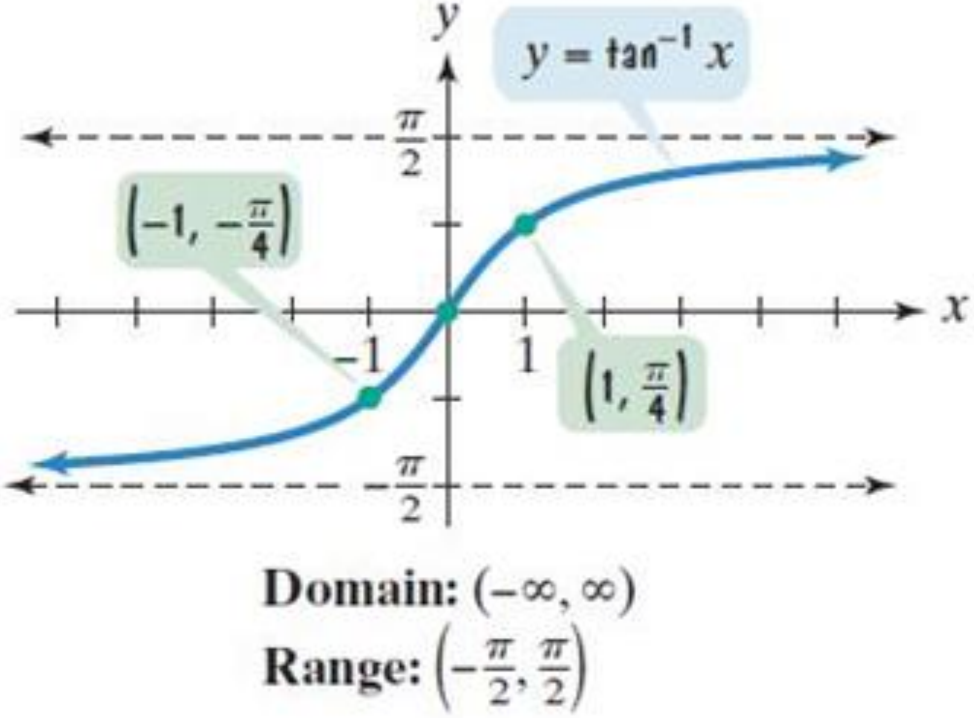
The Inverse Tangent Function (2 of 2)

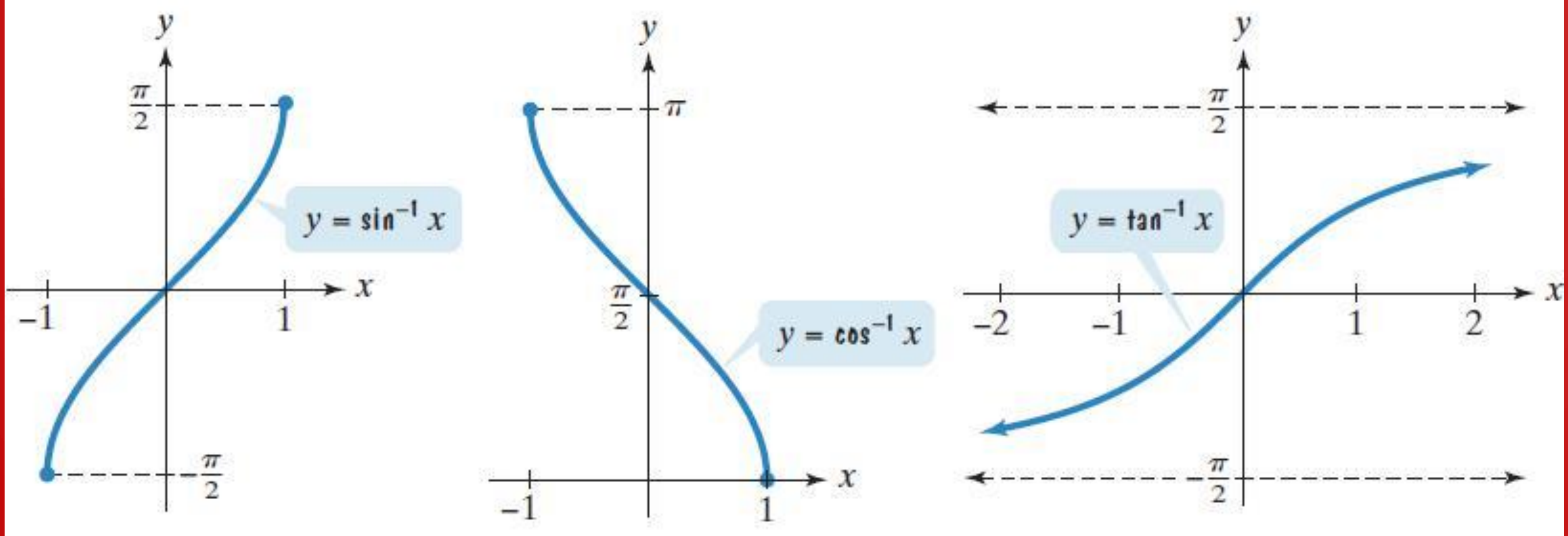
The inverse Tangent function, denoted by $\tan^{-1}$, is the inverse of the restricted Tangent function $y = \tan x, -\frac{\pi}{2} < x < \frac{\pi}{2}$. Thus

$$y = \tan^{-1} x \text{ means } \tan y = x$$

Where $-\frac{\pi}{2} < y < \frac{\pi}{2}$ and $-\infty \leq x \leq \infty$.


y
y = tan x, −π/2 < x < π/2
(π/4, 1)
1
x
−π/2
π/2
(−π/4, −1)
Domain: (−π/2, π/2)
Range: (−∞, ∞)


y
y = tan⁻¹ x
π/2
(−1, −π/4)
x
−1
1
(1, π/4)
−π/2
Domain: (−∞, ∞)
Range: (−π/2, π/2)



Domain: $[-1, 1]$
Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Domain: $[-1, 1]$
Range: $[0, \pi]$

Domain: $(-\infty, \infty)$
Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Inverse Properties

The Sine Function and Its Inverse

$\sin(\sin^{-1} x) = x$ for every x in the interval

$\sin^{-1}(\sin x) = x$ for every x in the interval

The Cosine Function and Its Inverse

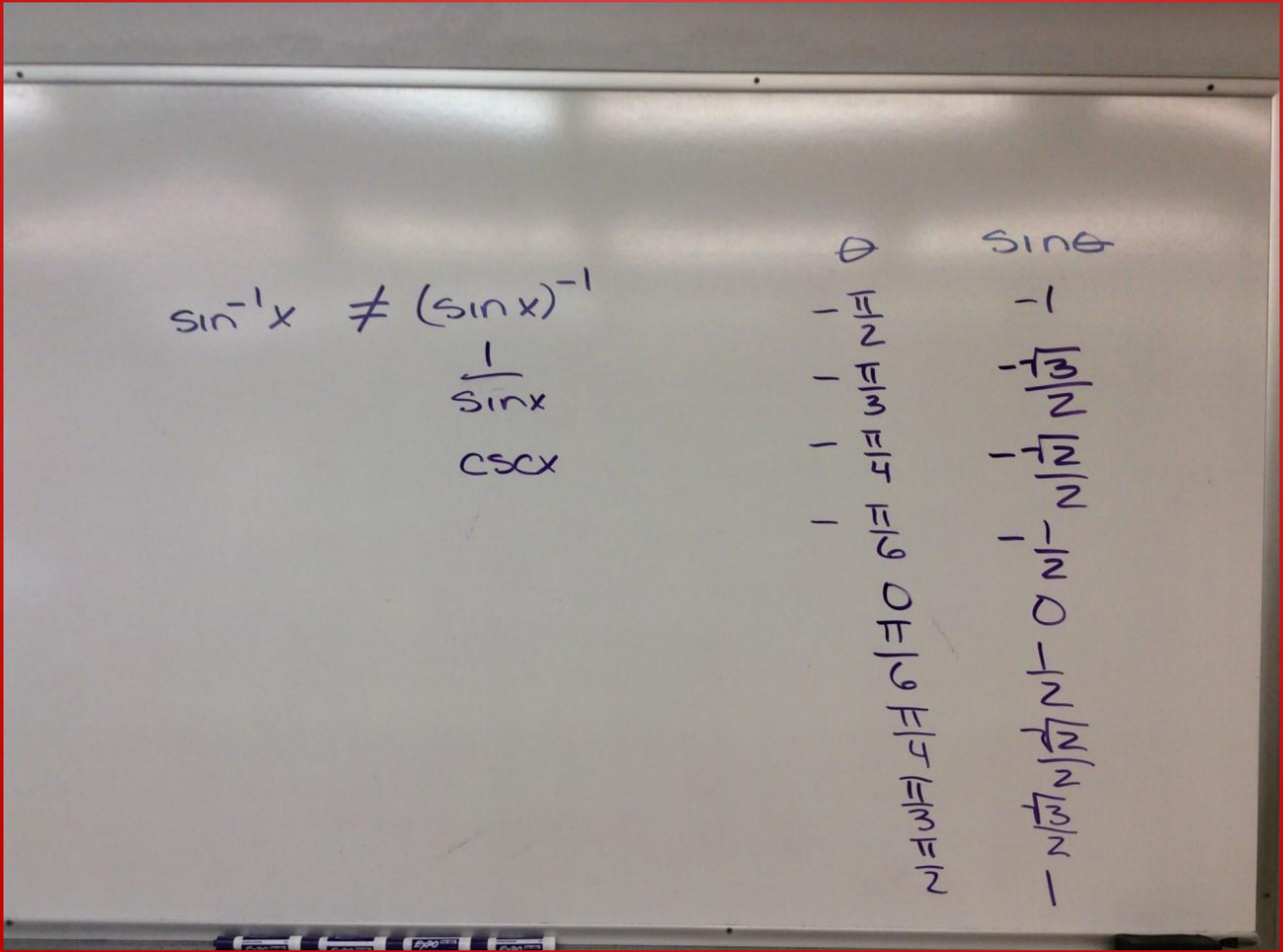
$\cos(\cos^{-1} x) = x$ for every x in the interval

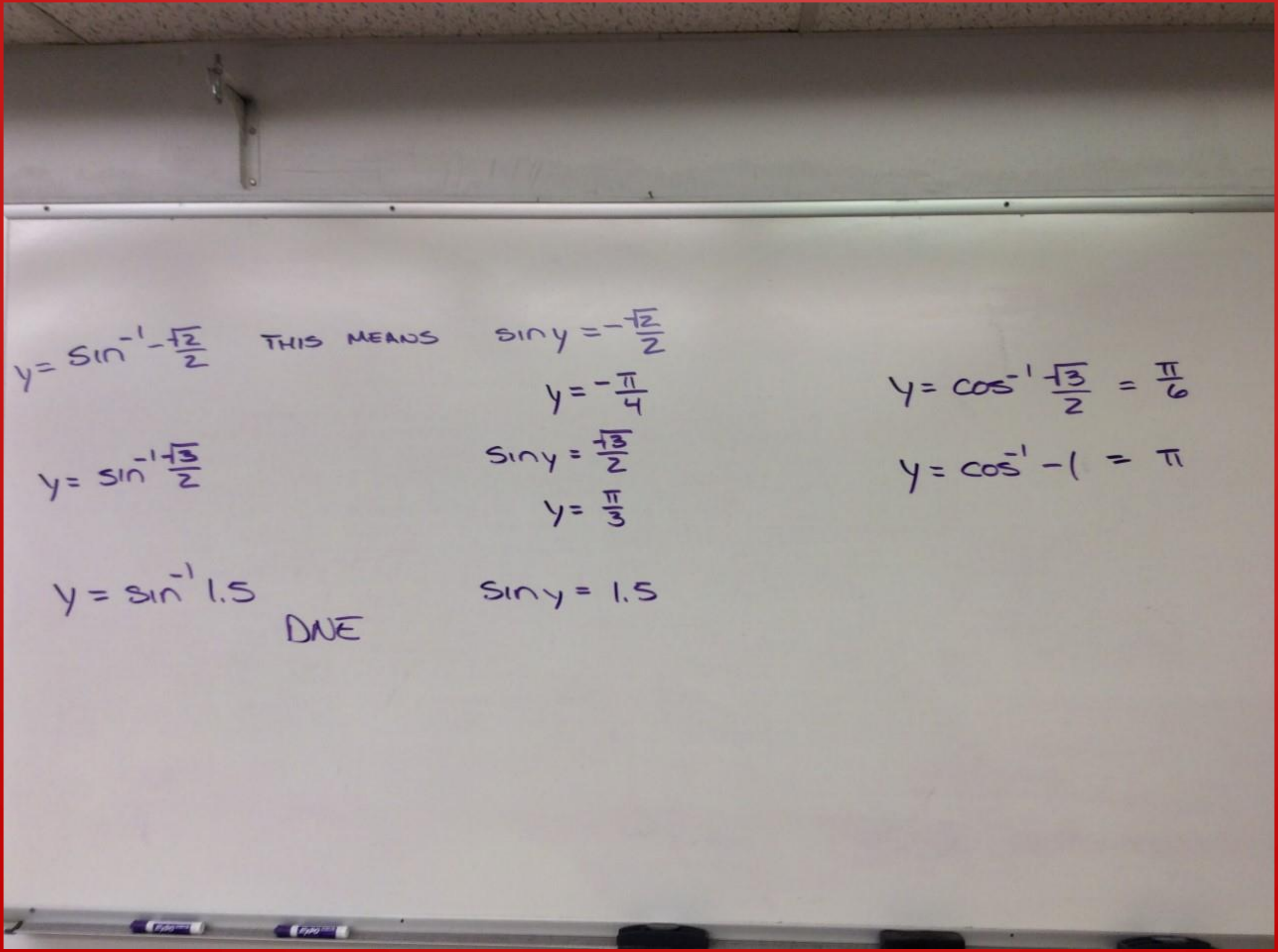
$\cos^{-1}(\cos x) = x$ for every x in the interval

The Tangent Function and Its Inverse

$\tan(\tan^{-1} x) = x$ for every real number x

$\tan^{-1}(\tan x) = x$ for every x in the interval


sin⁻¹x ≠ (sin x)⁻¹
1/sinx
cscx
θ sinθ
-π/2 -1
-π/3 -√3/2
-π/4 -√2/2
-π/6 -1/2
0 0
π/6 1/2
π/4 √2/2
π/3 √3/2
π/2 1


$y = \sin^{-1} -\frac{\sqrt{2}}{2}$
THIS MEANS
$\sin y = -\frac{\sqrt{2}}{2}$
$y = -\frac{\pi}{4}$
$y = \sin^{-1} \frac{\sqrt{3}}{2}$
$\sin y = \frac{\sqrt{3}}{2}$
$y = \frac{\pi}{3}$
$y = \sin^{-1} 1.5$
$\sin y = 1.5$
DNE
$y = \cos^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{6}$
$y = \cos^{-1} -1 = \pi$