BOARD NOTES

21 FEBRUARY 2019

CC TRIGONOMETRY
CHAPTER 2 – GRAPHS OF THE TRIGONOMETRIC FUNCTIONS; INVERSE TRIGONOMETRIC FUNCTIONS

SECTION 2.4 - Applications of Trigonometric Functions

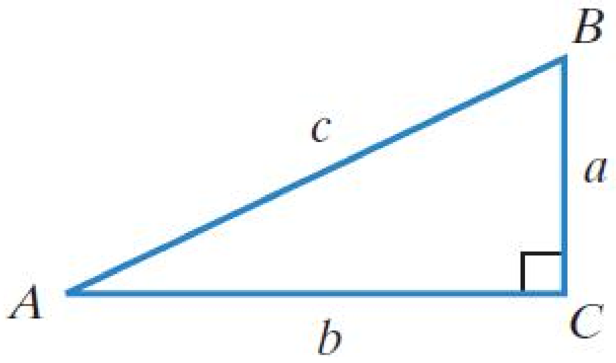
Objectives:

- Solve a right triangle.
- Solve problems involving bearings.
- Model simple harmonic motion.

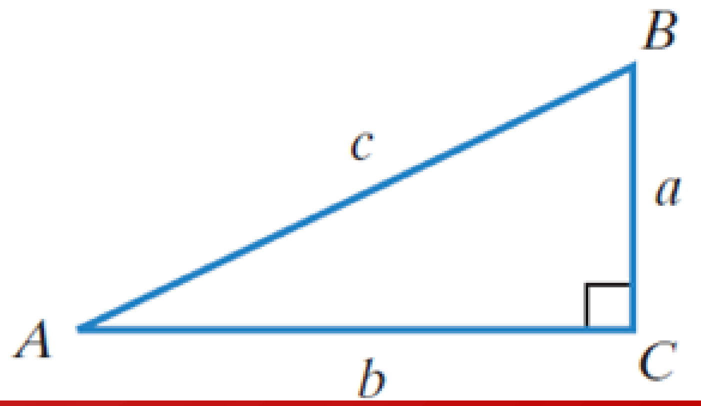
Solving Right Triangles

Solving a right triangle means finding the missing lengths of its sides and the measurements of its angles. We will label right triangles so that side a is opposite angle A, side b is opposite angle B, and side c, the hypotenuse, is opposite right angle C.

When solving a right triangle, we will use the sine, cosine, and tangent functions, rather than their reciprocals.

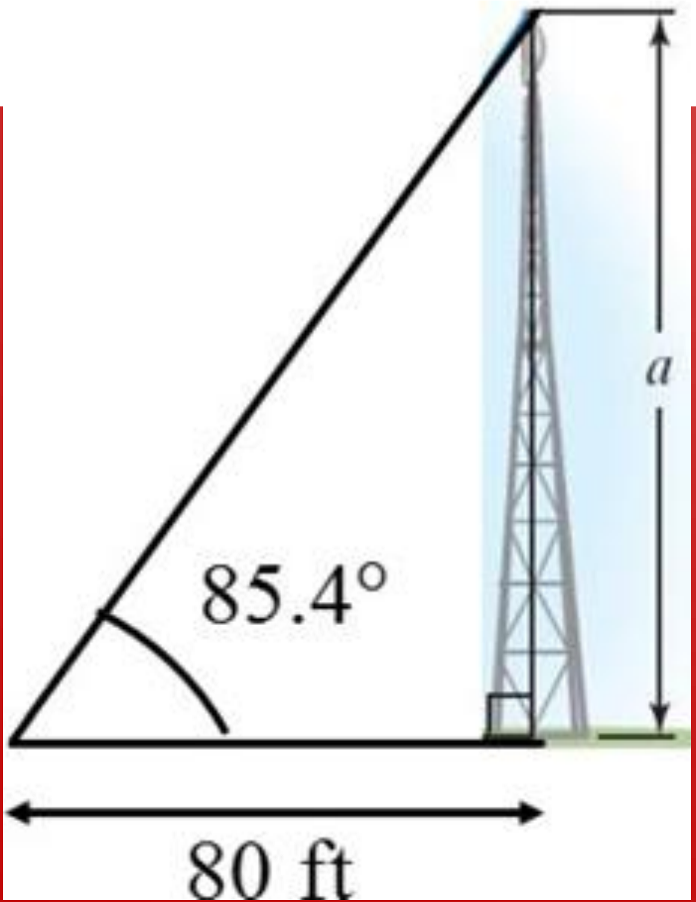


Let A = 62.7° and a = 8.4. Solve the right triangle, rounding lengths to two decimal places. Solution:

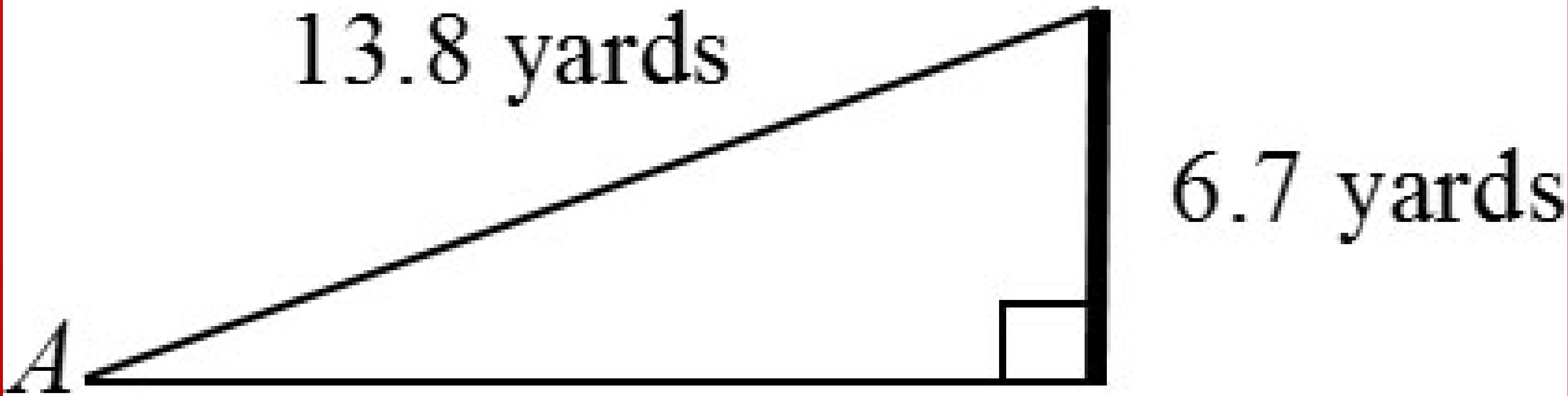


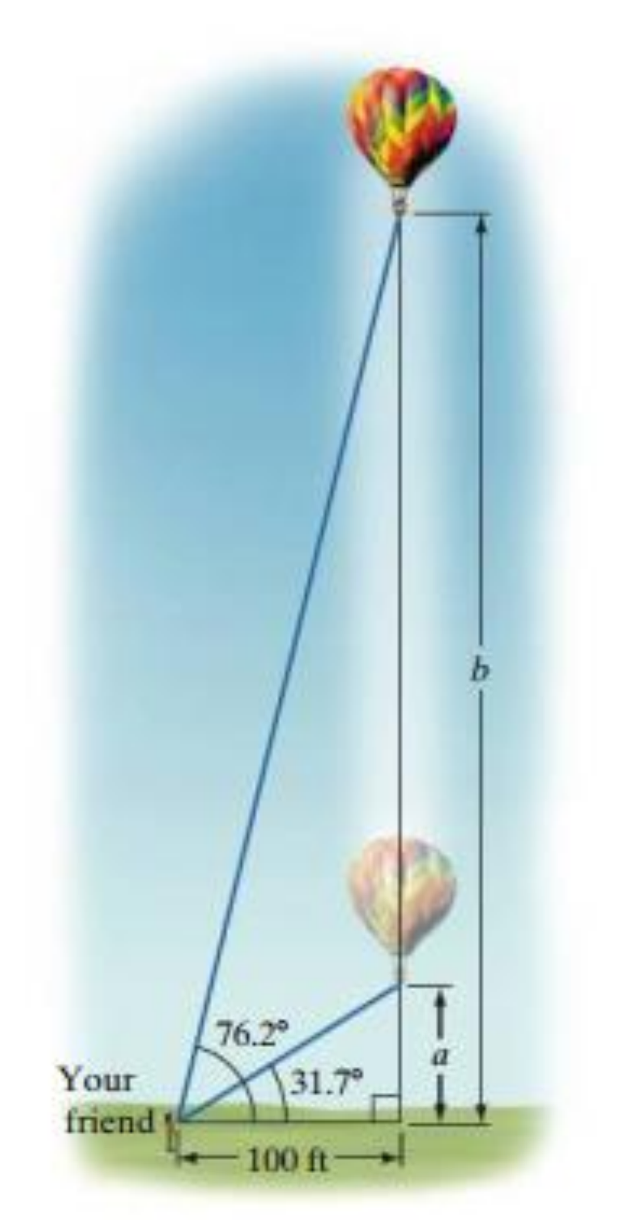
From a point on level ground 80 feet from the base of the Eiffel Tower, the angle of elevation is 85.4°. Approximate the height of the Eiffel Tower to the nearest foot.

Solution:



A guy wire is 13.8 yards long and is attached from the ground to a pole 6.7 yards above the ground. Find the angle, to the nearest tenth of a degree, that the wire makes with the ground.





You are taking your first hot-air balloon ride. Your friend is standing on level ground, 100 feet away from your launch point, making a video of your terrified face. How high did you travel in the air during the minute of ascent between a and b?

Trigonometry and Bearings

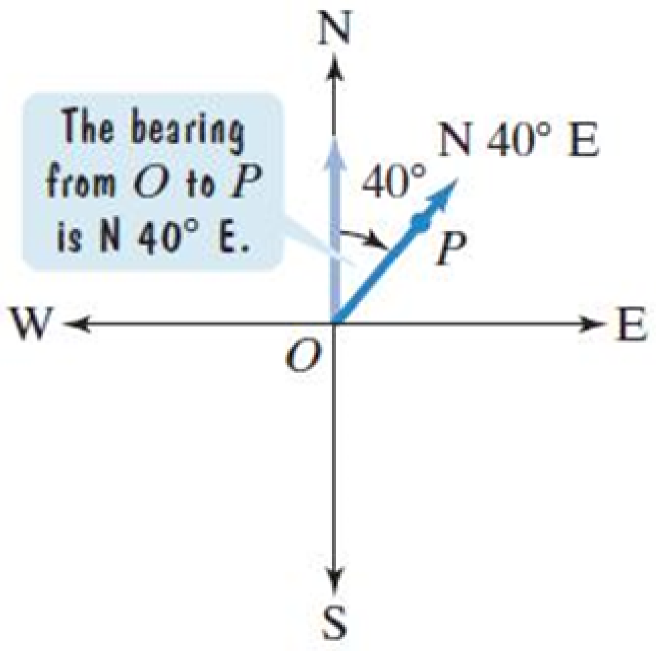
In navigation and surveying problems, the term **bearing** is used to specify the location of one point relative to another. The **bearing** from point *O* to point *P* is the acute angle, measured in degrees, between ray *OP* and a north-south line. The north-south line and the east-west line intersect at right angles. Each bearing has three parts: a letter (N or S), the measure of an acute angle, and a letter (E or W).

Trigonometry and Bearings

If the acute angle is measured from the **north side** of the north-south line, then we write N first.

Second, we write the measure of the acute angle.

If the acute angle is measured on the **east side** of the north-south line, then we write E last.

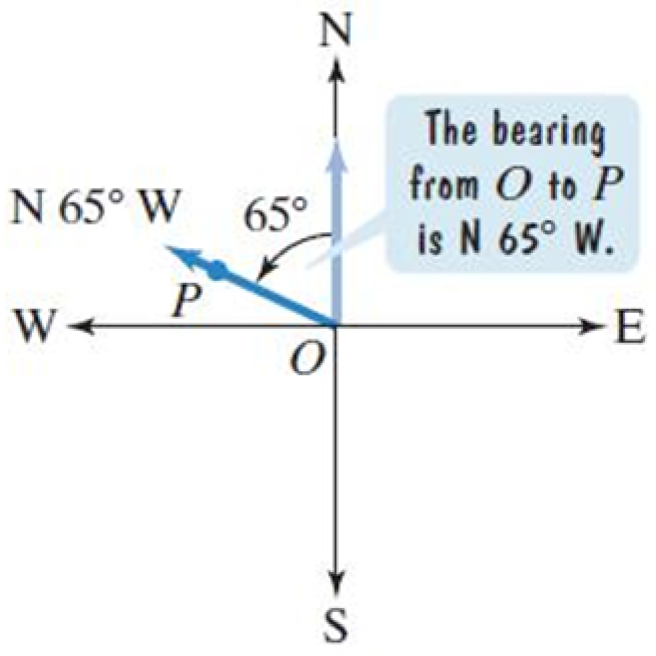


Trigonometry and Bearings (3 of 4)

If the acute angle is measured from the **north side** of the north-south line, then we write N first.

Second, we write the measure of the acute angle.

If the acute angle is measured on the **west side** of the north-south line, then we write W last.

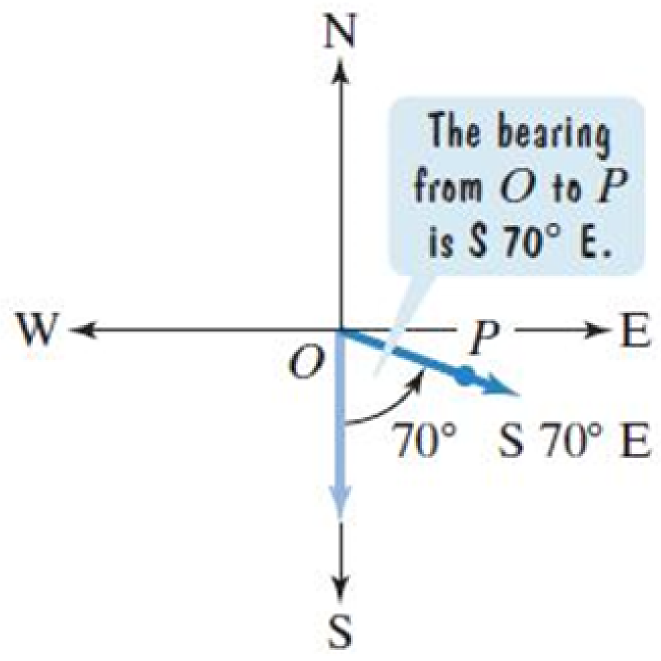


Trigonometry and Bearings (4 of 4)

If the acute angle is measured from the **south side** of the north-south line, then we write S first.

Second, we write the measure of the acute angle.

If the acute angle is measured on the **east side** of the north-south line, then we write E last.



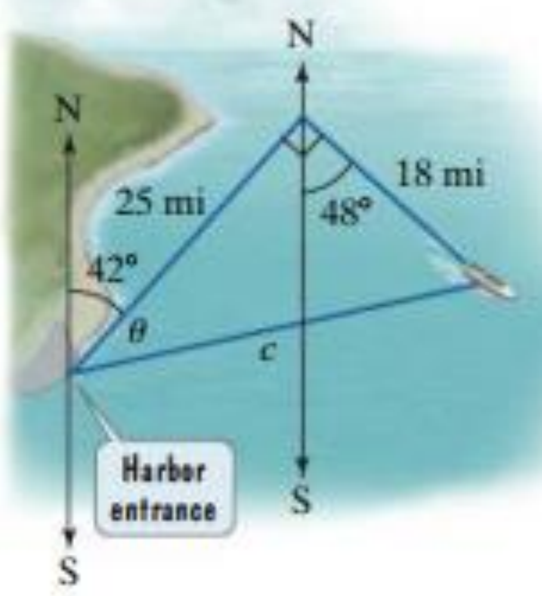


FIGURE 2.49 Finding a boat's bearing from the harbor entrance

A boat leaves the entrance to a harbor and travels 25 miles on a bearing of N 42° E. **Figure 2.49** shows that the captain then turns the boat 90° clockwise and travels 18 miles on a bearing of S 48° E. At that time:

- **a.** How far is the boat, to the nearest tenth of a mile, from the harbor entrance?
- **b.** What is the bearing, to the nearest tenth of a degree, of the boat from the harbor entrance?

Simple Harmonic Motion

An object that moves on a coordinate axis is in simple harmonic motion if its distance from the origin, d, at time t is given by either

$$d = a\cos\omega t \quad \text{or} \quad d = a\sin\omega t.$$

The motion has amplitude $|a|$, maximum displacement of the object from its rest position. The period of the motion is, $\frac{2\pi}{\omega}$ where $\omega > 0$.

The period gives the time it takes for the motion to go through one complete cycle.

In describing simple harmonic motion, the equation with the cosine function, $d = a\cos\omega t$, is used if the object if the greatest distance from rest position, the origion, at $t = 0$. By contrast, the equation with the sine function, $d = a\sin\omega t$, is used if the object is at its rest position, the origion, at $t = 0$.

A ball on a spring is pulled 6 inches below its rest position and then released. The period for the motion is 4 seconds. Write the equation for the ball's simple harmonic motion.

Frequency of an Object in Simple Harmonic Motion

An object in simple harmonic motion given by

$$d = a\cos\omega t \text{ or } d = \sin\omega t$$

has **frequency** f given by

$$f = \frac{\omega}{2\pi}, \omega > 0.$$

Equivalently,

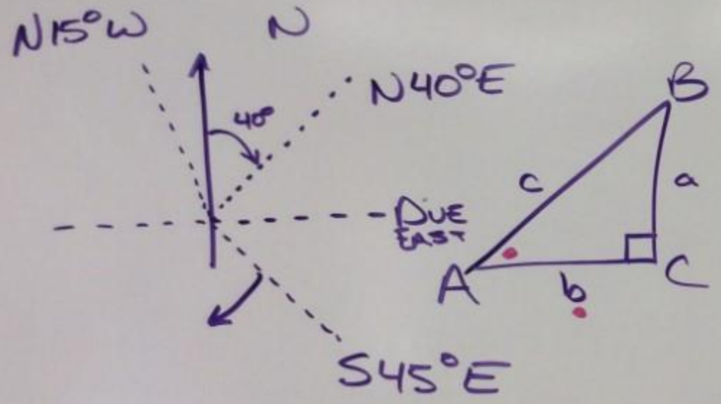
$$f = \frac{1}{\text{period}}.$$

An object moves in simple harmonic motion described by $d = 12\cos\frac{\pi}{4}t$, where t is measured in seconds and d in centimeters.

a) Find the maximum displacement.

b) Find the frequency.

c) Find the time required for one cycle.



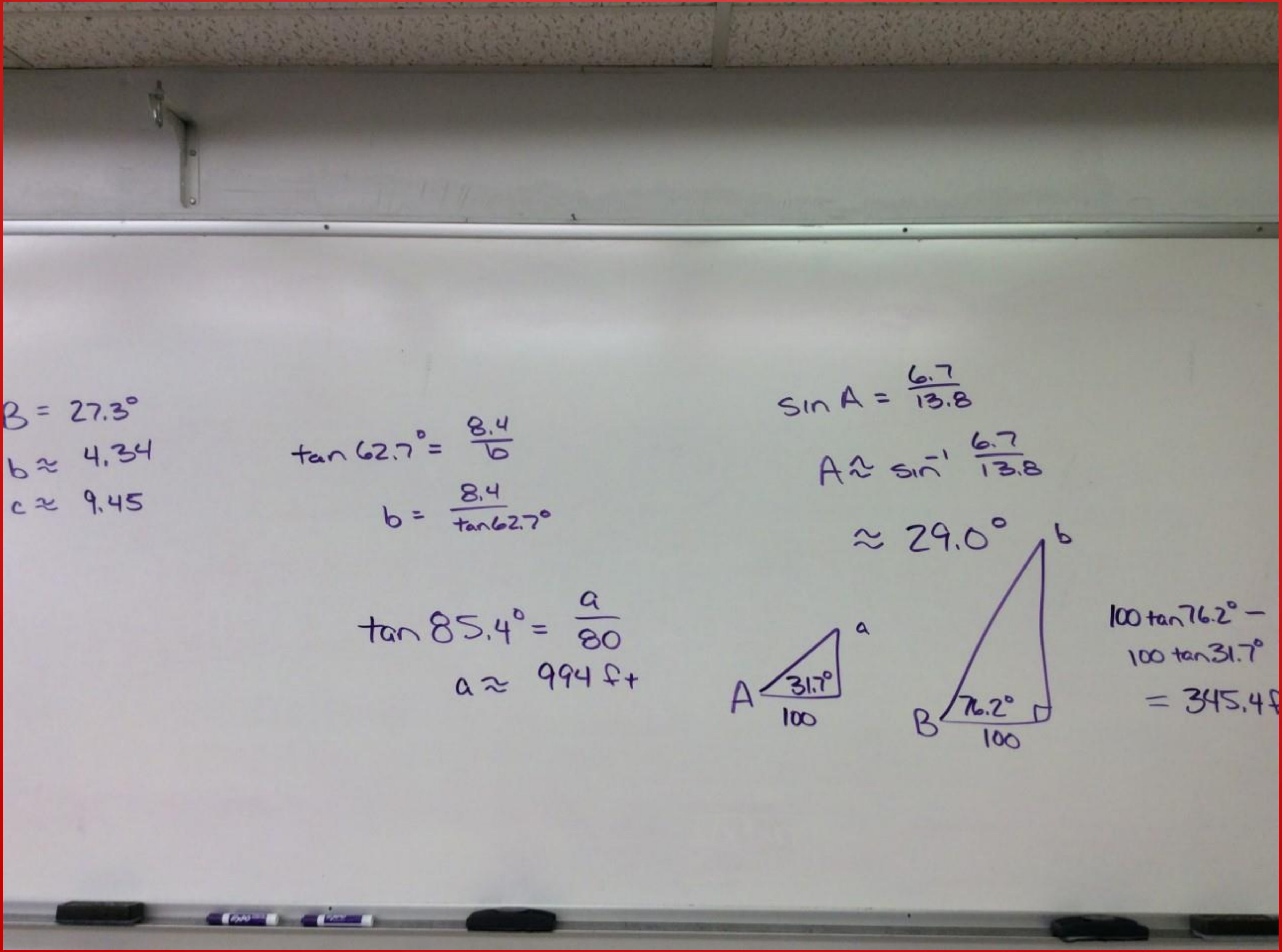
$A = 34.5°$ $\quad B = 90° - 34.5° = 55.5$

$b = 10.5$

$\tan 34.5° = \frac{a}{10.5}$

$a = 10.5 \tan 34.5° \approx 7.22$

$\cos 34.5° = \frac{10.5}{c}$ $\quad c = \frac{10.5}{\cos 34.5°} \approx 12.74$


B = 27.3°
b ≈ 4.34
c ≈ 9.45
tan 62.7° = 8.4/b
b = 8.4/tan 62.7°
tan 85.4° = a/80
a ≈ 994 ft
sin A = 6.7/13.8
A ≈ sin⁻¹ 6.7/13.8
≈ 29.0°
A
31.7°
100
a
B
76.2°
100
b
100 tan 76.2° −
100 tan 31.7°
= 345.4 f