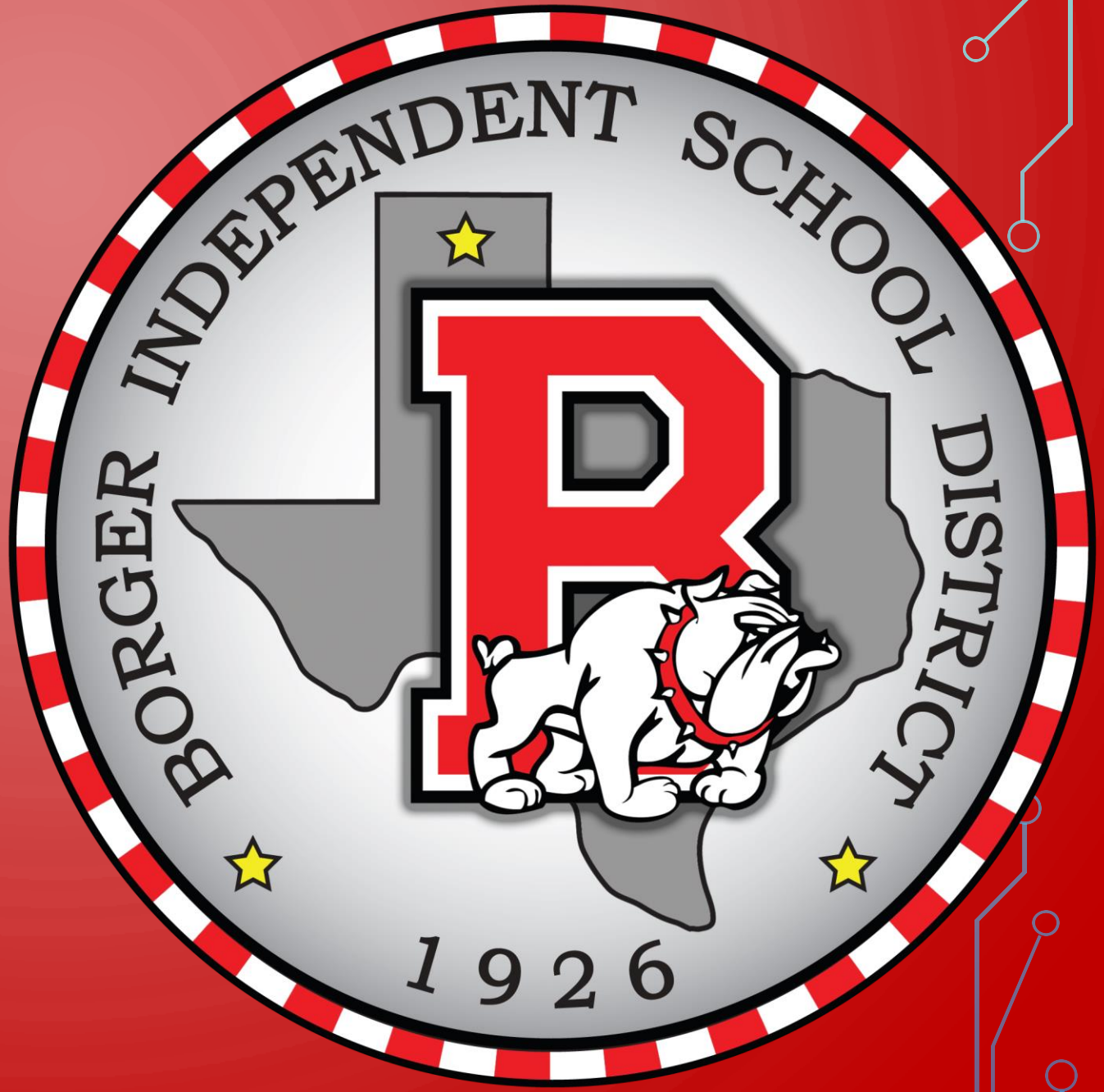
BOARD NOTES

28 FEBRUARY 2019



CC TRIGONOMETRY
CHAPTER 3 – TRIGONOMETRIC IDENTITIES AND EQUATIONS

SECTION 3.1 - Verifying Trigonometric Identities

Objectives:

- Use the fundamental trigonometric identities to verify identities.

The Fundamental Identities

- Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \quad \cos x = \frac{1}{\sec x} \quad \tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x} \quad \sec x = \frac{1}{\cos x} \quad \cot x = \frac{1}{\tan x}$$

- Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

- Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1 \quad 1 + \tan^2 x = \sec^2 x \quad 1 + \cot^2 x = \csc^2 x$$

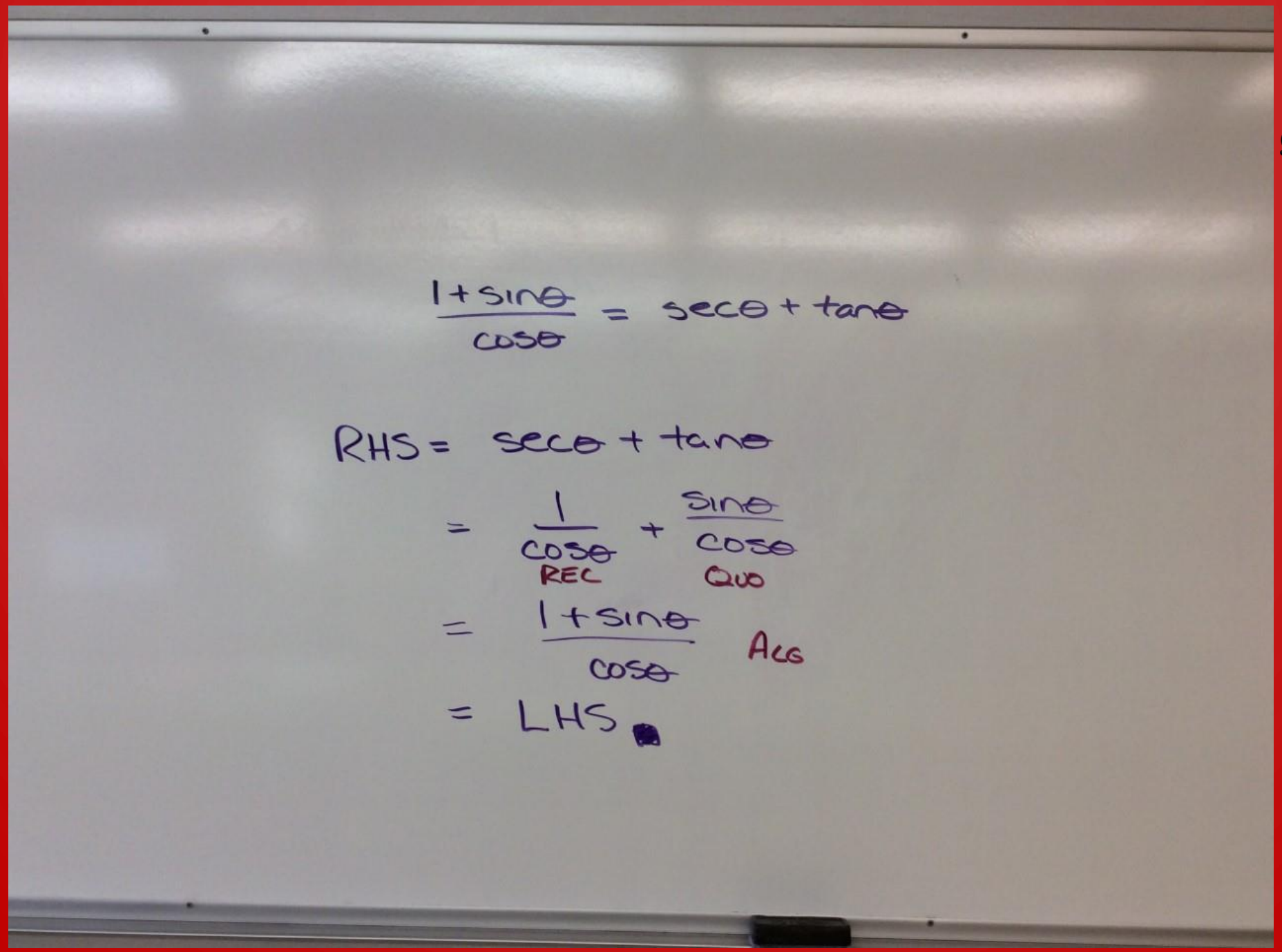
- Even-Odd Identities

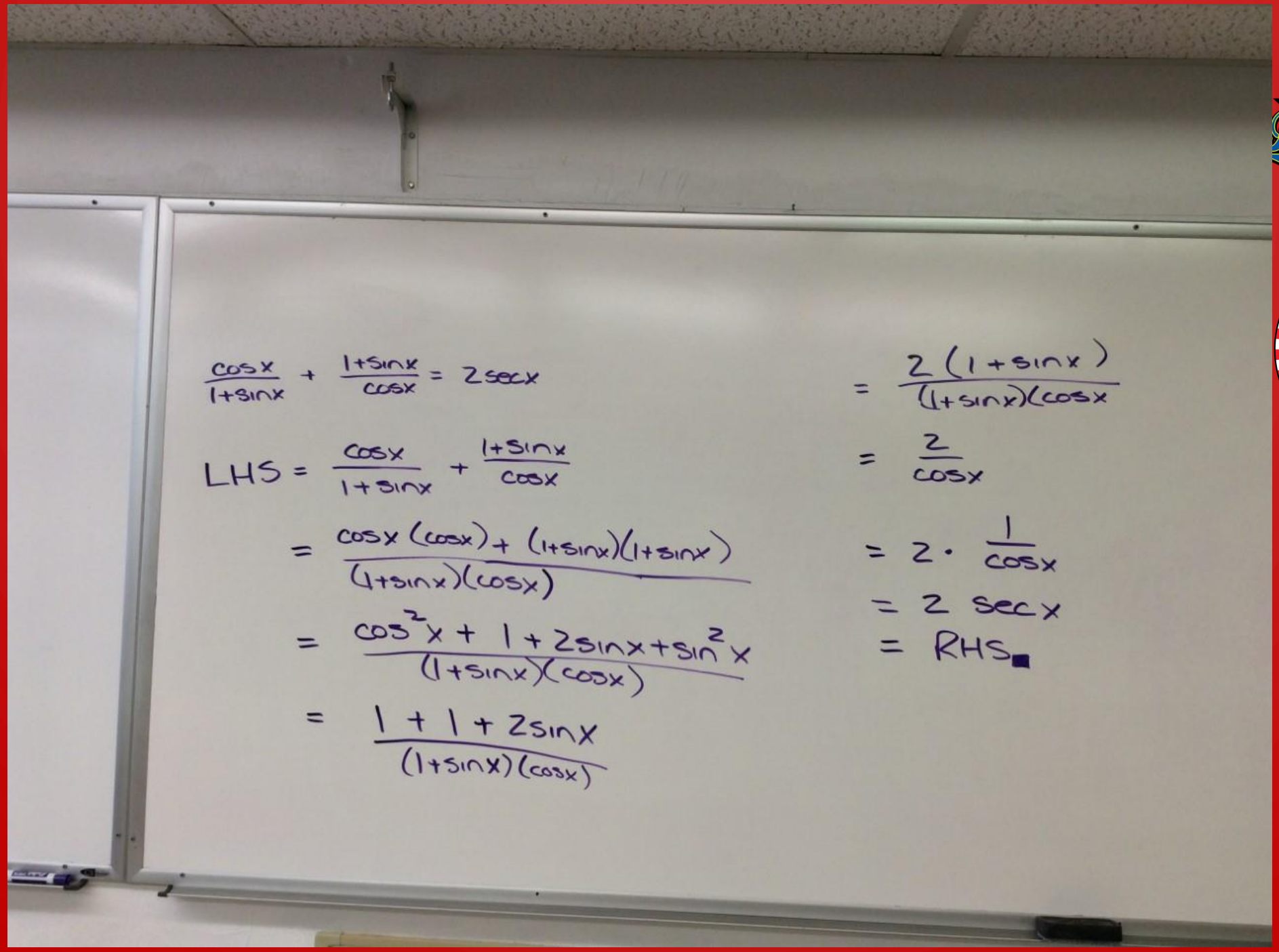
$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x \quad \tan(-x) = -\tan x$$

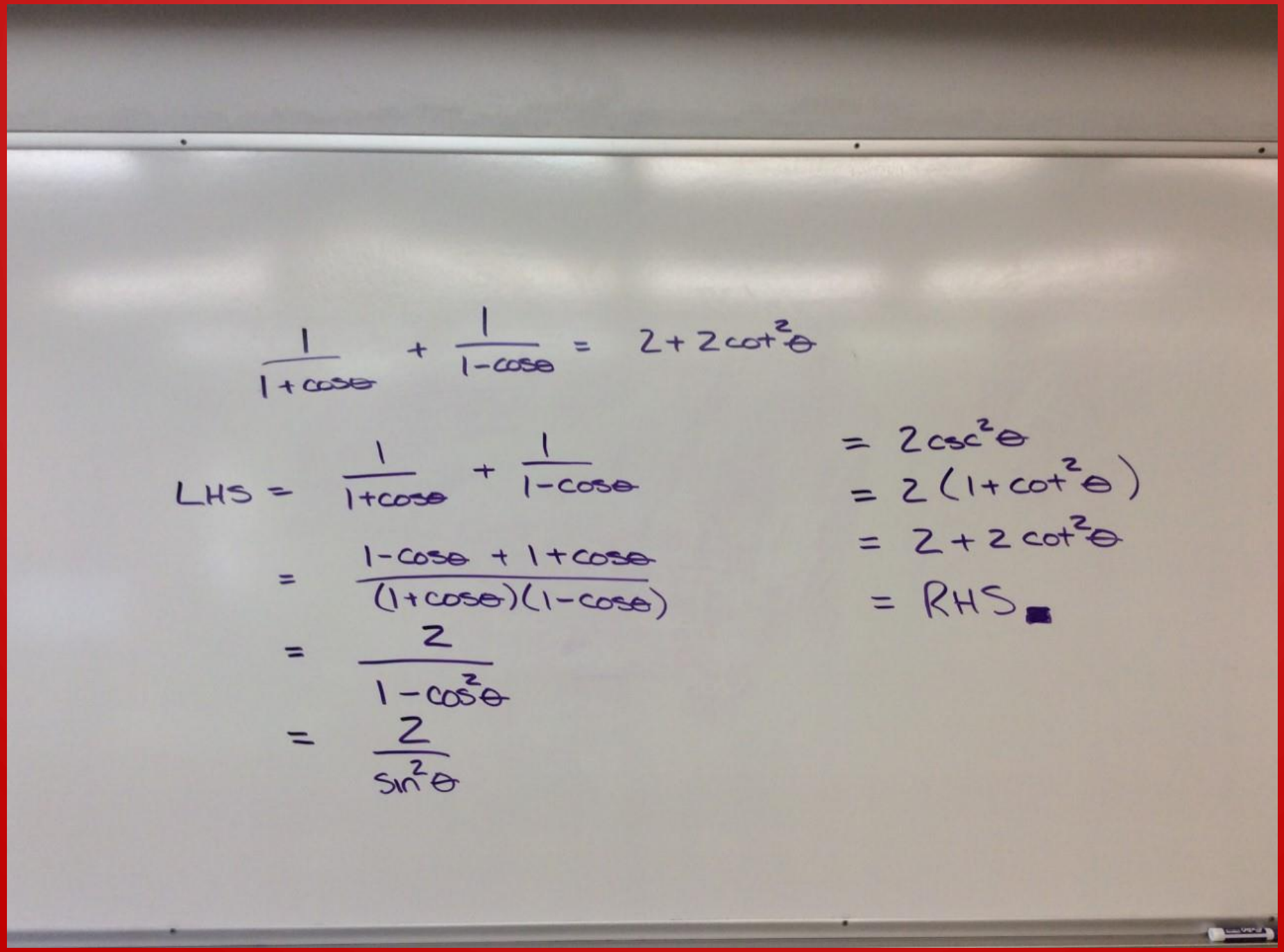
$$\csc(-x) = -\csc x \quad \sec(-x) = \sec x \quad \cot(-x) = -\cot x$$

Using Fundamental Identities to Verify Other Identities

To **verify an identity**, we show that one side of the identity can be simplified so that it is identical to the other side. Each side of the equation is manipulated independently of the other side of the equation. Start with the side containing the more complicated expression. If you substitute one or more of the fundamental identities on the more complicated side, you will often be able to rewrite it in a form identical to that of the other side.


$\frac{1+\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$
RHS = $\sec\theta + \tan\theta$
$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$
REC
QUO
$= \frac{1+\sin\theta}{\cos\theta}$
ALG
= LHS


$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 2\sec x$
LHS $= \frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x}$
$= \frac{\cos x(\cos x) + (1+\sin x)(1+\sin x)}{(1+\sin x)(\cos x)}$
$= \frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{(1+\sin x)(\cos x)}$
$= \frac{1 + 1 + 2\sin x}{(1+\sin x)(\cos x)}$
$= \frac{2(1+\sin x)}{(1+\sin x)(\cos x)}$
$= \frac{2}{\cos x}$
$= 2 \cdot \frac{1}{\cos x}$
$= 2\sec x$
$=$ RHS ■


$\frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta} = 2+2\cot^2\theta$
LHS $= \frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta}$
$= \frac{1-\cos\theta+1+\cos\theta}{(1+\cos\theta)(1-\cos\theta)}$
$= \frac{2}{1-\cos^2\theta}$
$= \frac{2}{\sin^2\theta}$
$= 2\csc^2\theta$
$= 2(1+\cot^2\theta)$
$= 2+2\cot^2\theta$
$=$ RHS ■