BOARD NOTES

28 FEBRUARY 2019

The Fundamental Identities

- Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \quad \cos x = \frac{1}{\sec x} \quad \tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x} \quad \sec x = \frac{1}{\cos x} \quad \cot x = \frac{1}{\tan x}$$

- Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

- Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1 \quad 1 + \tan^2 x = \sec^2 x \quad 1 + \cot^2 x = \csc^2 x$$

- Even-Odd Identities

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x \quad \tan(-x) = -\tan x$$

$$\csc(-x) = -\csc x \quad \sec(-x) = \sec x \quad \cot(-x) = -\cot x$$

Using Fundamental Identities to Verify Other Identities

To **verify an identity**, we show that one side of the identity can be simplified so that it is identical to the other side. Each side of the equation is manipulated independently of the other side of the equation. Start with the side containing the more complicated expression. If you substitute one or more of the fundamental identities on the more complicated side, you will often be able to rewrite it in a form identical to that of the other side.

CC TRIGONOMETRY
CHAPTER 3 – TRIGONOMETRIC IDENTITIES AND EQUATIONS

SECTION 3.2 - Sum and Difference Formulas

Objectives:

- Use the formula for the cosine of the difference of two angles.
- Use sum and difference formulas for cosines and sines.
- Use sum and difference formulas for tangents.

Sum and Difference Formulas for Cosines and Sines

1. $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
2. $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
3. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
4. $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

Sum and Difference Formulas for Tangets

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\cos^4 t - \sin^4 t = 1 - 2\sin^2 t$$

$$\begin{aligned}
LHS &= (\cos^2 t - \sin^2 t)(\cos^2 t + \sin^2 t) \\
&= (\cos^2 t - \sin^2 t)(1) \\
&= \cos^2 t - \sin^2 t \\
&= (1 - \sin^2 t) - \sin^2 t \\
&= 1 - 2\sin^2 t \\
&= RHS \blacksquare
\end{aligned}$$

$$\cos 15^\circ = \cos(60^\circ - 45^\circ)$$
$$= \cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ$$
$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{2} + \sqrt{6}}{4}$$

$$\cos 80^\circ \cos 20^\circ + \sin 80^\circ \sin 20^\circ = \cos 60^\circ = \frac{1}{2}$$

$$\frac{\cos(\alpha - \beta)}{\cos\alpha \cos\beta} = 1 + \tan\alpha \tan\beta$$