BOARD NOTES

20 MARCH 2019

The Fundamental Identities

- Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \quad \cos x = \frac{1}{\sec x} \quad \tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x} \quad \sec x = \frac{1}{\cos x} \quad \cot x = \frac{1}{\tan x}$$

- Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

- Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1 \quad 1 + \tan^2 x = \sec^2 x \quad 1 + \cot^2 x = \csc^2 x$$

- Even-Odd Identities

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x \quad \tan(-x) = -\tan x$$

$$\csc(-x) = -\csc x \quad \sec(-x) = \sec x \quad \cot(-x) = -\cot x$$

Using Fundamental Identities to Verify Other Identities

To **verify an identity**, we show that one side of the identity can be simplified so that it is identical to the other side. Each side of the equation is manipulated independently of the other side of the equation. Start with the side containing the more complicated expression. If you substitute one or more of the fundamental identities on the more complicated side, you will often be able to rewrite it in a form identical to that of the other side.

Sum and Difference Formulas for Cosines and Sines

1. $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$
2. $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$
3. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$
4. $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

Sum and Difference Formulas for Tangets

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

CC TRIGONOMETRY
CHAPTER 3 – TRIGONOMETRIC IDENTITIES AND EQUATIONS

SECTION 3.3 - Double-angle, Half-angle, and Power Reducing Formulas

Objectives:

- Use the double-angle formulas.
- Use the power-reducing formulas.
- Use the half-angle formulas.

Double Angle Formulas

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Power-Reducing Formulas

$$\sin^2\theta = \frac{1-2\cos\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

Half-Angle Formulas

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

The ± symbol in each formula does not mean that there are two possible values for each function. Instead, the ± indicates that you must determine the sign of the trigonometric function, + or −, based on the quadrant in which the half-angle

$$\sin^4 x = (\cos^2 x)^2$$

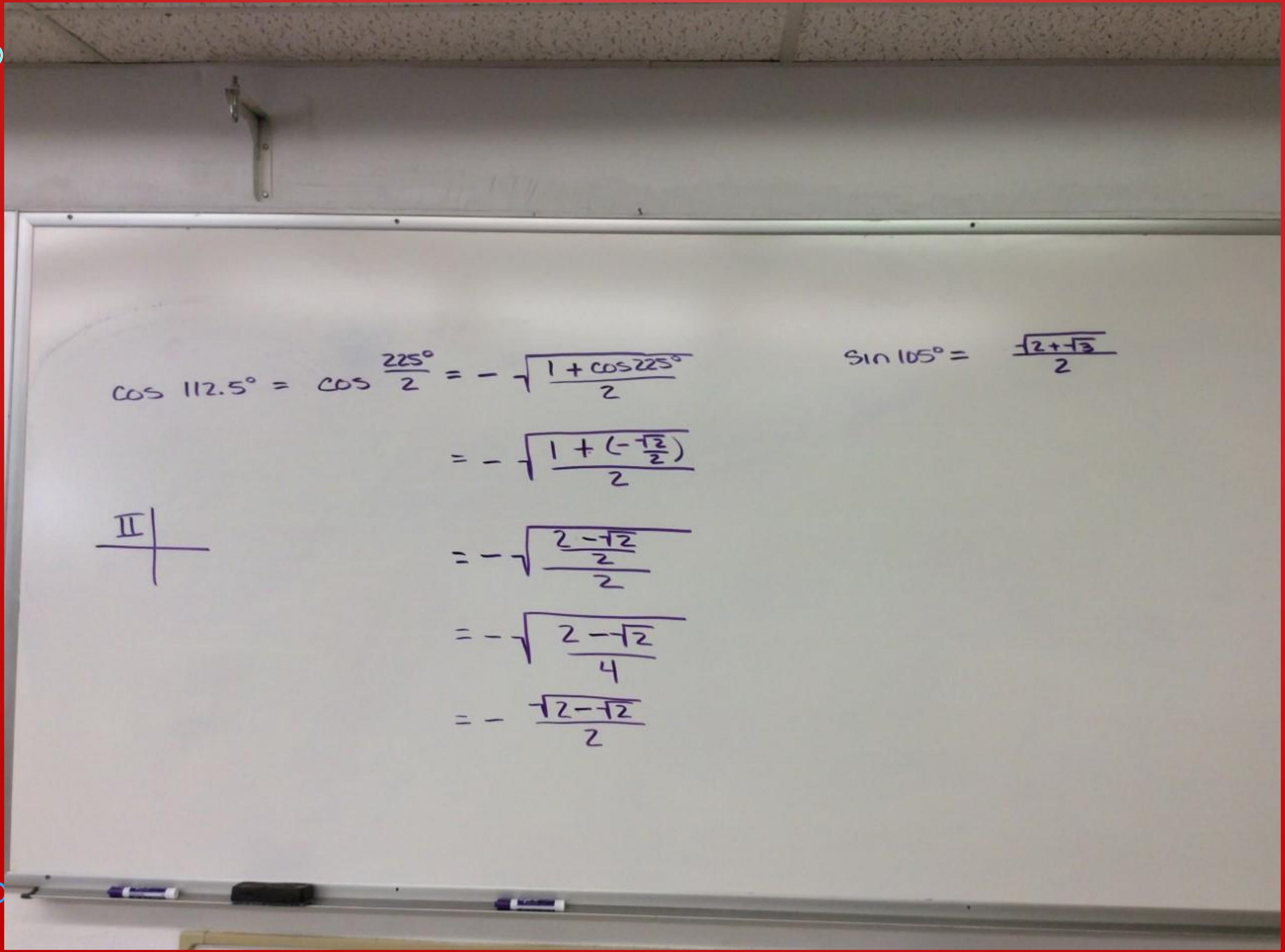
$$= \left(\frac{1-\cos 2x}{2}\right)^2$$

$$= \frac{1 - 2\cos 2x + \cos^2 2x}{4}$$

$$= \frac{1}{4} - \frac{1}{2}\cos 2x + \frac{1}{4}\left(\frac{1+\cos 2(2x)}{2}\right)$$

$$= \frac{1}{4} \quad \frac{1}{2}\cos 2x + \frac{1}{8} + \frac{1}{8}\cos 4x$$

$$= \frac{3}{8} - \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$$


$\cos 112.5^\circ = \cos \frac{225^\circ}{2} = -\sqrt{\frac{1+\cos 225^\circ}{2}}$
$= -\sqrt{\frac{1+\left(-\frac{\sqrt{2}}{2}\right)}{2}}$
$= -\sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}}$
$= -\sqrt{\frac{2-\sqrt{2}}{4}}$
$= -\frac{\sqrt{2-\sqrt{2}}}{2}$
II
$\sin 105^\circ = \frac{\sqrt{2+\sqrt{3}}}{2}$