BOARD NOTES

25 MARCH 2019

The Fundamental Identities

- Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \quad \cos x = \frac{1}{\sec x} \quad \tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x} \quad \sec x = \frac{1}{\cos x} \quad \cot x = \frac{1}{\tan x}$$

- Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

- Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1 \quad 1 + \tan^2 x = \sec^2 x \quad 1 + \cot^2 x = \csc^2 x$$

- Even-Odd Identities

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x \quad \tan(-x) = -\tan x$$

$$\csc(-x) = -\csc x \quad \sec(-x) = \sec x \quad \cot(-x) = -\cot x$$

Using Fundamental Identities to Verify Other Identities

To **verify an identity**, we show that one side of the identity can be simplified so that it is identical to the other side. Each side of the equation is manipulated independently of the other side of the equation. Start with the side containing the more complicated expression. If you substitute one or more of the fundamental identities on the more complicated side, you will often be able to rewrite it in a form identical to that of the other side.

Sum and Difference Formulas for Cosines and Sines

1. $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$
2. $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$
3. $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$
4. $\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$

Sum and Difference Formulas for Tangets

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

Double Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Power-Reducing Formulas

$$\sin^2 \theta = \frac{1-2\cos\theta}{2}$$

$$\cos^2 \theta = \frac{1+\cos 2\theta}{2}$$

$$\tan^2 \theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

Half-Angle Formulas

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

The ± symbol in each formula does not mean that there are two possible values for each function. Instead, the ± indicates that you must determine the sign of the trigonometric function, + or −, based on the quadrant in which the half-angle

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CHAPTER 3 – TRIGONOMETRIC IDENTITIES AND EQUATIONS

SECTION 3.4 - Product-to-Sum and Sum-to-Product Formulas

Objectives:

- Use the product-to-sum formulas.
- Use the sum-to-product formulas.

The Product-to-Sum Formulas

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$$

The Sum-to-Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

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SECTION 3.5 -

Trigonometric Equations

Objectives:

- Find all solutions of a trigonometric equation.
- Solve equations with multiple angles.
- Solve trigonometric equations quadratic in form.
- Use factoring to separate different functions in trigonometric equations.
- Use identities to solve trigonometric equations.
- Use a calculator to solve trigonometric equations.

Trigonometric Equations and Their Solutions

A **trigonometric equation** is an equation that contains a trigonometric expression with a variable, such as sin x.

The values that satisfy such an equation are its **solutions**. (There are trigonometric equations that have no solution.)

When an equation includes multiple angles, the period of the function plays an important role in ensuring that we do not leave out any solutions.

$$\sin\frac{x}{2} = \frac{\sqrt{3}}{2} \qquad 0 \le x < 2\pi$$

Let $u = \frac{x}{2}$

$$\sin u = \frac{\sqrt{3}}{2}$$

$$u = \frac{\pi}{3} + 2k\pi$$

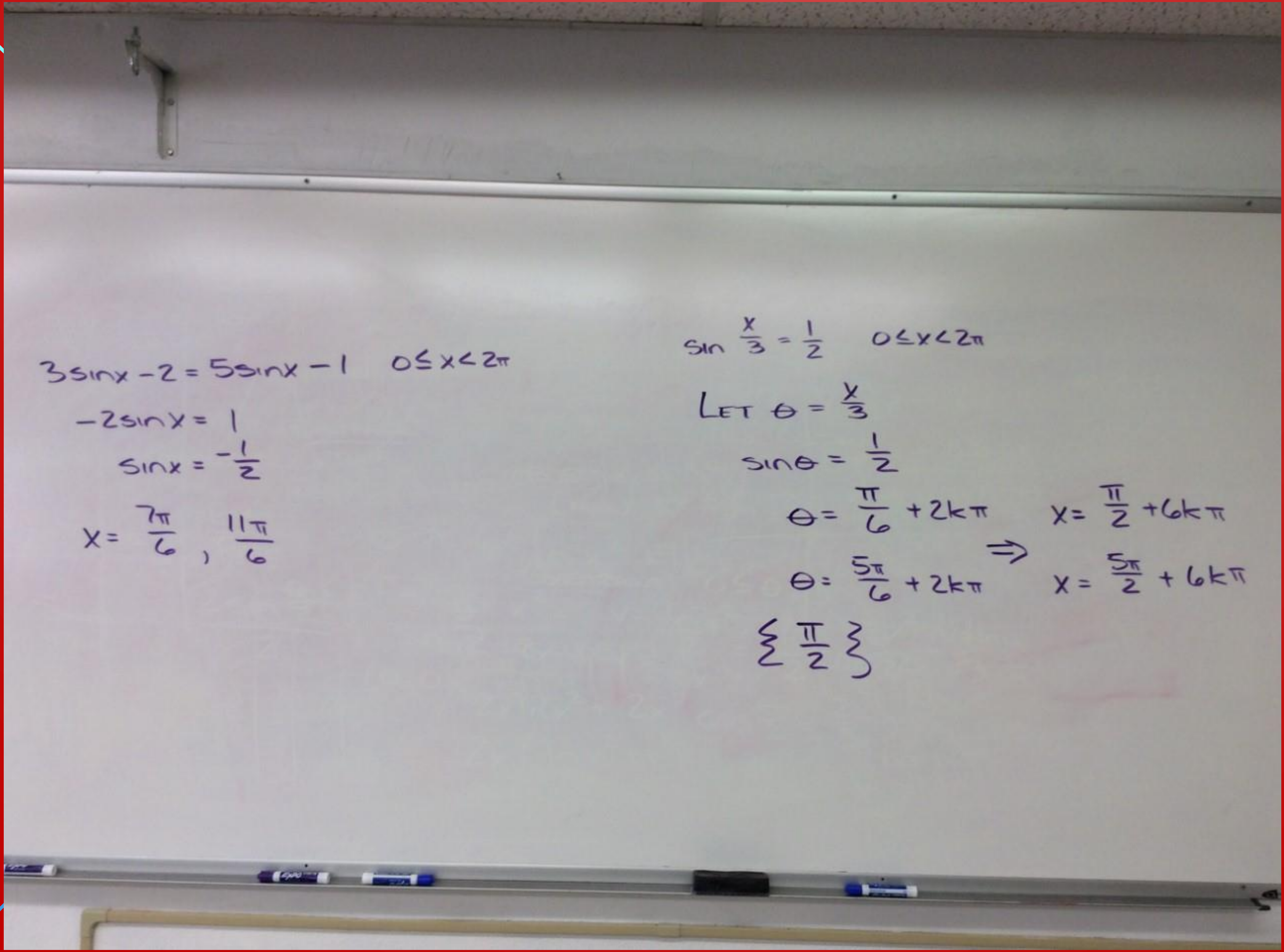
or

$$u = \frac{2\pi}{3} + 2k\pi$$

$$\Rightarrow \quad \frac{x}{2} = \frac{\pi}{3} + 2k\pi \Rightarrow x = \frac{2\pi}{3} + 4k\pi$$

$$x = \frac{4\pi}{3} + 4k\pi$$

$$\left\{ \frac{2\pi}{3}, \frac{4\pi}{3} \right\}$$


$3\sin x - 2 = 5\sin x - 1 \quad 0 \le x < 2\pi$
$-2\sin x = 1$
$\sin x = -\frac{1}{2}$
$x = \frac{7\pi}{6}, \frac{11\pi}{6}$
$\sin\frac{x}{3} = \frac{1}{2} \quad 0 \le x < 2\pi$
Let $\theta = \frac{x}{3}$
$\sin\theta = \frac{1}{2}$
$\theta = \frac{\pi}{6} + 2k\pi$
$\theta = \frac{5\pi}{6} + 2k\pi$
$\Rightarrow$
$x = \frac{\pi}{2} + 6k\pi$
$x = \frac{5\pi}{2} + 6k\pi$
$\{\frac{\pi}{2}\}$